

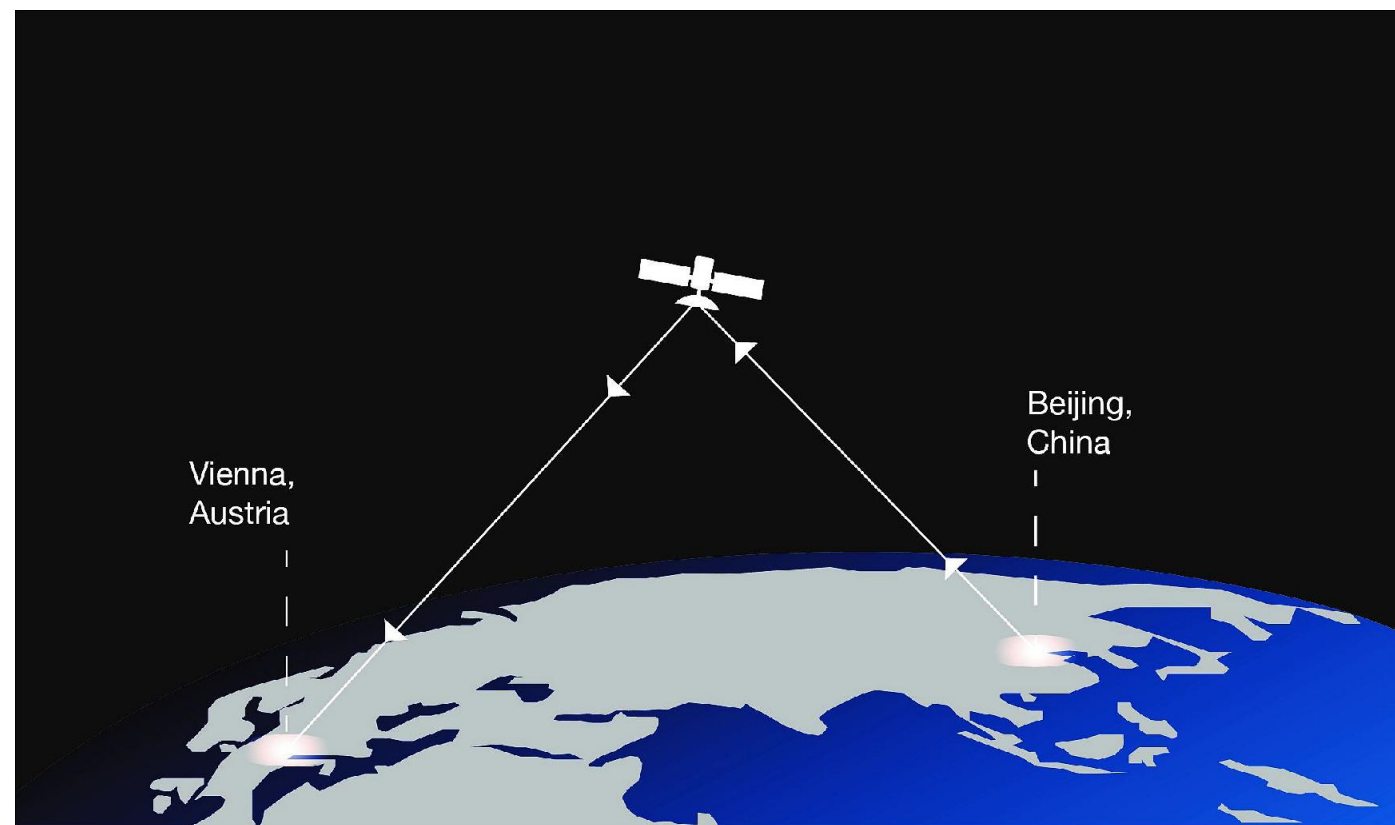
# Secure Quantum Computation over Classical Networks

Atul Mantri | February 15th, 2021

The Joint Center for Quantum Information and Computer Science  
(QuICS)  
University of Maryland

Classical devices cannot efficiently simulate quantum systems!

## Important Milestones!



Secure Communication over 7600km (2017)

Entanglement-based secure quantum cryptography over 1,120 kilometres

### Article

## Quantum supremacy using a programmable superconducting processor

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The promise of quantum computers is that certain computational tasks might be executed exponentially faster on a quantum processor than on a classical processor<sup>1</sup>. A fundamental challenge is to build a high-fidelity processor capable of running quantum algorithms in an exponentially large computational space. Here we report the use of a processor with programmable superconducting qubits<sup>2,7</sup> to create quantum states on 53 qubits, corresponding to a computational state-space of dimension  $2^{53}$  (about  $10^{16}$ ). Measurements from repeated experiments sample the resulting probability distribution, which we verify using classical simulations. Our Sycamore processor takes about 200 seconds to sample one instance of a quantum circuit a million times—our benchmarks currently indicate that the equivalent task for a state-of-the-art classical supercomputer would take approximately 10,000 years. This dramatic increase in speed compared to all known classical algorithms is an experimental realization of quantum supremacy<sup>8–14</sup> for this specific computational task, heralding a much-anticipated computing paradigm.

Google (2019)

### QUANTUM COMPUTING

## Quantum computational advantage using photons

Han-Sen Zhong<sup>1,2\*</sup>, Hui Wang<sup>1,2\*</sup>, Yu-Hao Deng<sup>1,2\*</sup>, Ming-Cheng Chen<sup>1,2\*</sup>, Li-Chao Peng<sup>1,2</sup>, Yi-Han Luo<sup>1,2</sup>, Jian Qin<sup>1,2</sup>, Dian Wu<sup>1,2</sup>, Xing Ding<sup>1,2</sup>, Yi Hu<sup>1,2</sup>, Peng Hu<sup>3</sup>, Xiao-Yan Yang<sup>3</sup>, Wei-Jun Zhang<sup>3</sup>, Hao Li<sup>3</sup>, Yuxuan Li<sup>4</sup>, Xiao Jiang<sup>1,2</sup>, Lin Gan<sup>4</sup>, Guangwen Yang<sup>4</sup>, Lixing You<sup>3</sup>, Zhen Wang<sup>3</sup>, Li Li<sup>1,2</sup>, Nai-Le Liu<sup>1,2</sup>, Chao-Yang Lu<sup>1,2,†</sup>, Jian-Wei Pan<sup>1,2,†</sup>

Quantum computers promise to perform certain tasks that are believed to be intractable to classical computers. Boson sampling is such a task and is considered a strong candidate to demonstrate the quantum computational advantage. We performed Gaussian boson sampling by sending 50 indistinguishable single-mode squeezed states into a 100-mode ultralow-loss interferometer with full connectivity and random matrix—the whole optical setup is phase-locked—and sampling the output using 100 high-efficiency single-photon detectors. The obtained samples were validated against plausible hypotheses exploiting thermal states, distinguishable photons, and uniform distribution. The photonic quantum computer, *Jiuzhang*, generates up to 76 output photon clicks, which yields an output state-space dimension of  $10^{30}$  and a sampling rate that is faster than using the state-of-the-art simulation strategy and supercomputers by a factor of  $\sim 10^{14}$ .

USTC, China (2020)

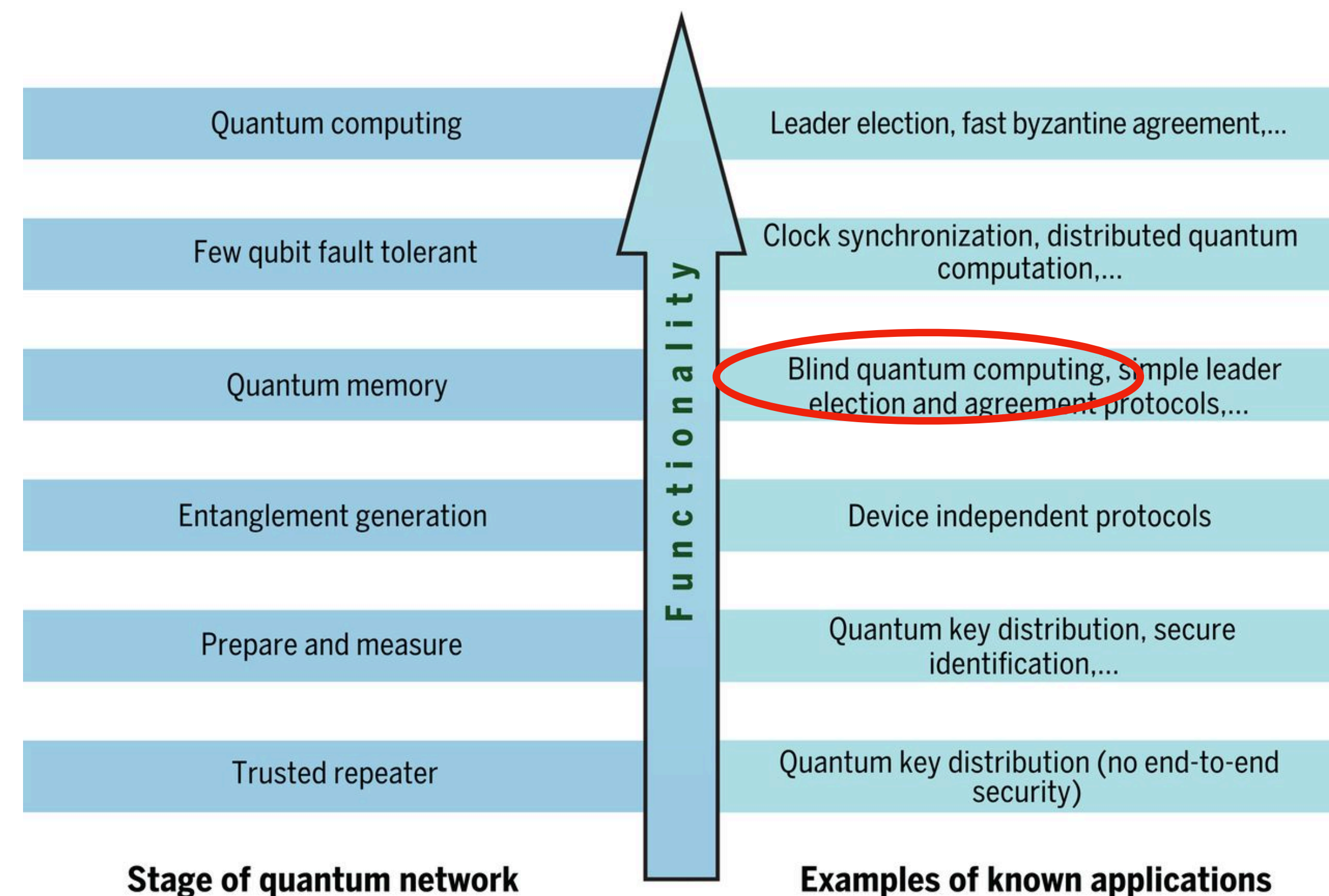
Building quantum computers is very hard but not impossible!  
Hardware is working, what's next..?

## Future of Internet



Quantum Internet promises to provide radically new internet technologies.

Some maybe not possible to accomplish on the modern internet.



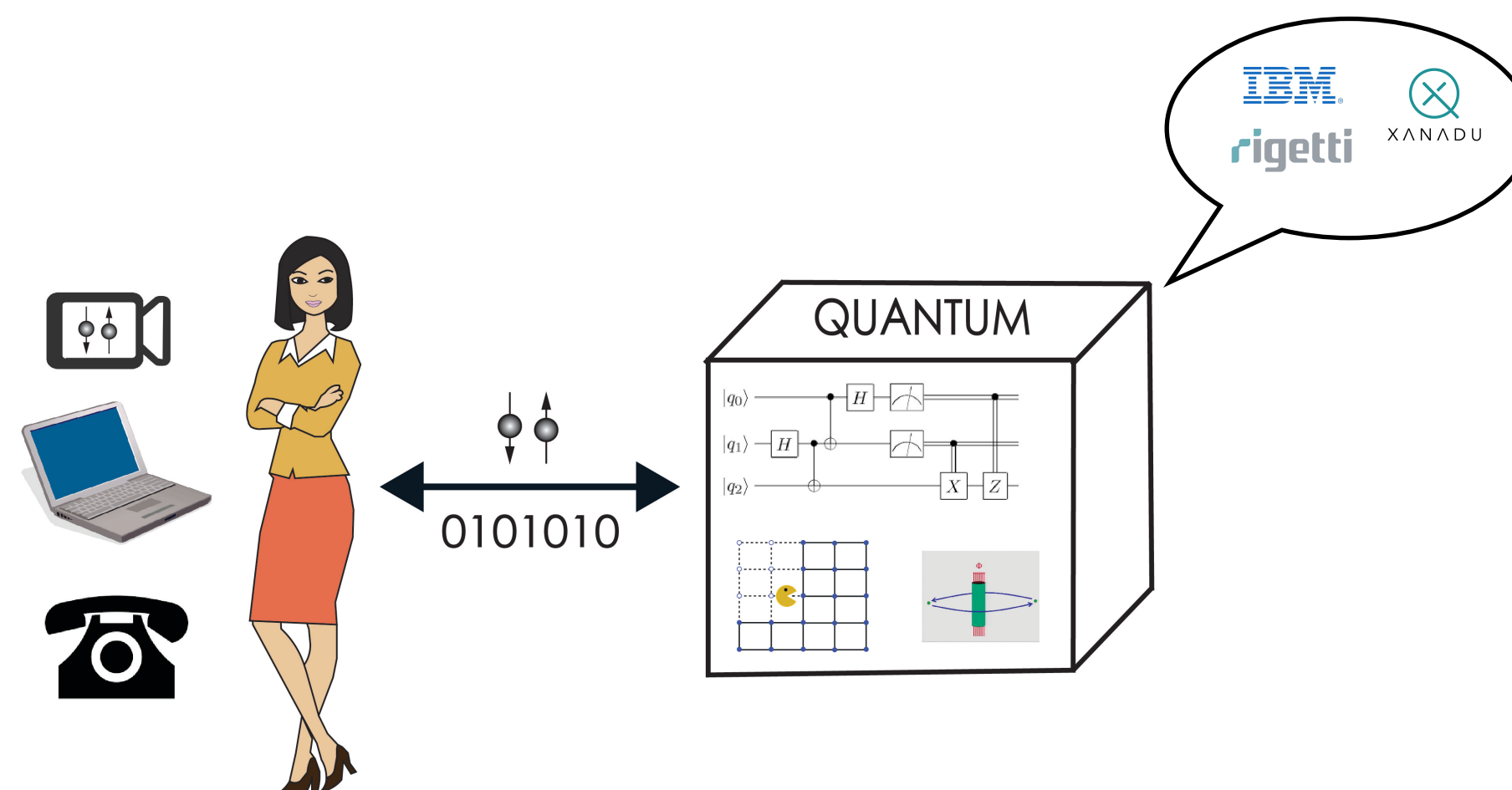
Wehner, S., Elkouss, D., & Hanson, R. (2018). Quantum internet: A vision for the road ahead. *Science*, 362(6412).

A more realistic setting is where both quantum and classical channel co-exists!

# Secure Cloud Quantum Computing

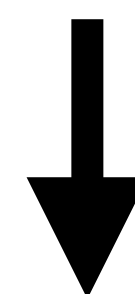
## Client

- Has limited computational resources.
- Wants to use quantum computer.
- Doesn't want to reveal the data.
- Problems may involve confidential data or be commercially sensitive.



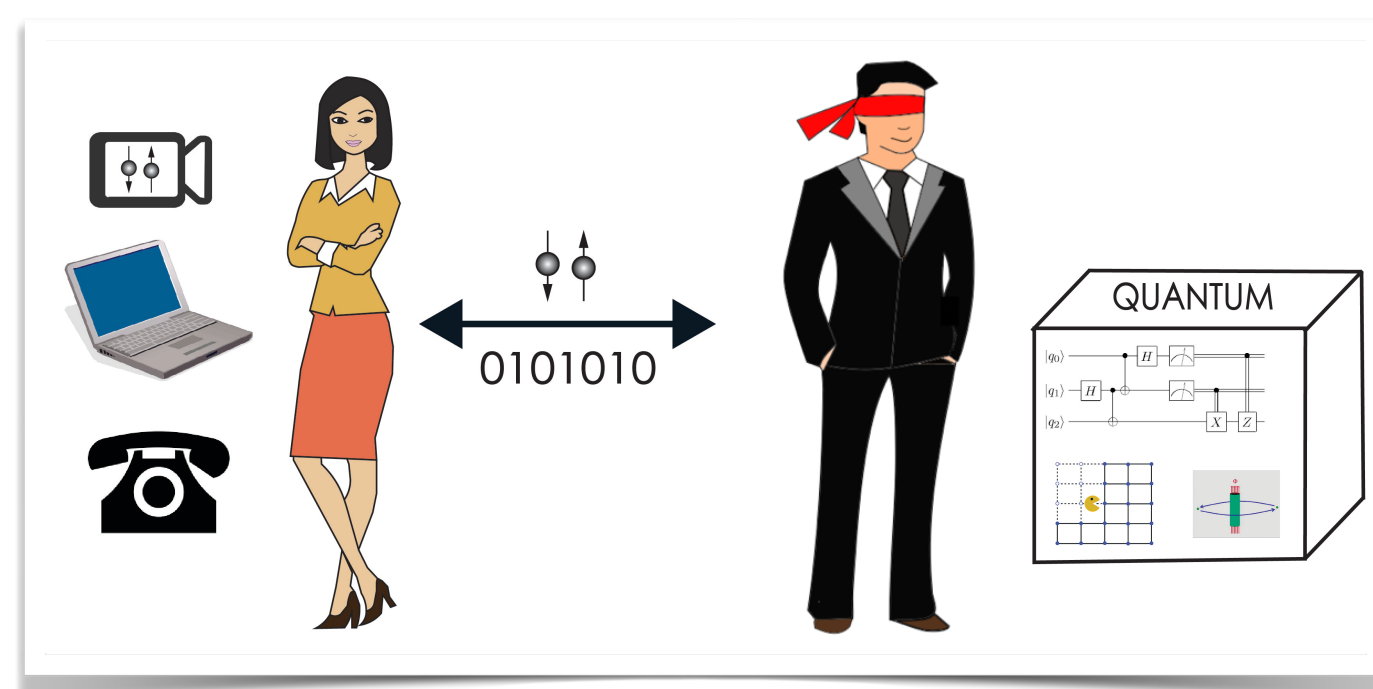
## Server

- Has a full quantum computer.
- Willing to help and provide cloud-based services.
- Cannot be trusted.



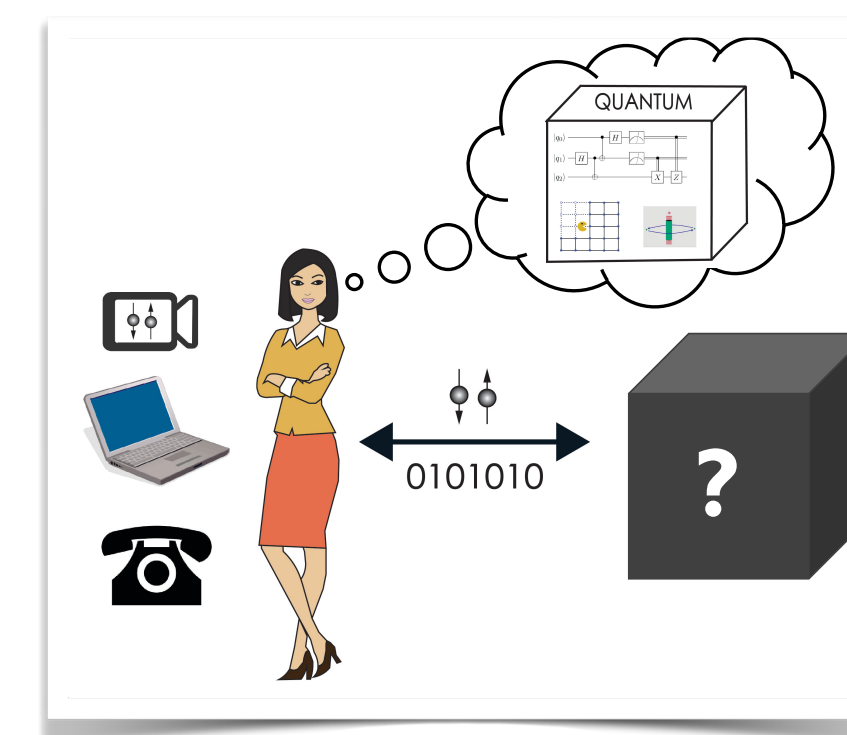
## Blindness

privacy of client's computation is preserved.

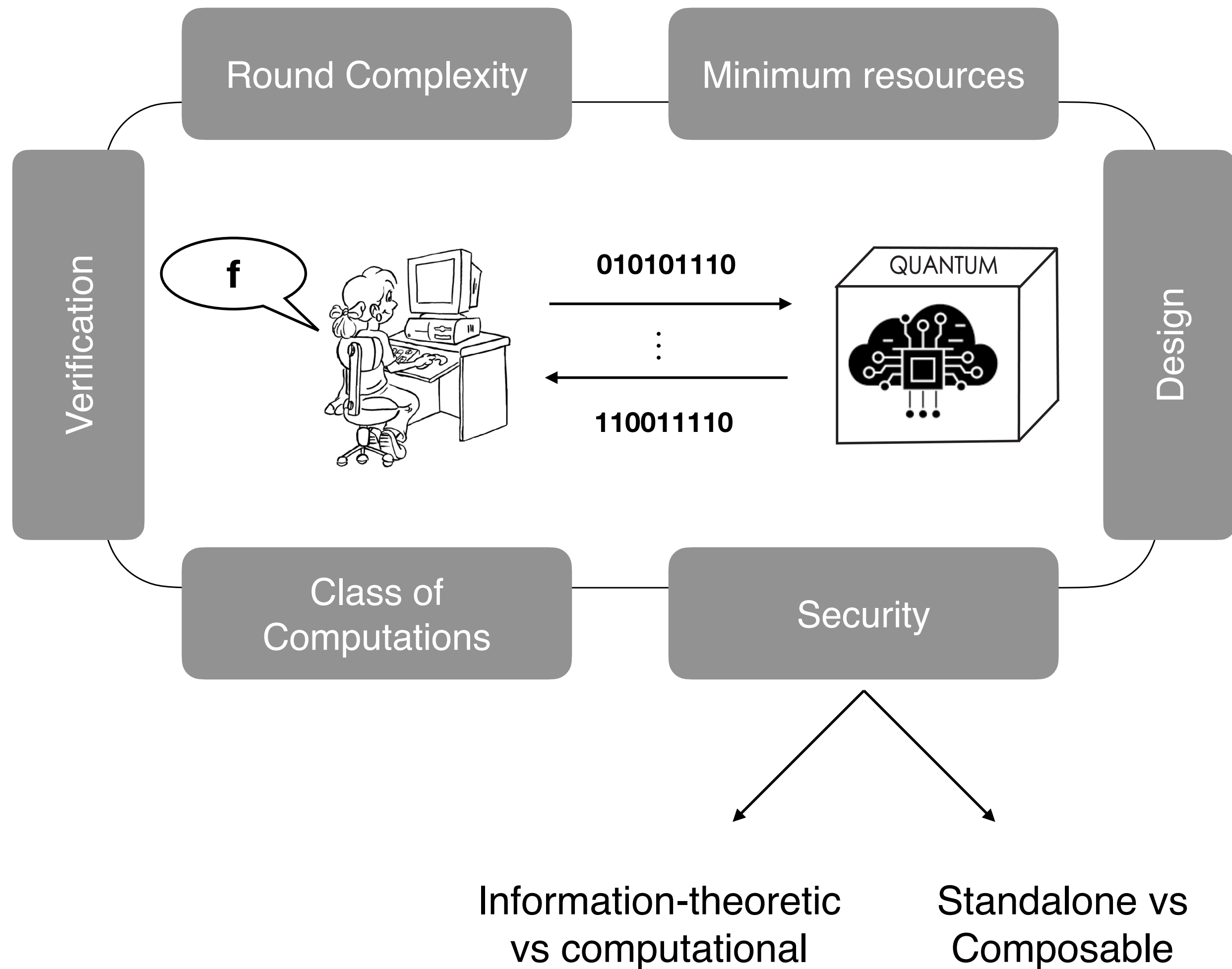


## Verification

Integrity of the desired computation is maintained

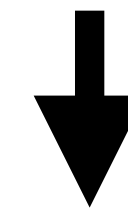


# Requirements



## Classical User - Classical Server Abadi, Feigenbaum and Kilian (1987)

computing NP-hard function securely



Unlikely consequences in complexity theory\*

\*polynomial hierarchy collapses at the third level

functions that are harder to compute are harder to encrypt...

Fully homomorphic encryption (FHE) schemes!

Quantum functions?

Information-theoretic — Against Unbounded Adversaries (Ideal)

Computational — Against Computationally-Bounded Adversaries  
(say, Quantum poly-time adversaries)

# Classical vs Quantum Computing

Bits

Qubits

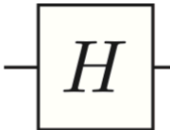
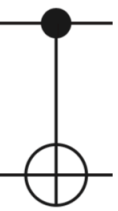

AND/OR/NOT Gates

Unitary Quantum Gates

Reading the output bit    Measuring: final state -> classical output

Single qubit gates

Two qubit gates -  
entangling gates

Name	Matrix	Circuit Element
Hadamard	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	
CNOT	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	
CZ	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	

Single qubit state

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Superposition

$$\alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

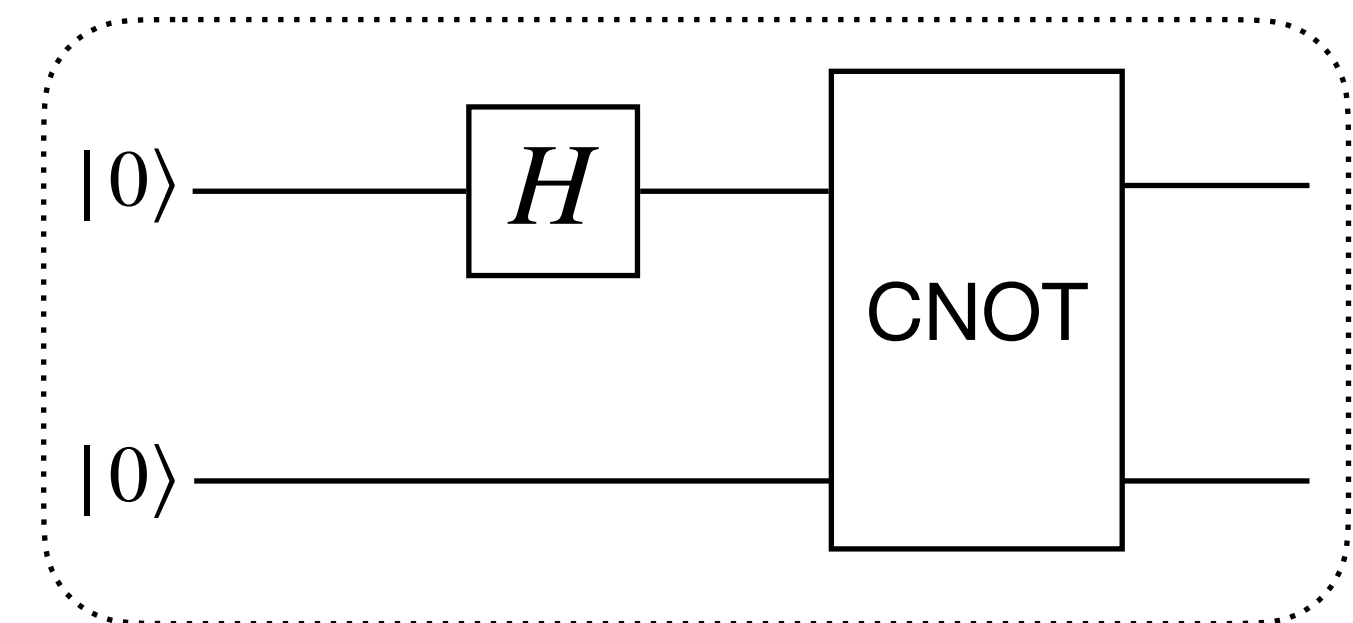
n-qubit state

$$|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha_x |x\rangle$$

$$\sum_{x \in \{0,1\}^n} |\alpha_x|^2 = 1$$

Example

Input state



Output state

$$|00\rangle \rightarrow \frac{|00\rangle + |10\rangle}{\sqrt{2}} \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Entanglement

# Models for Quantum Computing

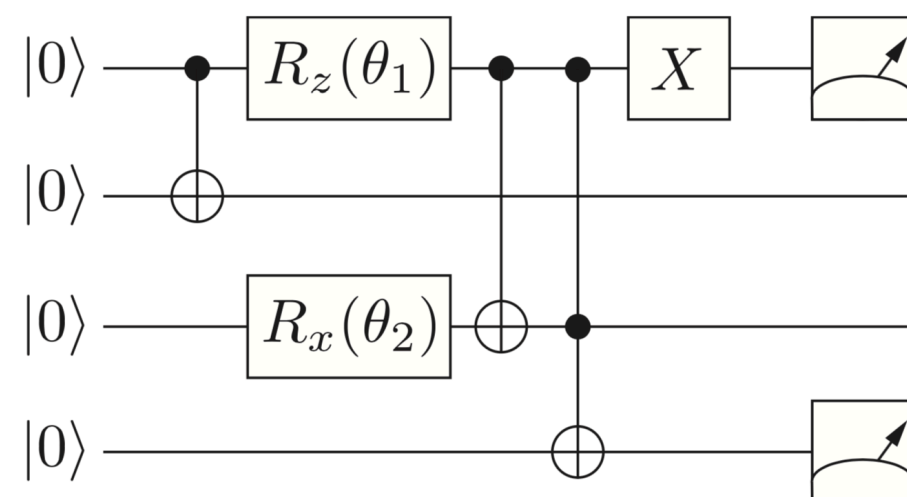
## Quantum Circuit Model

- Prepare a quantum state in the computational basis.
- Apply a sequence of unitary operations.
- Perform a measurement of one or more of the qubits.



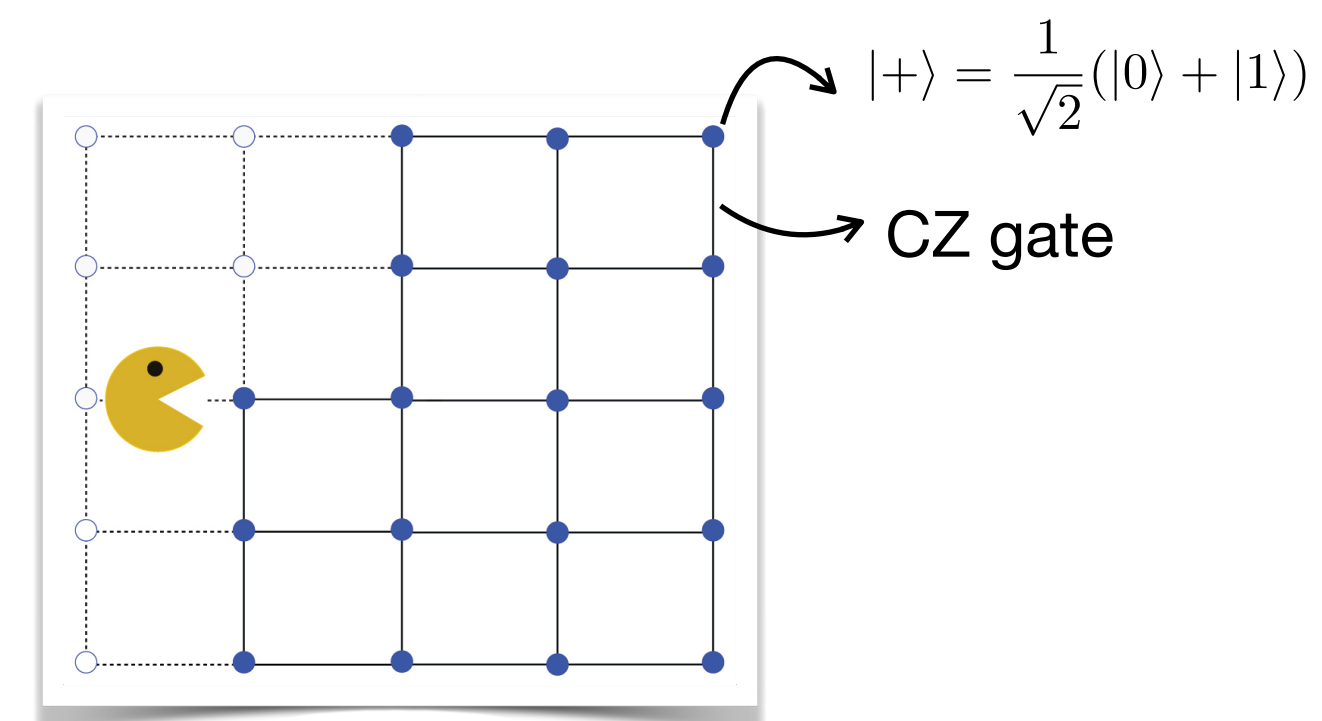
Adding stone by stone to make a sculpture

### Example

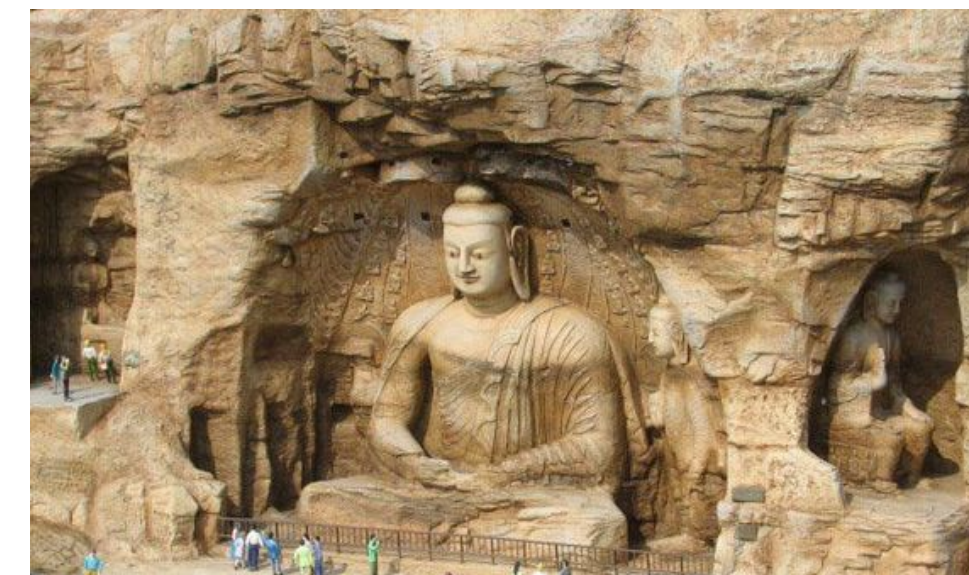


## Measurement-Based Model

A computation is performed by means of **single-qubit projective measurements** that drive the quantum information across a **highly entangled state**.



single qubit measurements

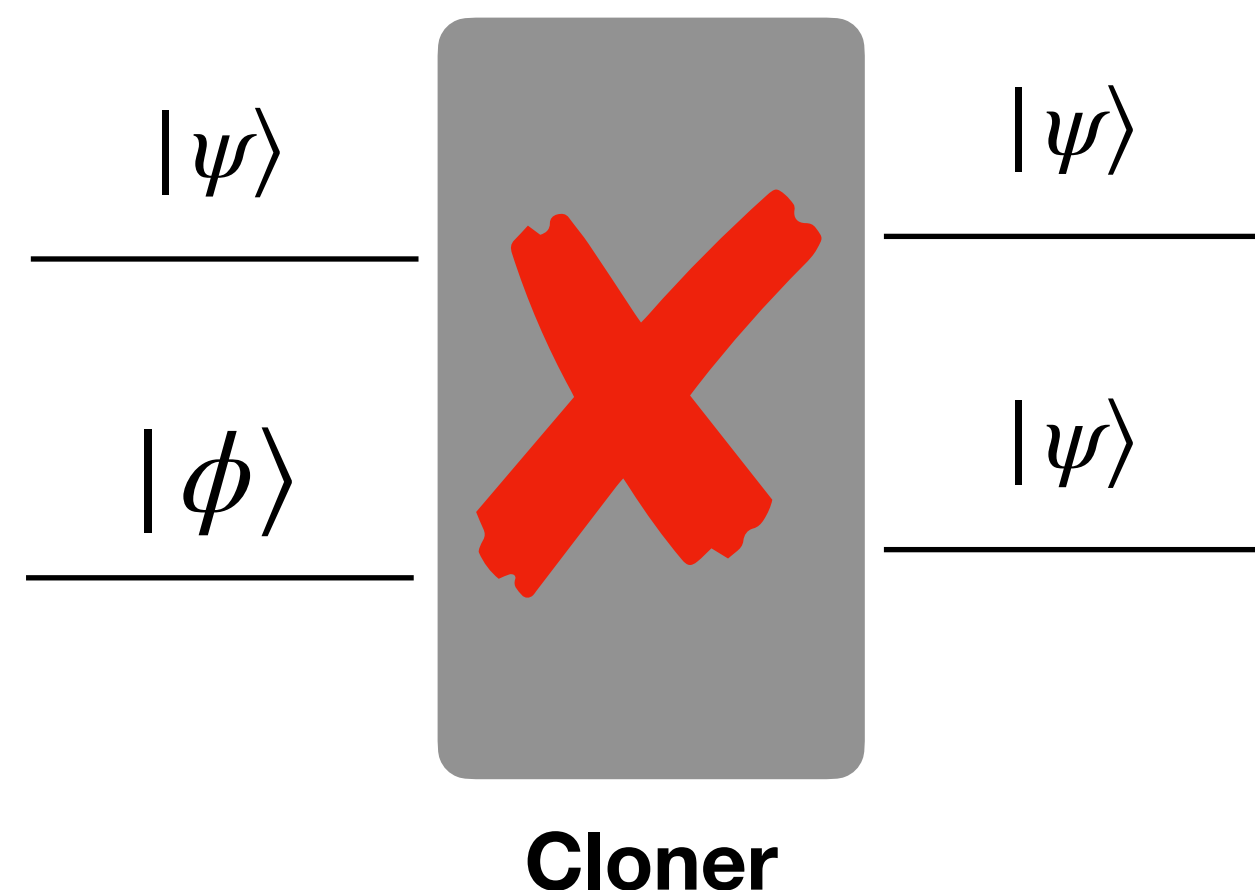


Rock cut sculpture

Highly Entangled State + (Adaptive) Measurements

# Quantum No-Cloning

There does not exist any physical device that can output two perfectly identical copies of an unknown quantum state  $|\psi\rangle$  when given a single copy of  $|\psi\rangle$ .

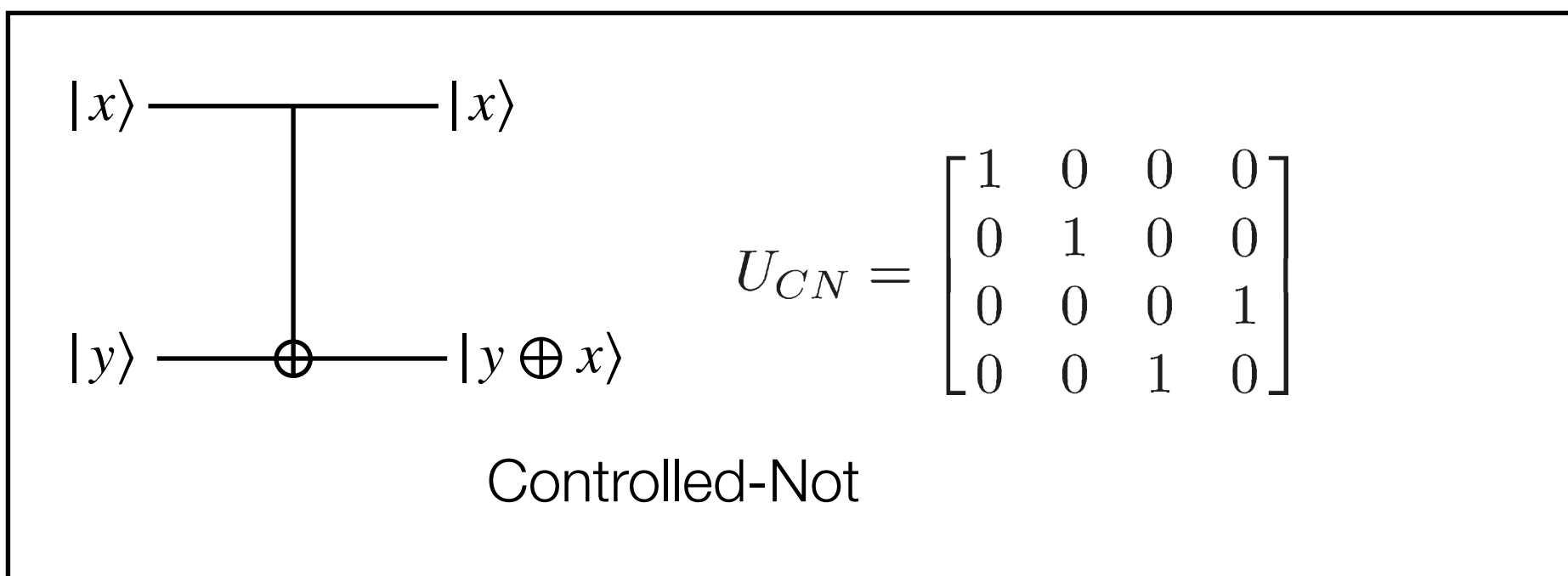
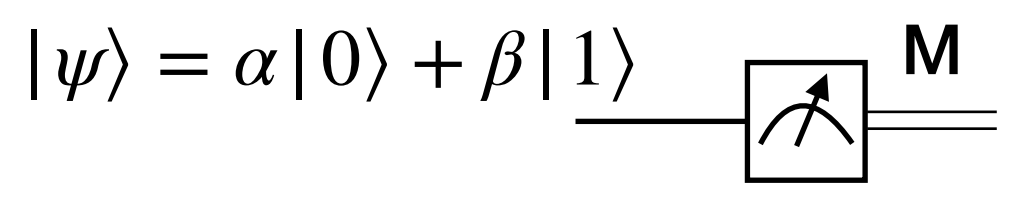
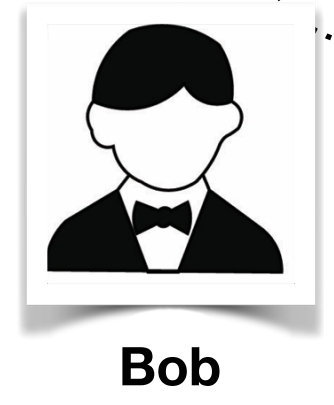
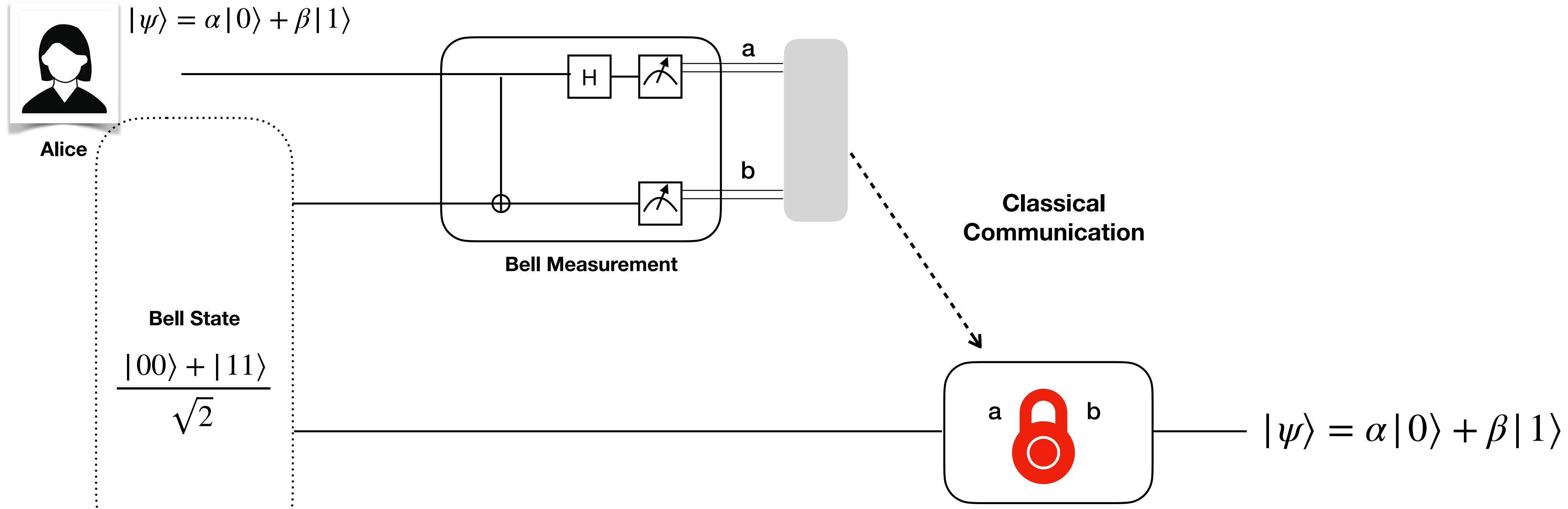


Useful in Quantum Key Distribution, Quantum Money, etc..

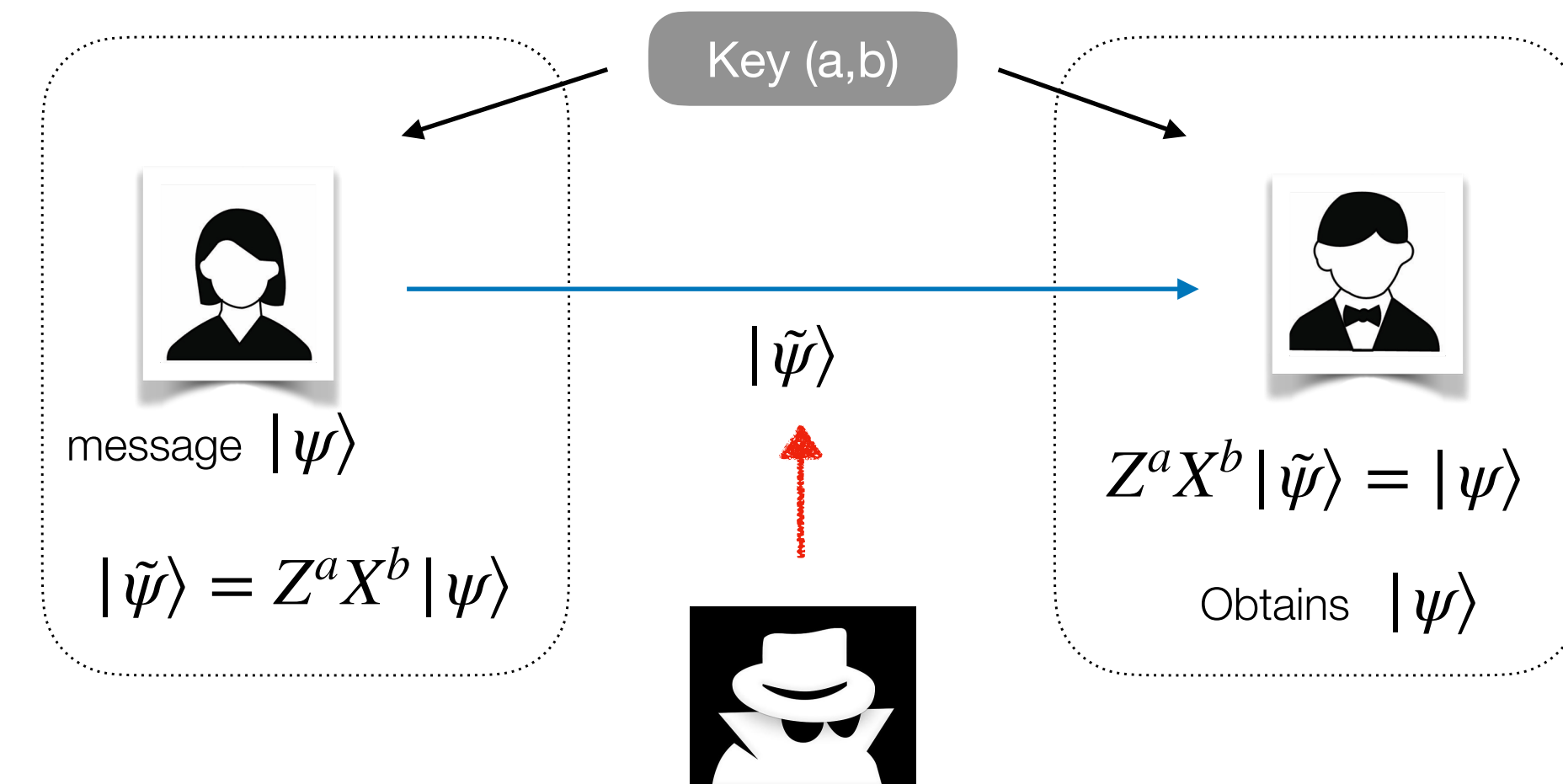
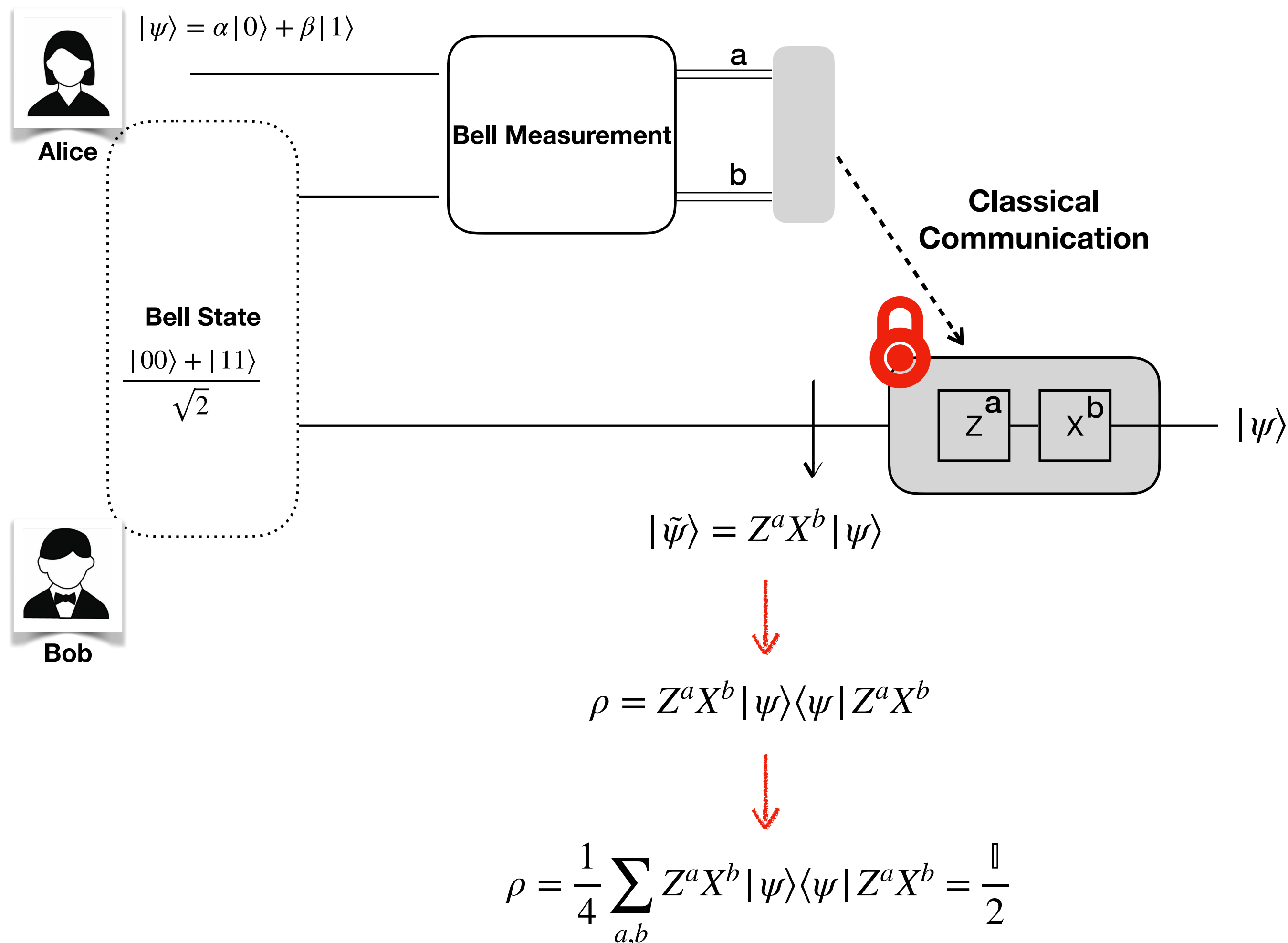
We will see security limitation of two-party quantum cryptographic primitives based on no-cloning theorem



# Quantum Teleportation



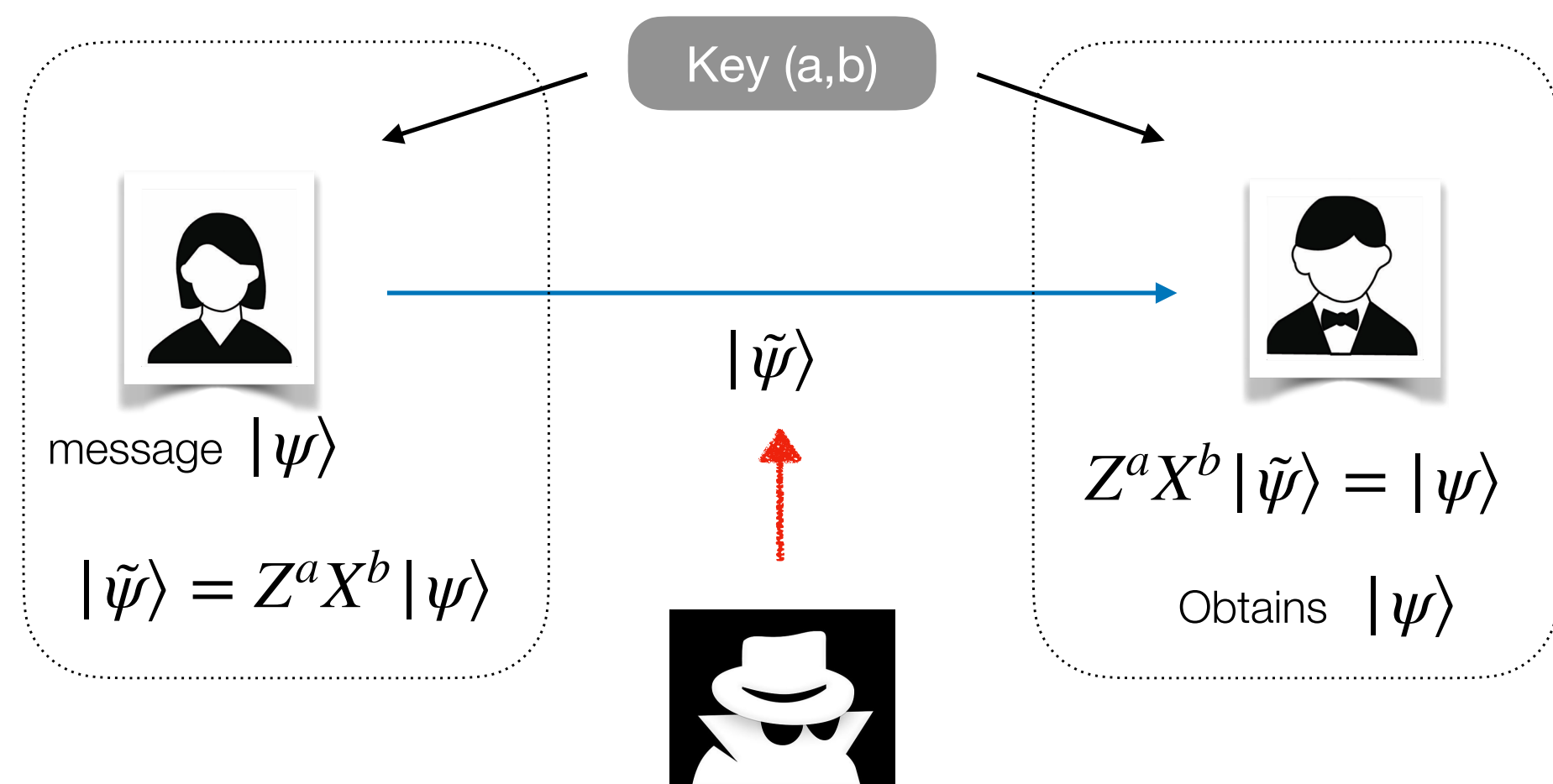
# Quantum Teleportation || Quantum One-Time Pad



**Correctness:** Bob can read the message  
**Security:** Eve gains no information i.e. state is completely mixed and hence independent of message

Information-theoretically secure

# Classical vs Quantum One-time Pad



**Correctness:** Bob can read the message

**Security:** Eve gains no information i.e. state is completely mixed and hence independent of message

Information-theoretically secure

**Encrypt**

$$\tilde{m} = m \oplus b$$

$$|\tilde{m}\rangle = X^b |m\rangle$$

X is quantum operation that does the bit flip

**Decrypt**

$$m = \tilde{m} \oplus b = (m \oplus b) \oplus b$$

$$|m\rangle = X^b |\tilde{m}\rangle = X^b (X^b |m\rangle)$$

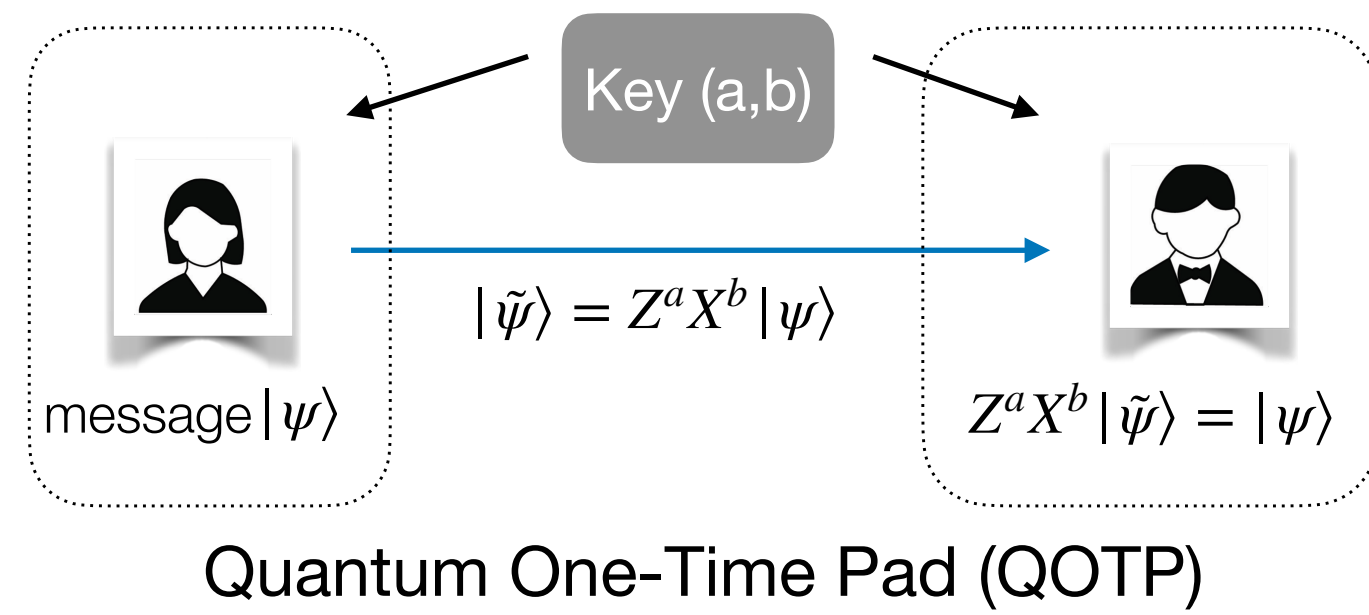
$$X|m\rangle = |m \oplus 1\rangle$$

$$Z|m\rangle = (-1)^m |m\rangle$$

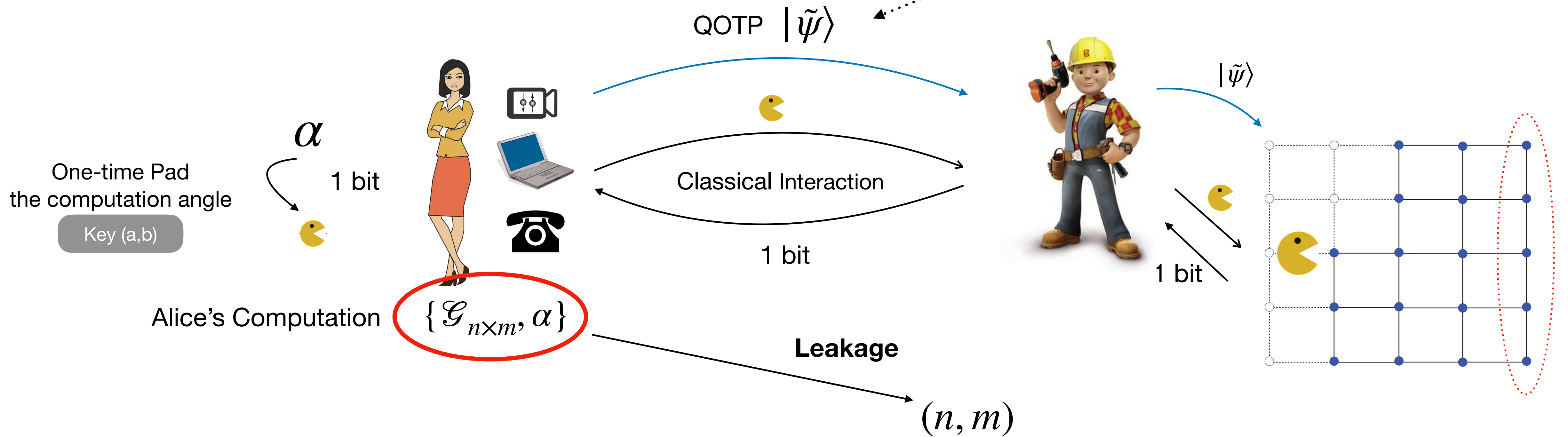
Quantum One-time pad: Bit flips in both basis

# Delegated Quantum Computation

Broadbent-Fitzsimons-Kashefi (BFK) scheme



Single-qubit quantum states  $|\tilde{\psi}\rangle := \frac{|0\rangle + e^{i\theta}|1\rangle}{\sqrt{2}}$



BFK Scheme consists of **Quantum + Classical Interaction**

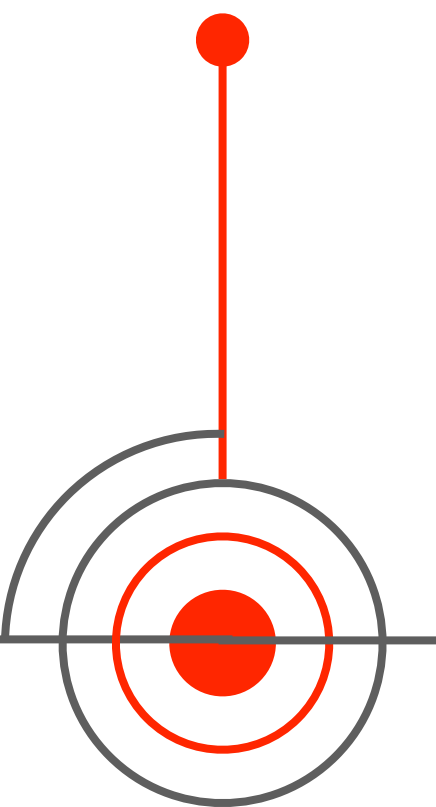
### Weak Quantum Client

### Optimal Scheme for Cloud QC

Broadbent-Fitzsimons-Kashefi (FOCS)  
Aharonov-Ben-or-Eban (ICS)

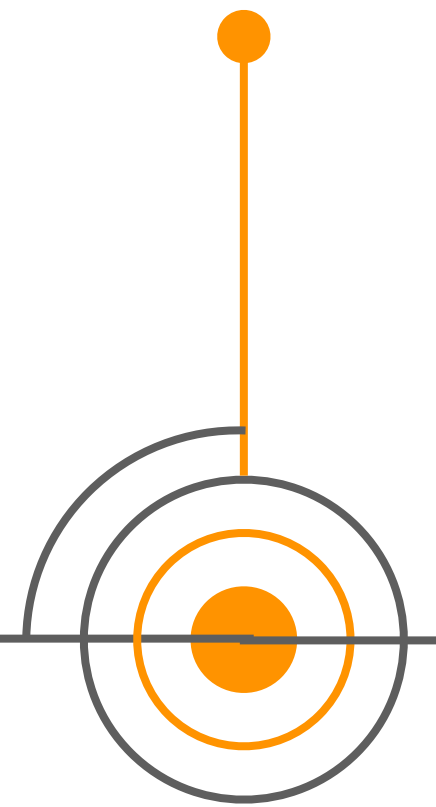
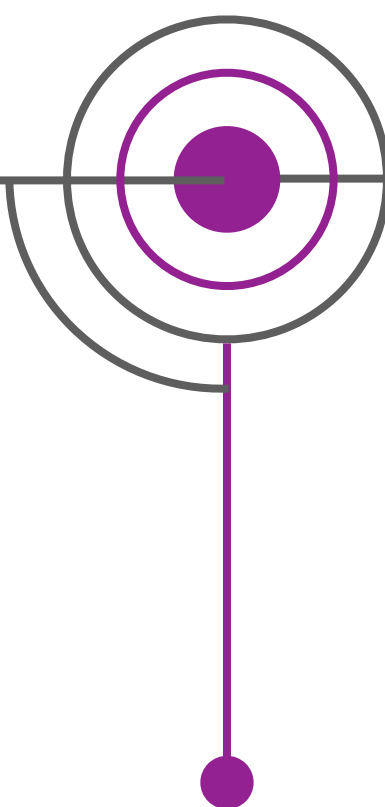
MDPF(PRL)  
GMMR(PRL)

2001



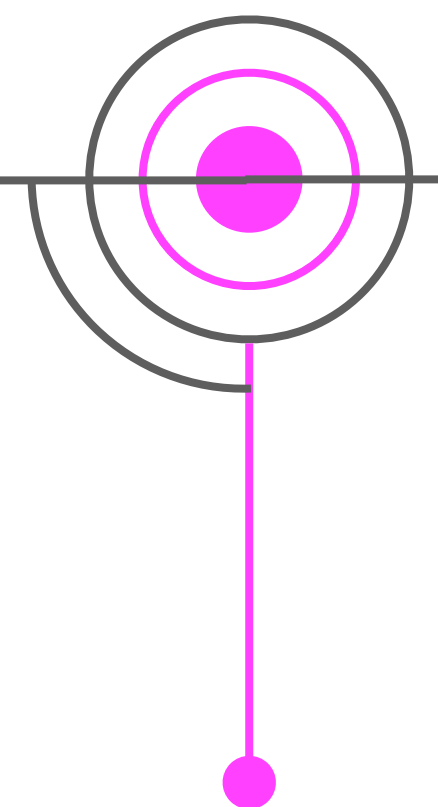
2009

2012



2013

2016



Powerful Quantum Client

Childs' Scheme (QIC)

Classical Client  
2 entangled servers

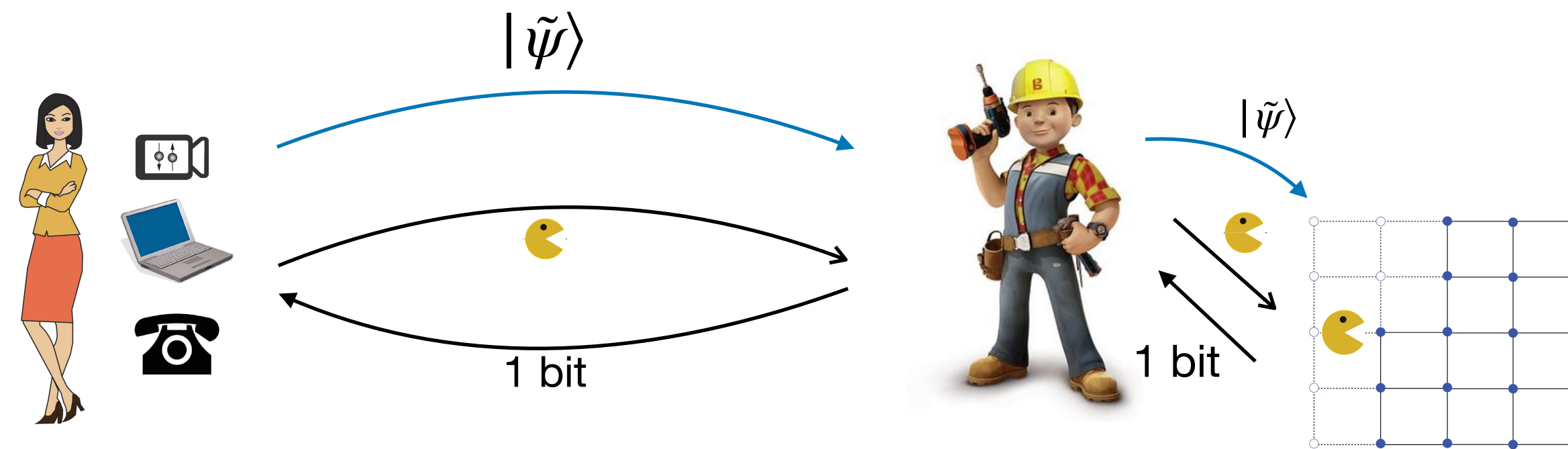
Reichardt-Unger-Vazirani  
(Nature)

First Scheme  
Classical Client - Single server

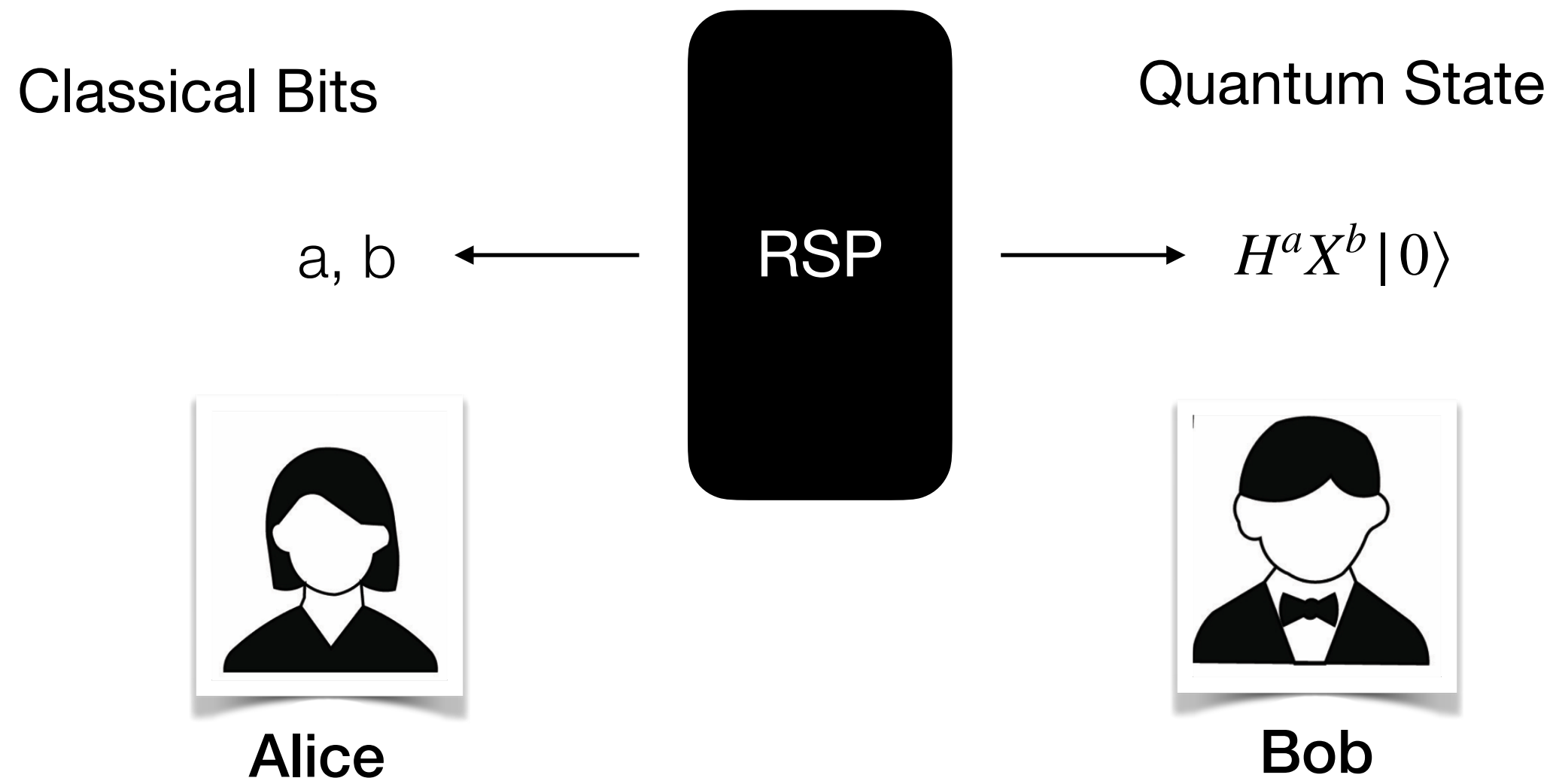
MDMF (PRX)

Any questions so far?

Can we “dequantize” the **quantum interaction** from the BFK scheme?



# Remote State Preparation (RSP)



a	b	$H^a X^b  0\rangle$
0	0	$ 0\rangle$
0	1	$ 1\rangle$
1	0	$ +\rangle$
1	1	$ -\rangle$

Easy and secure if Alice and Bob share quantum resources

1. Alice could perform Quantum Teleportation
2. Alice could prepare and send the state via Quantum Channel



Quantum Satellite

Maybe not so easy and secure only if Quantum Satellite doesn't collude with Bob

**What if Alice and Bob share a classical channel and they don't trust Quantum Satellite?**

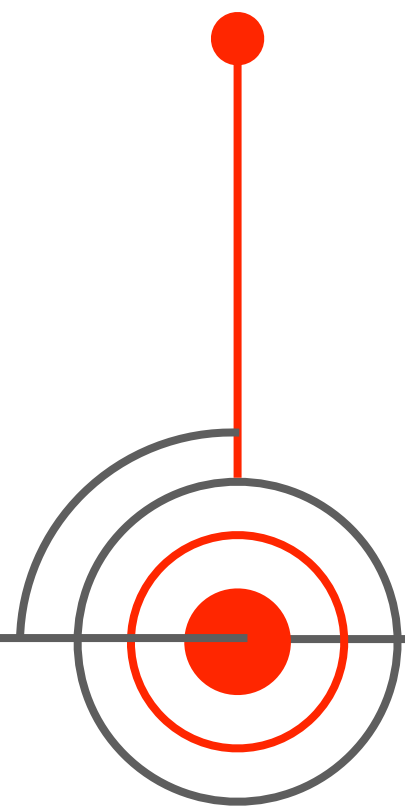
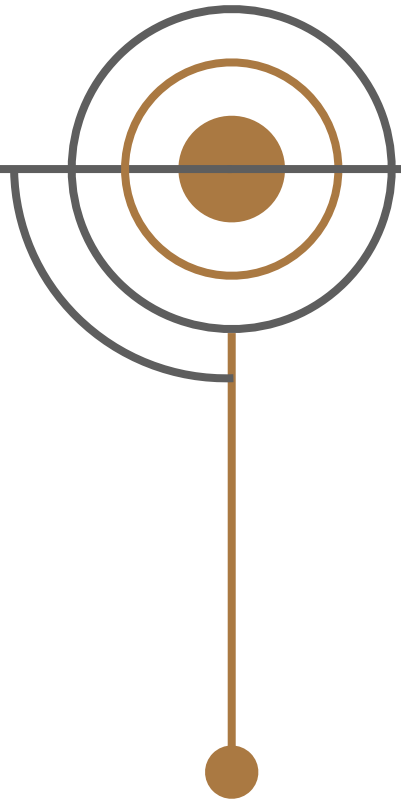
# Classical User - Quantum Server

FHE for Quantum Circuits  
Verification of Quantum Comp.  
Mahadev (FOCS)

Modular approach, simpler  
cryptographic assumption  
and security limitations  
BCCKMW ASIACRYPT

Computational Security

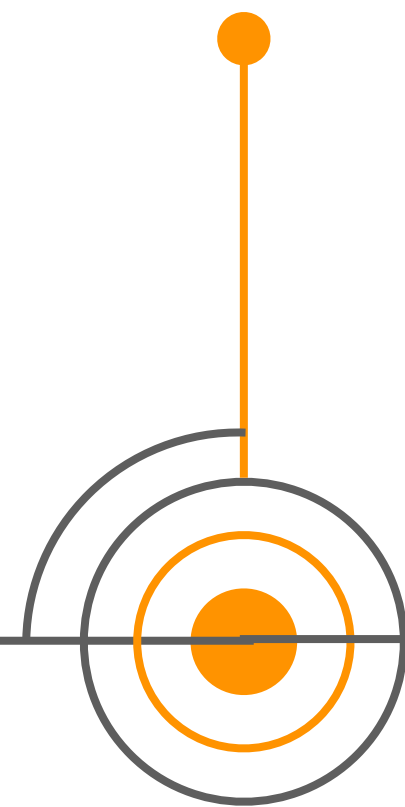
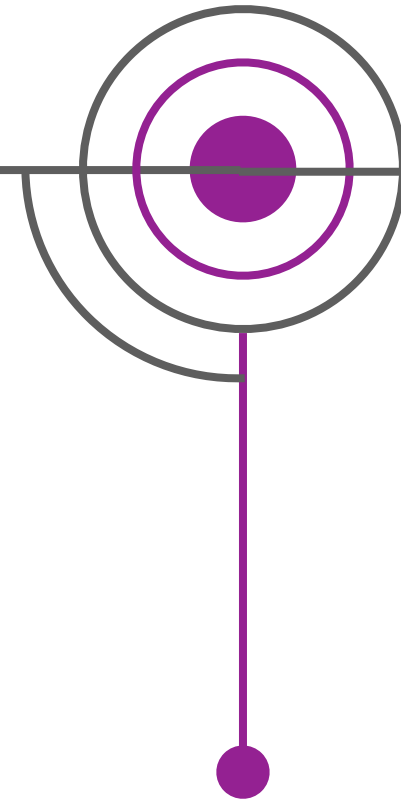
2017



2018

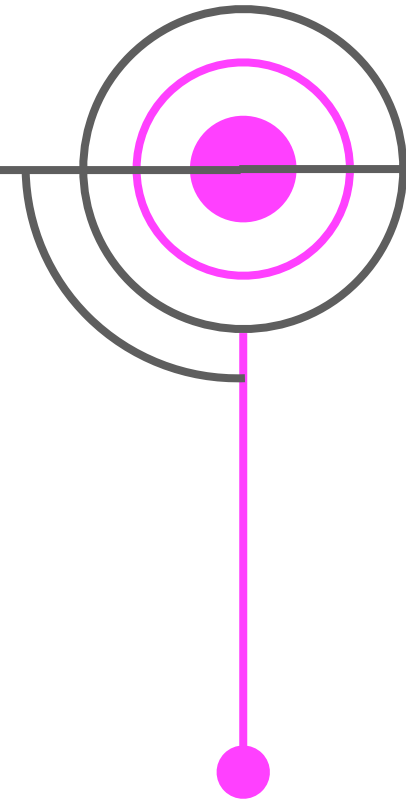


2019



2020

2020



Complexity-theoretic limitations

Aaronson et al. (ICALP)

Secure RSP

Gheorghiu and Vidick (FOCS)  
Cojocaru et al. (ASIACRYPT)

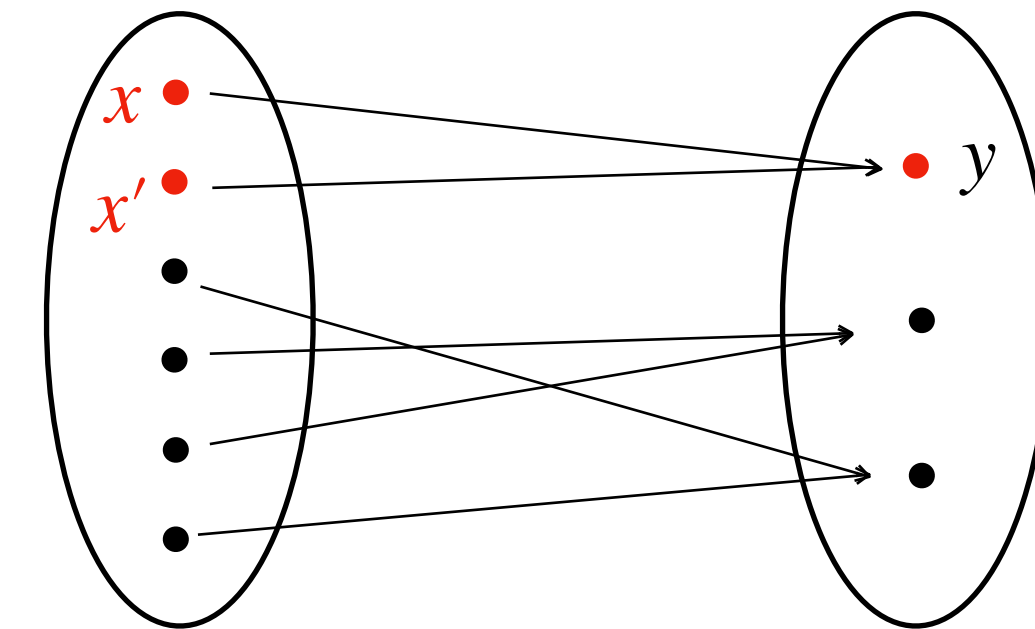
First Secure Q2PC based on  
Classical Channel

CCKM (arXiv: 2010.07925)



## Type I: Using trapdoor claw-free functions (TCFF)

Brakerski et al., Mahadev (FOCS 2018), Gheorghiu-Vidick (FOCS 2019)



- $f$  is one-way, hard to invert
- 2-to-1 function
- Collision resistant i.e. hard to find claws: pairs  $(x, x')$  such that  $f(x) = f(x') = y$  without trapdoor
- With trapdoor it is easy to invert  $y$  and find  $(x, x')$

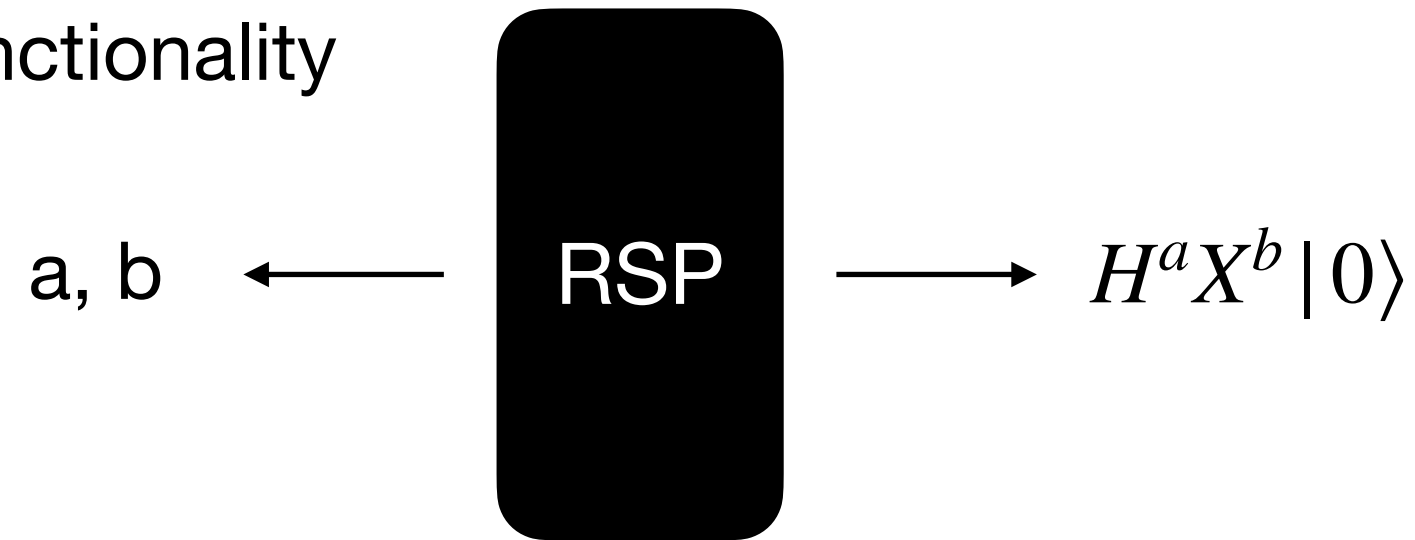
## Type II: Using homomorphic trapdoor injective OWF

Cojocaru et al. (Asiacrypt 2019)

← RSP Construction in this talk are based on Type II

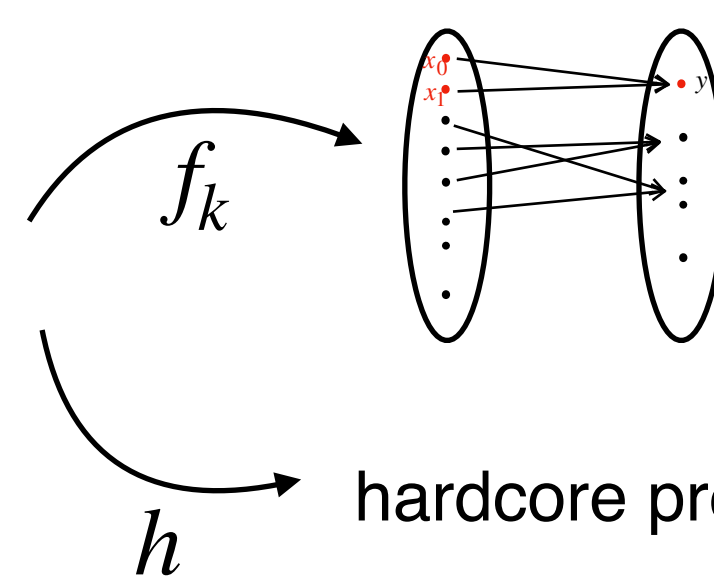
# High-Level Idea

Ideal Functionality



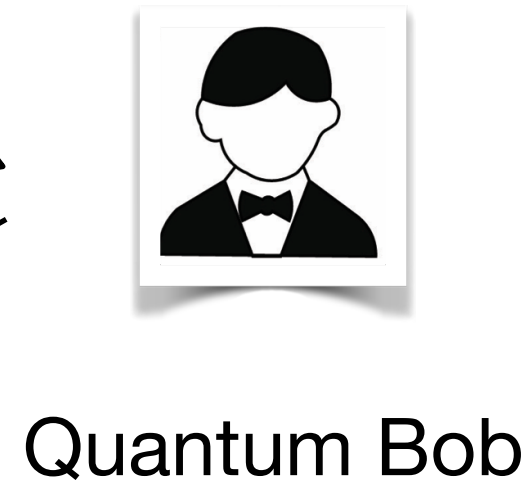
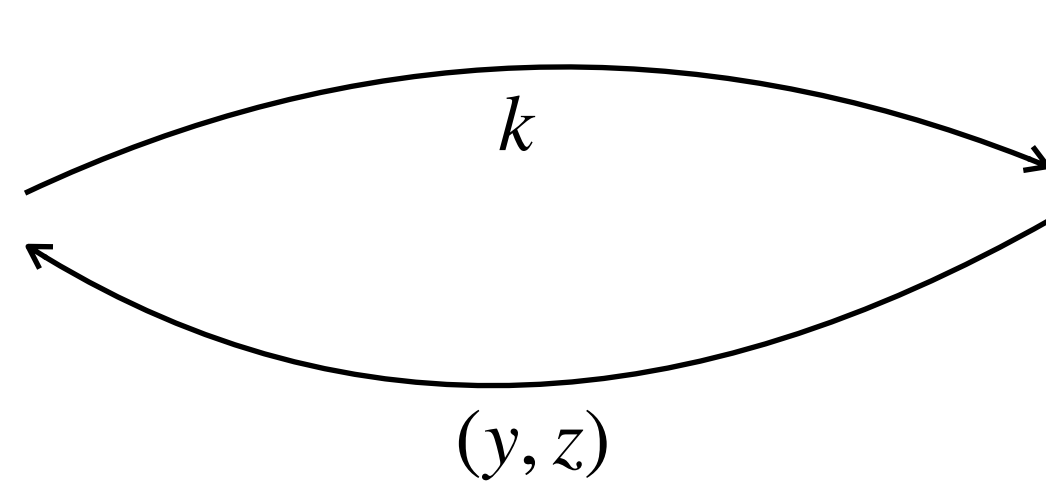
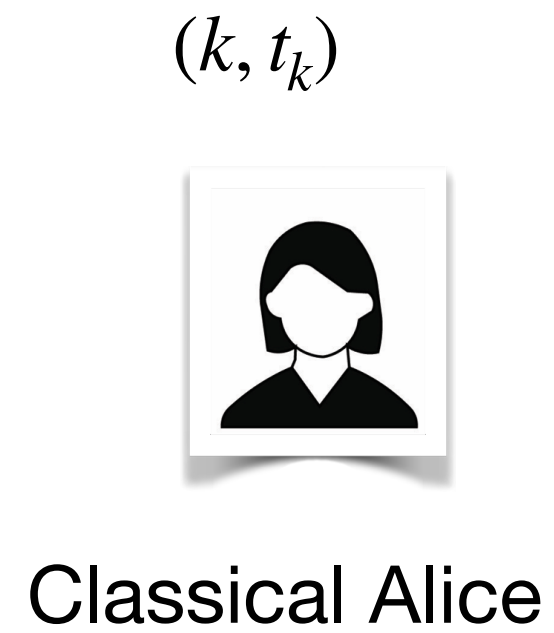
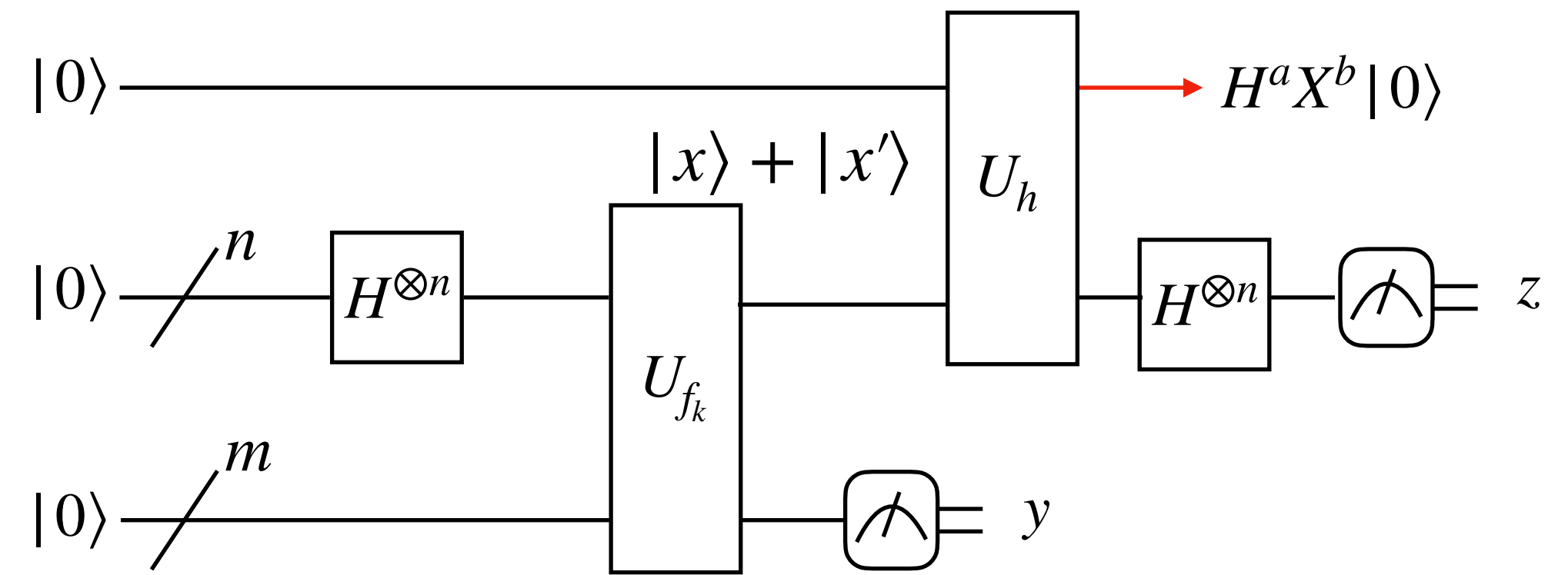
Trapdoor, Injective, homomorphic OWF

$g_k$



hardcore predicate (w.r.t function g)

## Concrete Protocol



$(t_k, y, z) \rightarrow (a, b)$

Prepare superposition over all possible inputs  $x$

Evaluate the function  $f$  in another register

Measure the second register (image) to obtain the outcome  $y$

Evaluate the function  $h$

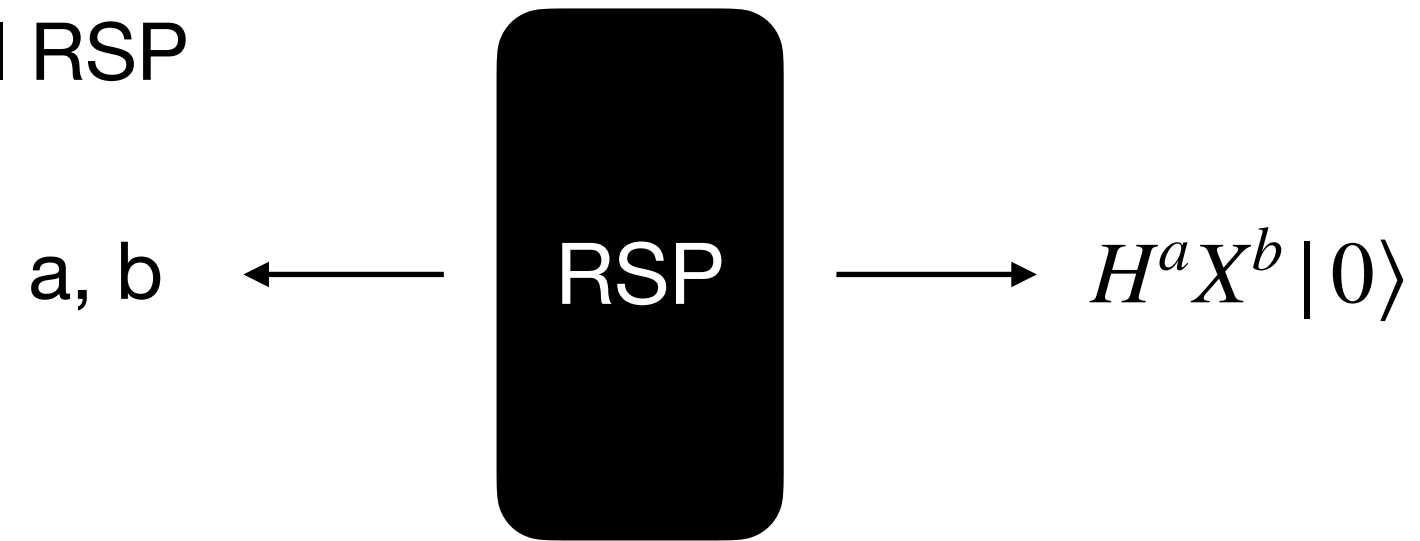
Measure all but one qubit and obtain  $z$

**Security: The bit “a” is a hard-core predicate => Bob cannot guess the bit “a” any better than 1/2**

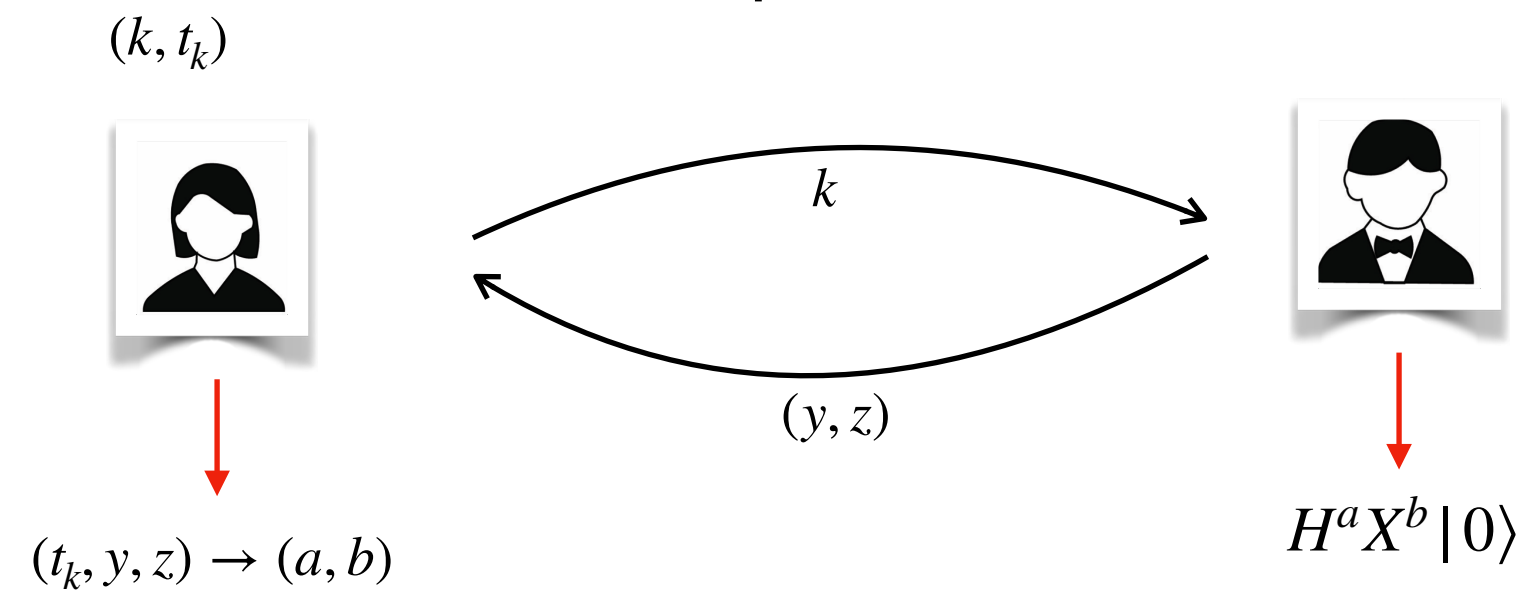
# Abstract Cryptography

A framework for Composable security

Ideal RSP



Concrete protocol RSP

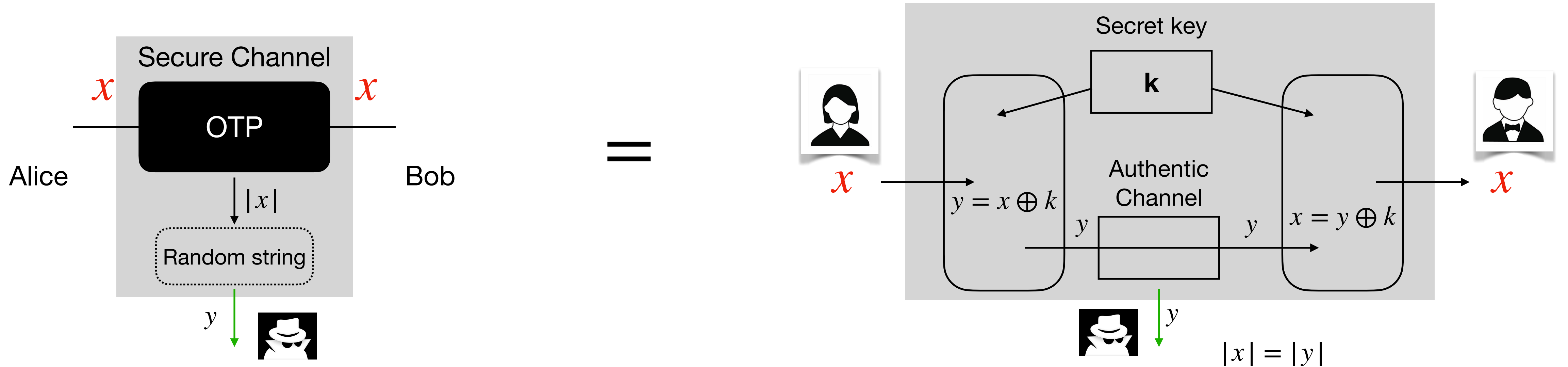


Secure by definition

Cryptography can be regarded as a resource theory!

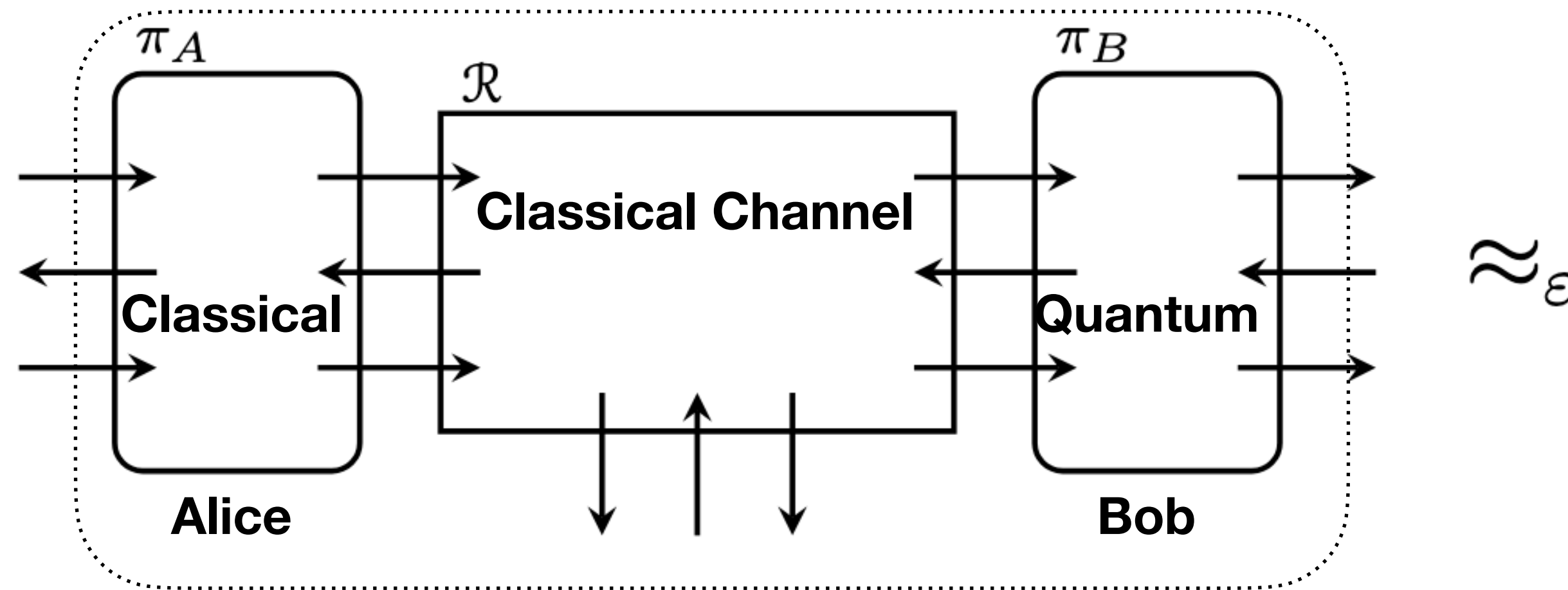
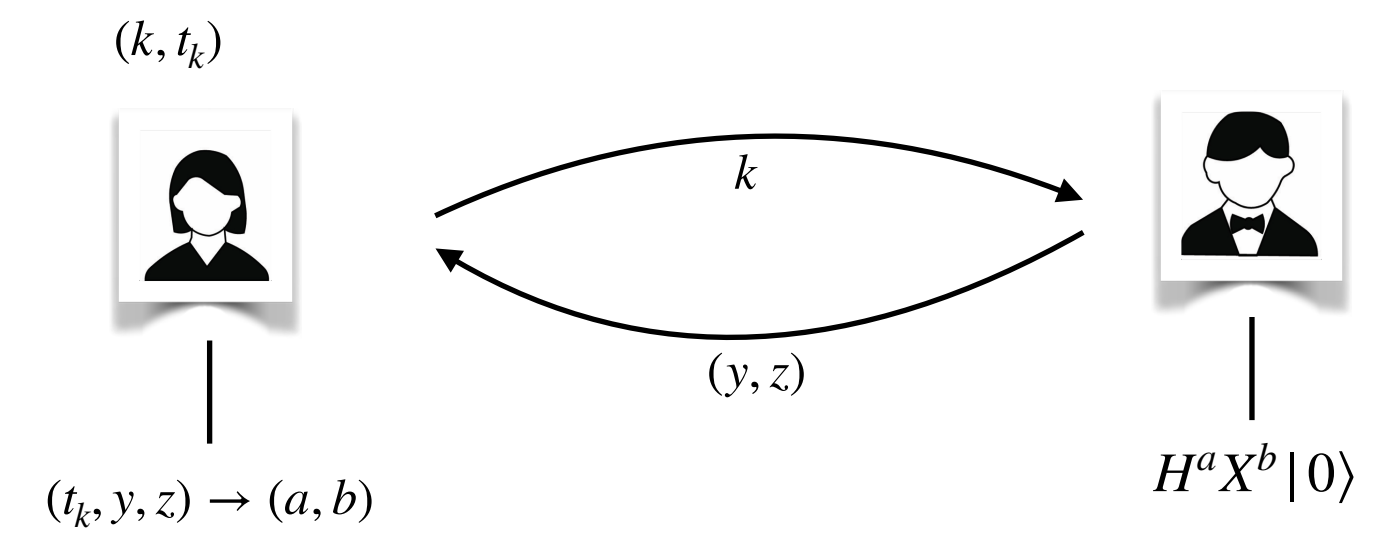
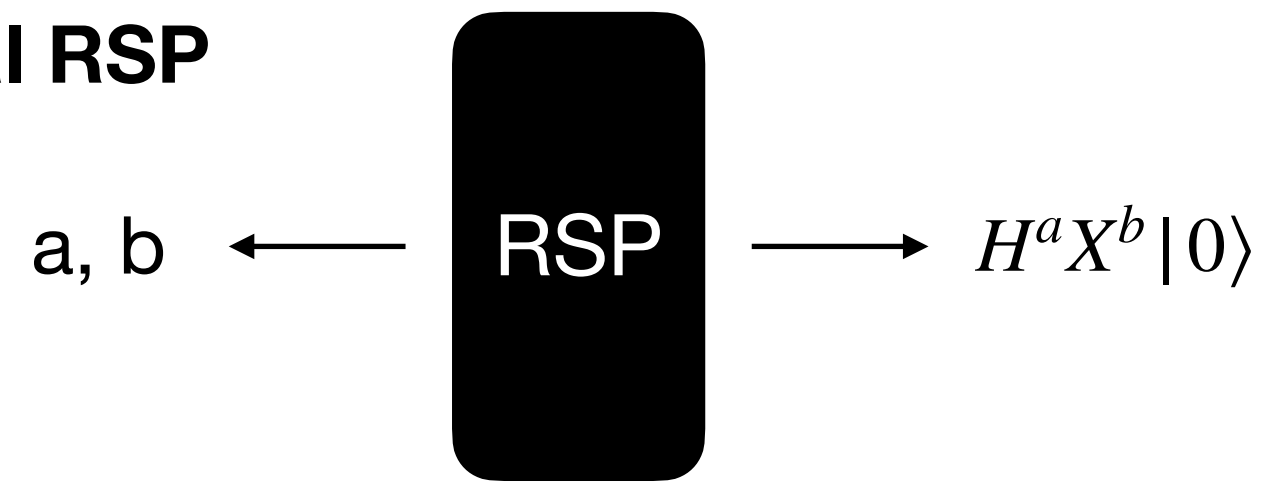
Aim is to construct desired resources from a set of given resources

Example: One-time Pad

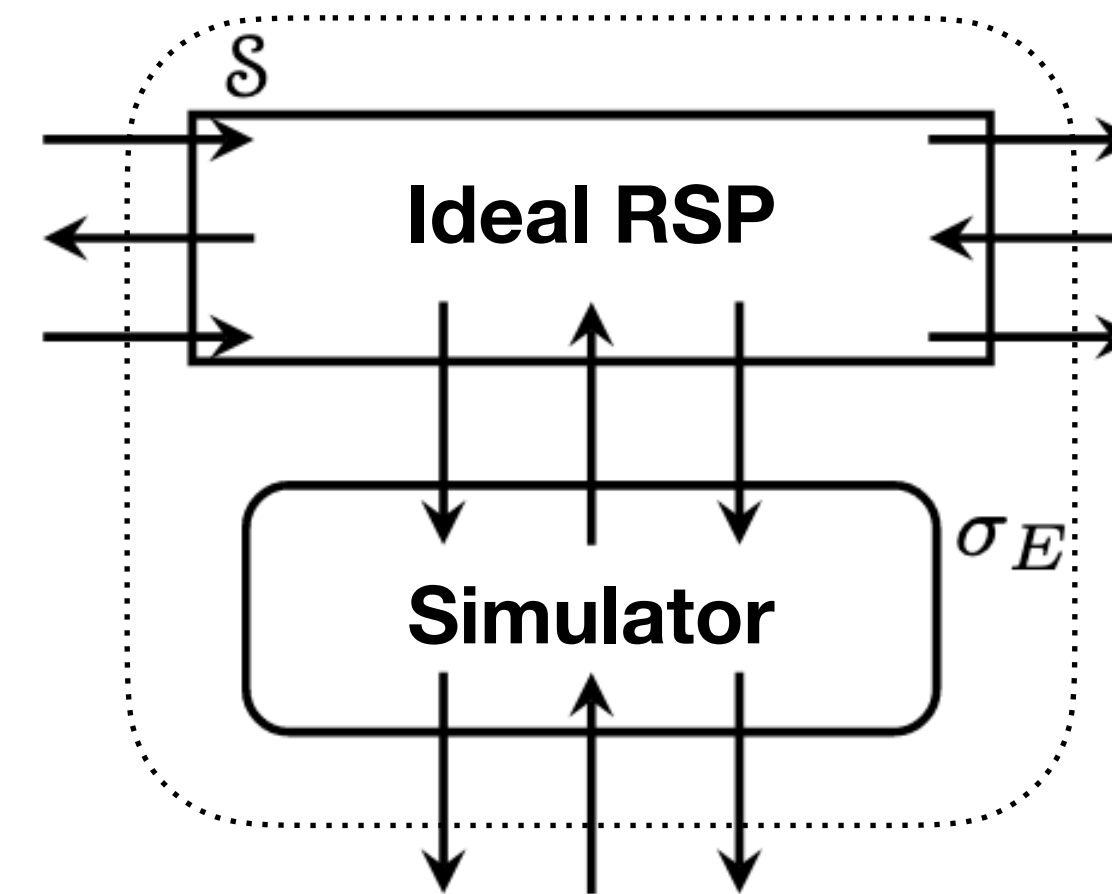


# Security of RSP

Ideal RSP

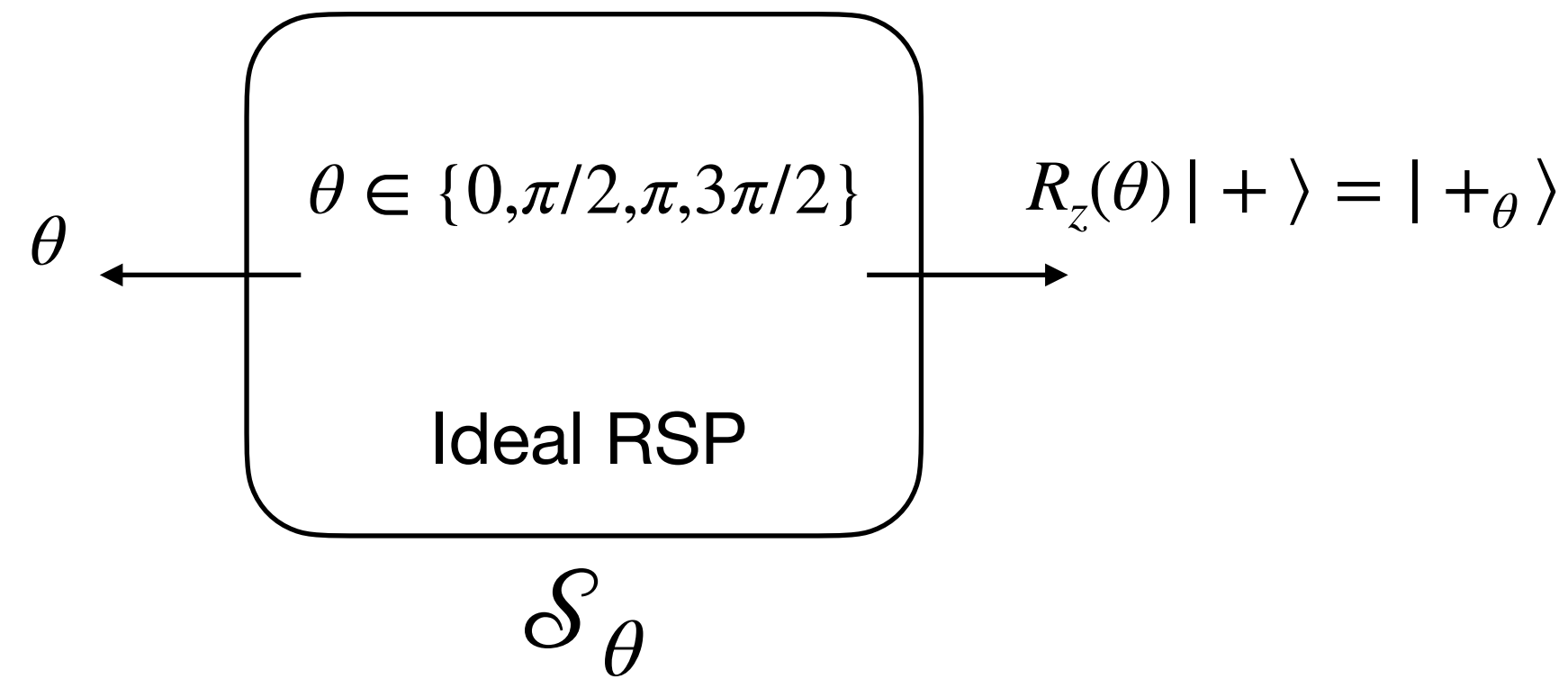


$\approx_\epsilon$

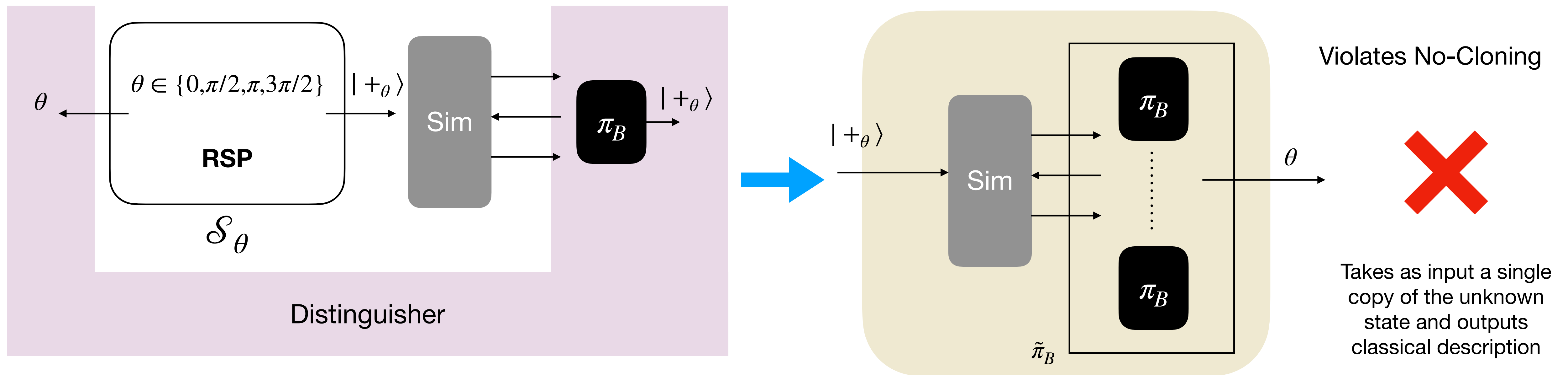


Result: Classical-client RSP protocols **cannot be secure in composable setting.**

# Proof Sketch

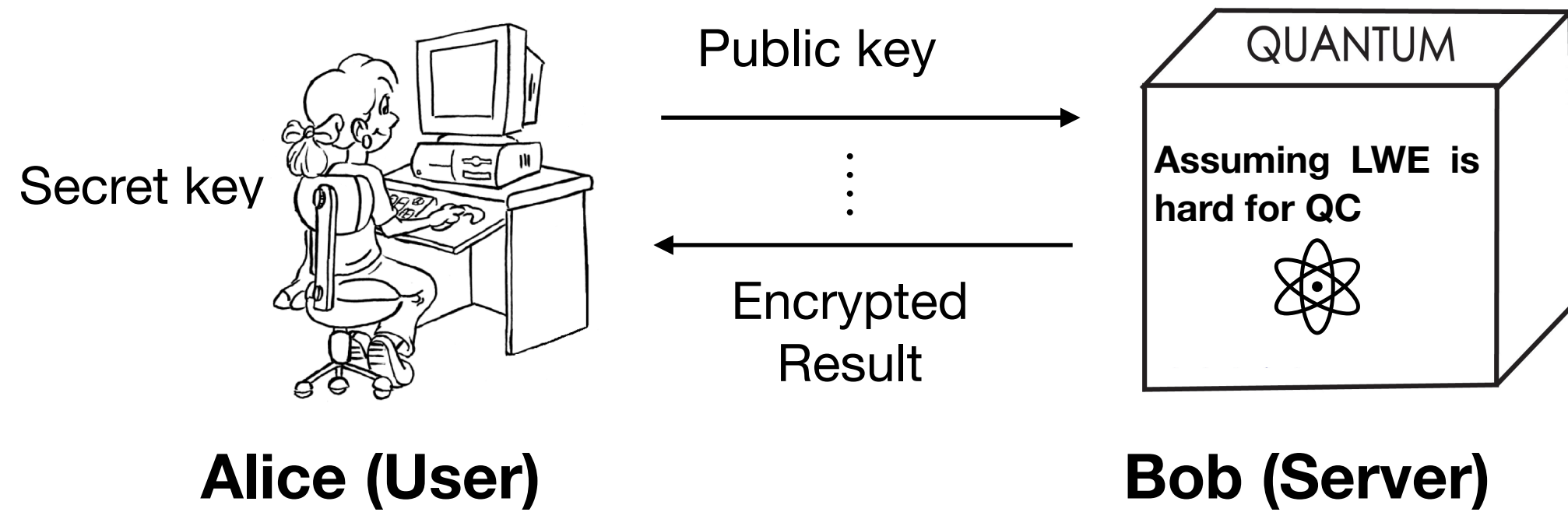


$\pi_B$  : Bob's local protocol



Does that mean RSP is not useful at all?

# Applications



Modular classical client delegation scheme (based on computational assumptions)

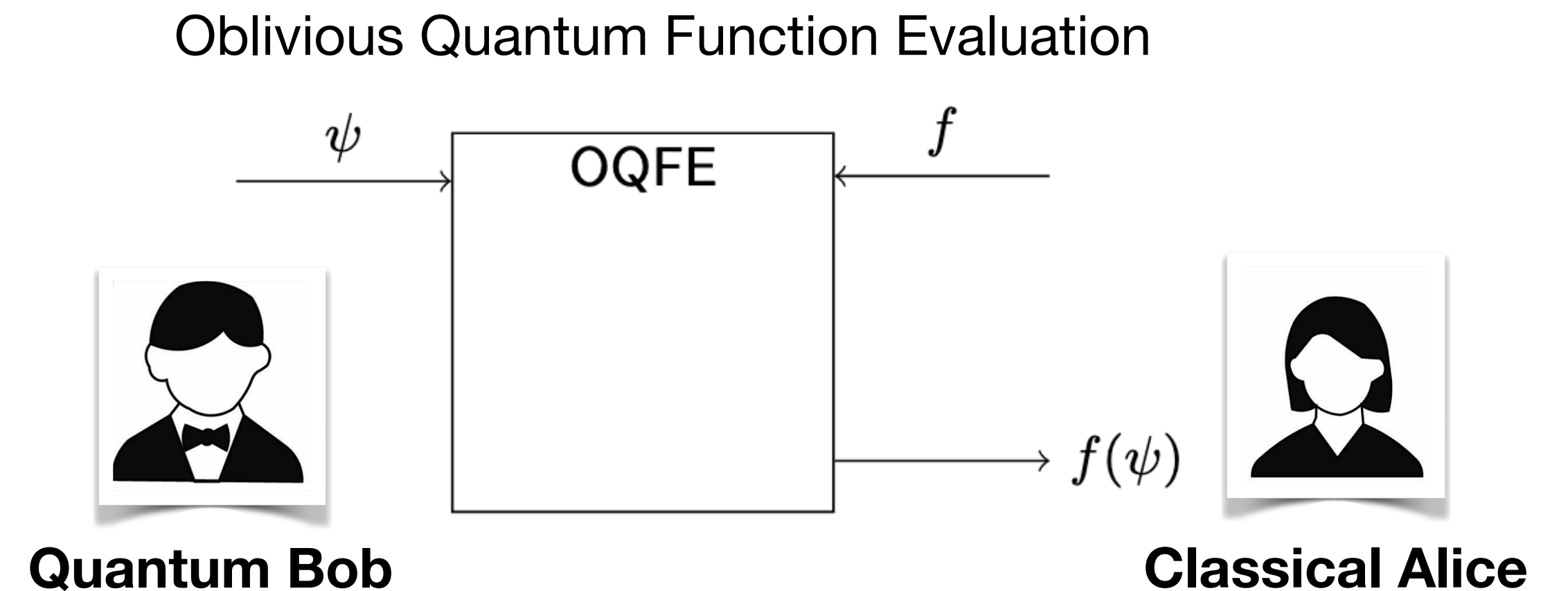
Using [remote state preparation](#) to replace quantum channel in BFK scheme

Assumptions: Trapdoor homomorphic Injective OWFs

Security: Game-based vs composable

**Open Questions:** Composable Verifiable Delegated Quantum Computation? Other relaxations?

## Quantum two-party Computation over Classical Channel

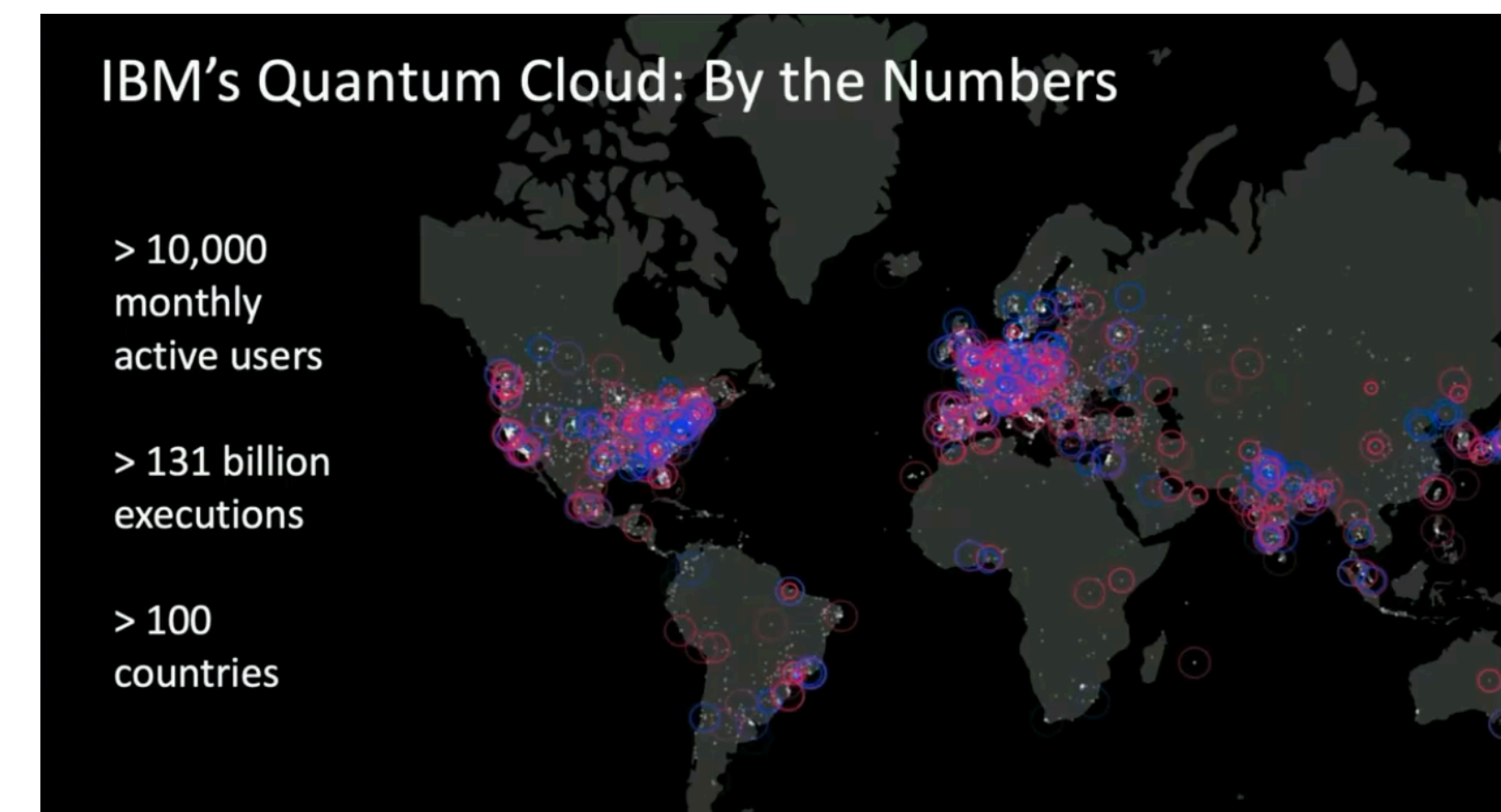
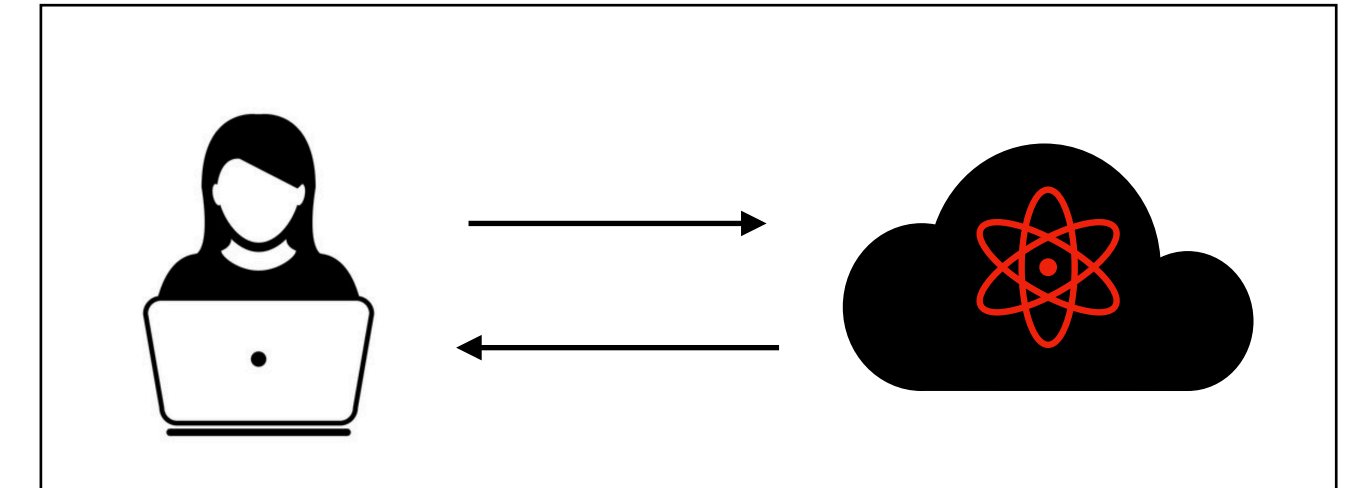


- Construction: Using RSP and ideas from BFK scheme!
- Security: Simulation-based security against Malicious Alice and Privacy against Quantum Bob
- Limitation with fully Black-Box simulation ~ Classical Proofs of Quantum Knowledge
- **Open Questions:** MPC over hybrid classical-quantum networks? Non Black-Box simulation?

# Summary

- Securely delegating quantum functions is indeed possible.
- Tradeoff: Information-theoretic security and Computational Security
  - Perfect Security is possible but requires quantum channel.
  - Protocols based on Classical Networks are possible at the cost of (weaker) security i.e. against Quantum Servers.
  - Open Problems: Other applications of secure remote state preparation?  
Can they be based on weaker cryptographic assumptions?

Quantum Computers are getting distributed around the world and applications/algorithms would require privacy.



Data from early 2020

*Thank you for listening!*