There are many graphs, for example cliques, with chromatic number close to the maximum degree $\Delta$ and almost all of them contain many triangles. Vizing [9] asked whether the bound in Brooks' Theorem can be improved significantly for triangle-free graphs. The first non-trivial result was independently due to Borodin and Kostochka [1], Catlin [2] and Lawrence [6], who showed that if a graph $G$ has no triangles then the chromatic number of $G$, $\chi(G)$, is at most $\frac{3}{4}(\Delta(G)+2)$. Later, Kostochka (see [3]) showed that for a triangle-free graph $G$, $\chi(G) \leq \frac{2}{3}\Delta(G)+2$. This was the best bound known for triangle-free graphs until Johansson's [4] following result.

**Theorem 1.** *There exists $\Delta_0$ such that every triangle-free graph $G$ with maximum degree $\Delta \geq \Delta_0$, has $\chi(G) \leq \frac{160\Delta}{ln\ \Delta}$.*

I will present the proof of this result as described in [7]. The approach used here is applying several iterations of the so-called *Naive Coloring Procedure* introduced first by Kahn in [5]. First a partial coloring is constructed and instead of completing it greedily, a series of refinements are produced using the Naive Coloring Procedure. This is a special case of the following general technique. First an object $X$ is constructed via a series of partial objects $X_1, \ldots, X_t{=}X$. At each step, the existence of an extension of $X_i$ to a suitable $X_{i+1}$ is shown by considering a random choice for that extension and applying the probabilistic method to several incremental random choices rather than to a single random choice of $X$. This technique is often referred to as the *semi-random method*, the *pseudo-random method* or the *Rödl Nibble*, where the latter name comes from a series of well known papers beginning with [8].

## References


[1] O.V. Borodin and A.V. Kostochka. On an upper bound of a graph's chromatic number, depending on the graph's degree and density. *Journal of Combinatorial Theory, Series B*, 23(2-3):247–250, 1977.

[2] P.A. Catlin. A bound on the chromatic number of a graph. *Discrete Mathematics*, 22(1):81–83, 1978.

[3] TR Jensen and B. Toft. *Graph Coloring Problems.* Wiley, New York, 1995.

[4] A. Johansson. Asymptotic Choice Number for Triangle Free Graphs. *DIMACS Technical Report*, pages 91–95, 1996.

[5] J. Kahn. Asymptotically good list-colorings. *Journal of Combinatorial Theory, Series A*, 73(1):1–59, 1996.

[6] J. Lawrence. Covering the vertex set of a graph with subgraphs of smaller degree. *Discrete Mathematics*, 21(1):61–68, 1978.

[7] M.S. Molloy and B.A. Reed. *Graph colouring and the probabilistic method.* Springer Verlag, 2002.

[8] V. Rödl. On a packing and covering problem. *European Journal of Combinatorics*, 5(1):69–78, 1985.

[9] VG Vizing. Some unsolved problems in graph theory. *Russian Mathematical Surveys*, 23:125, 1968.