BAYESIAN INVERSE PROBLEMS

Andrew Stuart
Email: a.m.stuart@warwick.ac.uk, D2.01 Zeeman building

Inverse problems appear widely throughout the applied sciences. Typically we need to recover some hidden information u from indirect noisy measurements $y \approx G(u)$. Examples may be found in geophysics, weather forecasting, image processing, medical imaging, traffic flow, econometrics etc.

Typcially dimension of the reconstruction space, where u lies, is larger than dimension of the measurement space, where y lives; and indeed it may be useful to think of the former as infinite dimensional. In this setup the inverse problem is under-determined and ill-posed. Common methods for dealing with ill-posed problems are regularization and Bayesian inference. These methods are linked: some maximum a posteriori (MAP) estimates of Bayesian or hierarchical Bayesian models correspond to regularized minimization problems [1],[4]. However regularization methods give only a point estimate, whilst the Bayesian point of view can also quantify uncertainty, given some prior information about it, such as smoothness (Tikhonov regularization [1], [3]) or sharp edges (Mumford-Shah regularization [2], [4]).

Bayesian methods have been applied widely to practical inverse problems but there remain a large number of open questions. One class of problems is the study of discretization invariant algorithms: ones that can be used consistently and efficiently under refinement of discretization towards the infinite dimensional setting. Studying this issue for heirarchical Bayesian inversion is a particular open area of interest. In this context many articles consider only linear forwards operator G, and a major challenging and interesting area is to study nonlinear G.

This project will address the study of uncertainty in different hierarchical Bayesian models for infinite dimensional linear inverse problems with different priors [3], [4], and their generalizations to nonlinear case. The project will involve a mix of analytic, statistical and numerical techniques, and will lead to many problems that are interesting from both the applied and theoretical points perspectives.

References

[1] A.M.Stuart *The Bayesian Approach to Inverse Problems* http://arxiv.org/pdf/1302.6989v1.pdf

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[3] S.Agapiou, J.M.Bardsley, O.Papaspiliopoulos, A.M.Stuart *Analysis of Gibbs sampler for hierarchical inverse problems* http://arxiv.org/pdf/1311.1138v1.pdf

[4] T.Helin, M.Lassas *Hierarhical models in statistical inverse oroblems and the Mumford-Shah functional* http://arxiv.org/pdf/0908.3396v2.pdf