A recently made analogy between the well-studied problems in gambling and typical thermodynamic systems involving feedback allows for deeper insights into the functioning of information-based molecular machines. We consider that the information used by a well informed gambler (or stock-trader) is analogous to the information processing performed by a feedback mechanism - a la Maxwell’s Daemon. Both systems use an information source in order to maximise some quantity, wealth in the case of the gambler; work extraction in the case of the Daemon and his real-world counterparts. The current research hopes to strengthen this analogy and use it to study the role of information processing in physics and nature.

The Szilard engine is one of the first examples of an ‘information engine’, and has been used to study the effects of Maxwell’s Daemon. The engine works in the following cycle:

1. A particle moves freely in a box at equilibrium with a heat bath
2. The box is partitioned, separating it into two volumes. The side that the particle is now located on is denoted $X \in \{L, R\}$.
3. The particle position is measured and the result is recorded in $Y \in \{L, R\}$.
4. The divider is moved quasistatically until the volumes on the left and right are $V_f^L$ and $V_f^R$.
5. The divider is removed from the box and the particle equilibrates, returning to 1.

From ideal gas dynamics, we can show that the work extracted by the operation of a cycle is:

$$W(X | Y) = k_B T \ln \frac{V_f(X | Y)}{V_0(X)} = k_B T \ln \frac{V_f(X | Y)}{P(X)}$$

where $k_B$ is the Boltzmann constant. The optimal $V_f$ is then found by:

$$V_f^*(X | Y) = \text{argmax}(W(X | Y)) = P(X | Y)$$

and after $n$ cycles, the maximum amount of work extracted will be:

$$\text{max}(W_n) = nk_B T \left( \ln \frac{P(X | Y)}{P(X)} \right) = nk_B T I(X; Y)$$

The work extracted after $n$ cycles is directly proportional to the mutual information between the measurement and the actual state of the system.

Let $p(X = x)$ be the probability that horse $x$ wins a given race between $m$ horses, where $X \in \{1, \ldots, m\}$. Let $b(x) \geq 0$ be the fraction of wealth a gambler invests in horse $x$, where $\sum b(x) = 1$. If horse $x$ wins, the gambler receives $o(x)$ times his investment and all other bets are lost. After each race, the gambler’s wealth is multiplied by $b(X) + o(X) = S(X)$. After $n$ races, his wealth is

$$S_n = 2^{nW(X)}$$

where $W$ is the ‘exponential doubling rate’, given by

$$W(X) = \langle \ln S(X) \rangle = \sum_x p(x) \ln b(x) + o(x)$$

Assuming ‘fair’ odds (that is, $\frac{1}{p(x)} = (x | y)$), the maximum wealth growth rate is 0 and the gambler can at best break even. The optimal bet is then proportional gambling (also known as Kelly Gambling) such that $b(x | y) = p(x | y)$.

Let $Y$ be a second variable which has a joint distribution $p(X = x, Y = y)$ and let $b(x | y) \geq 0$, with $\sum b(x | y) = 1$ be a conditional betting strategy depending on the value of the additional variable. The gambler seeks to optimise his betting strategy $b(x | y)$. Assuming proportional gambling and fair odds, the optimal wealth growth rate is then given by:

$$W(X | Y) = \langle \ln S(X | Y) \rangle = \sum_{x, y} p(x, y) \ln b(x | y)$$

which leads to

$$\langle \ln S_n \rangle = nI(X; Y)$$

that is in the presence of side information, the maximum wealth (obtained by the optimal gambling strategy) is directly proportional to the mutual information between the races and the second variable.

# References