Quantum impurity problem in ultracold gases: from dark solitons to quantum ferromagnets

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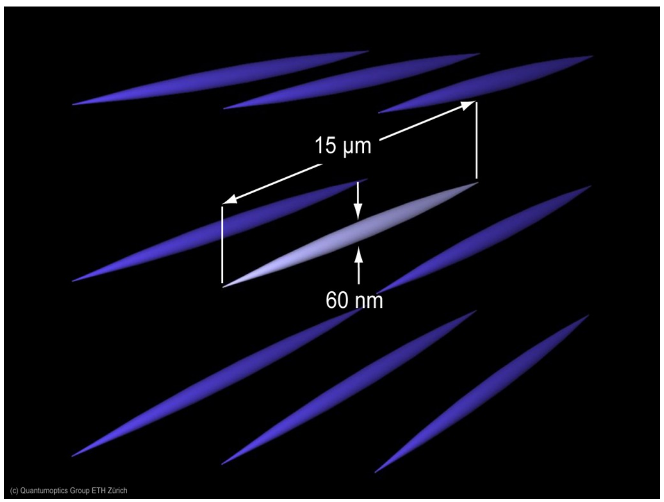
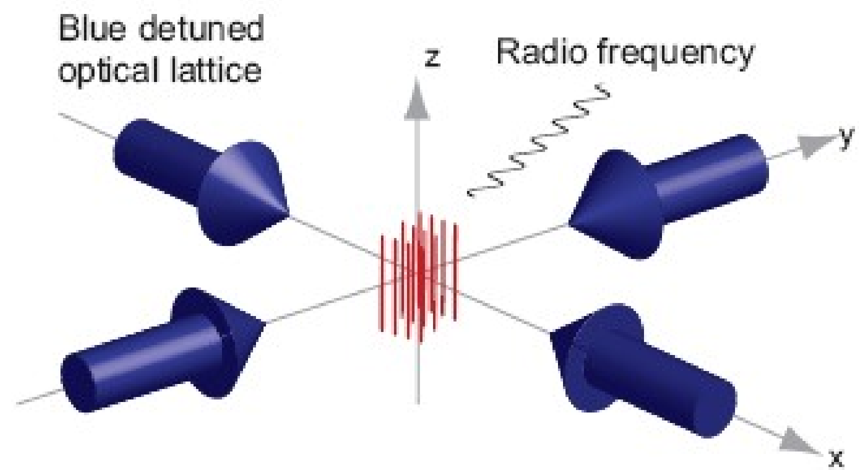
EPSRC

Experiments in 1D





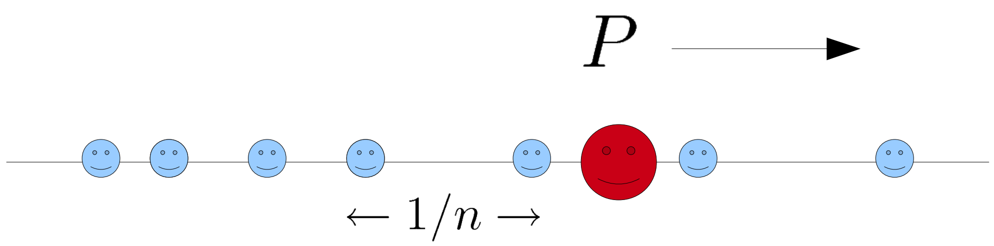
$N \sim 100$ atoms in each tube

1 dimensional regime $\hbar\omega_\perp \gg \mu, T$

Short-range interactions $V_{\text{int}} = g\delta(x_i - x_j)$ $g = 2\hbar a\omega_\perp$

Impurity in 1D

1 impurity of mass M making its way through the liquid....



Questions:

- How to characterise its interactions with background
- Dissipative processes
- Motion under slow external fields (harmonic trap)

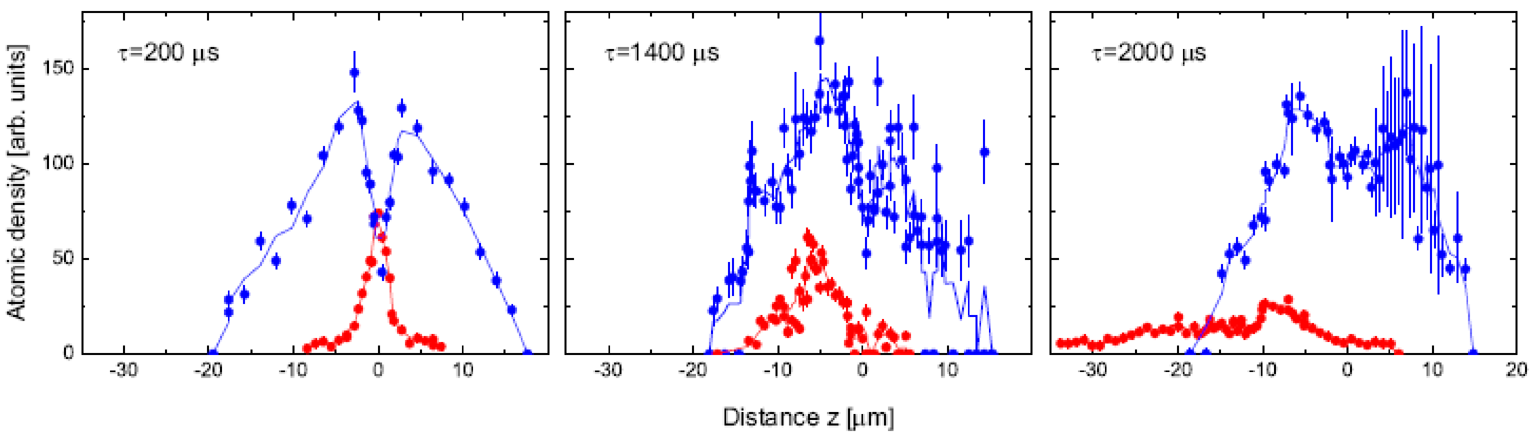
Bose spinor gas

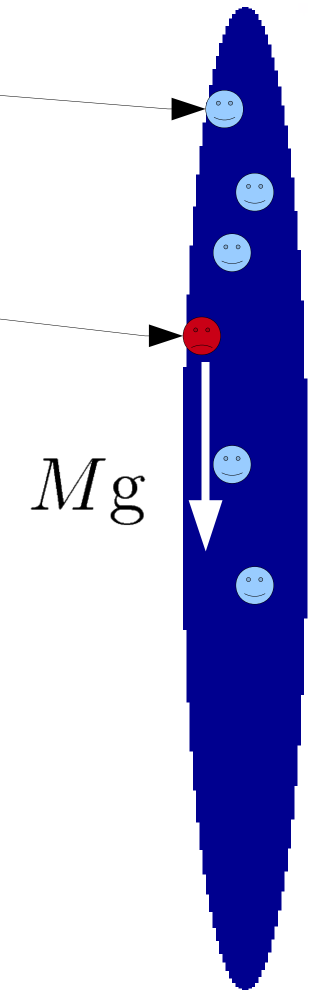
Rb^{87} $\quad |F=1, m_F=-1\rangle \rightarrow |\uparrow\rangle$

Magnetically trapped

$|F=2, m_F=0\rangle \rightarrow |\downarrow\rangle$

Untrapped





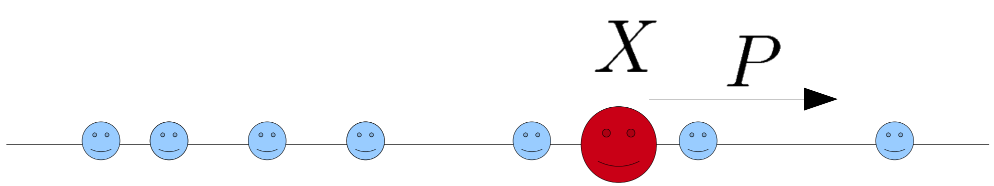
S. Palzer et al. PRL 2009

Constructing effective action

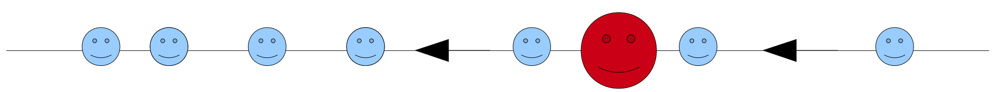
Impurity $P(t), X(t)$ Bose Liquid $\bar{\psi}(x,t), \psi(x,t)$

$$L = P\dot{X} - \frac{P^2}{2M} \qquad - \qquad G|\psi(X,t)|^2$$

$$+ \int dx \left(i\bar{\psi}\partial_t\psi - \frac{1}{2m}\partial_x\bar{\psi}\partial_x\psi - \frac{g}{2}\bar{\psi}\bar{\psi}\psi\psi \right)$$



co-moving frame



$$X = 0$$

$$L = P\dot{X} - \frac{1}{2M}\left(P - P(\bar{\psi},\psi)\right)^2 \quad -G|\psi(0,t)|^2$$

$$+\int dx \left(i\bar{\psi}\partial_t\psi - \frac{1}{2m}\partial_x\bar{\psi}\partial_x\psi - \frac{g}{2}\bar{\psi}\bar{\psi}\psi\psi \right)$$

P is TOTAL momentum

$$P(\bar{\psi},\psi) = \frac{1}{i}\int dx\, \bar{\psi}\partial_x\psi$$

Renormalised collective dispersion

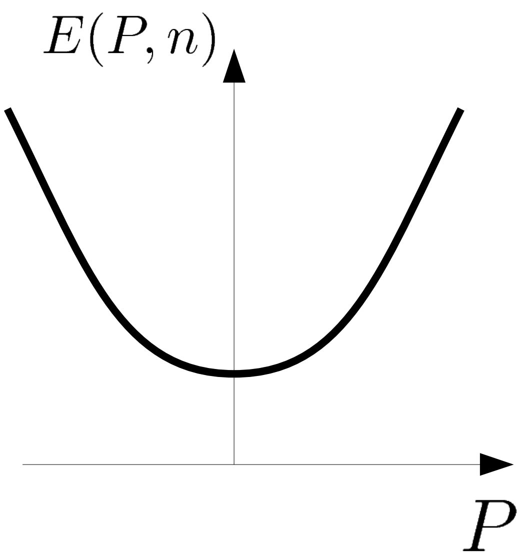
Minimise the total energy of impurity + liquid for given total momentum

ground state of $E(\bar{\psi}, \psi) - VP(\bar{\psi}, \psi)$

Control parameters: velocity V, background chemical pot. μ

$$E(V, \mu) \longrightarrow E(P, n)$$

$$\simeq \varepsilon_0(n) + P^2/2M^*$$



Dispersion periodicity

'superfluid' argument

Transfer momentum quantum $\dfrac{2\pi\hbar}{L}$ to each particle

No change to internal state!

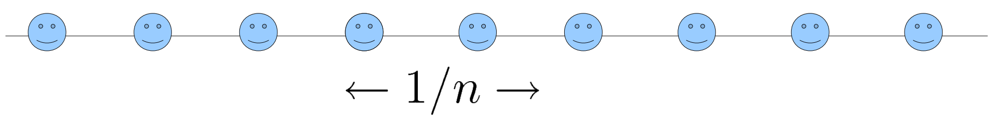
Total momentum $$P = P_{\mathrm{imp}} + N \times \frac{2\pi\hbar}{L} = P_{\mathrm{imp}} + 2\pi\hbar n$$

Total energy $$E(P) = E(P_{\mathrm{imp}}) + N \times \frac{1}{m}\left(\frac{2\pi\hbar}{L}\right)^2$$

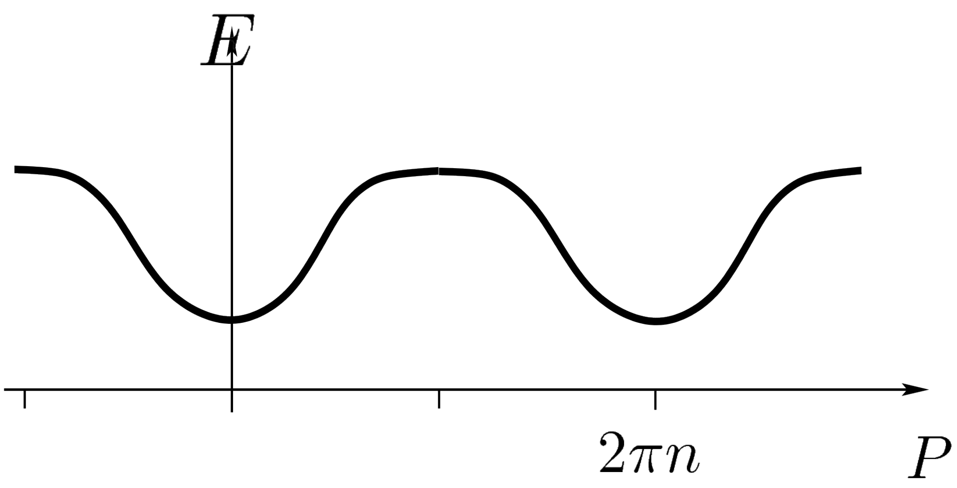
Periodic dispersion $$E(P) = E(P - 2\pi n\hbar)$$

Dispersion periodicity
'umklapp' argument

Assume no (quantum, thermal) fluctuations: perfect lattice



Periodic zone with period $\frac{2\pi\hbar}{a} = 2\pi\hbar n$



$E(P)$ contains contribution of all excitations with

$$k > k_c = \sqrt{m\mu}$$

Phonons

Phase-density representation for slow fields

$$\psi(x,t) = \sqrt{n + \rho(x,t)}\, e^{i\phi(x,t)}$$

Close to equibrium

$$L = P\dot{X} - E(P,n) + \int dx \left(-\rho \partial_t \phi - \frac{mc^2}{2n}\rho^2 - \frac{n}{2m}(\partial_x \phi)^2 \right)$$

+ interactions

Real processes: dissipation and viscous friction

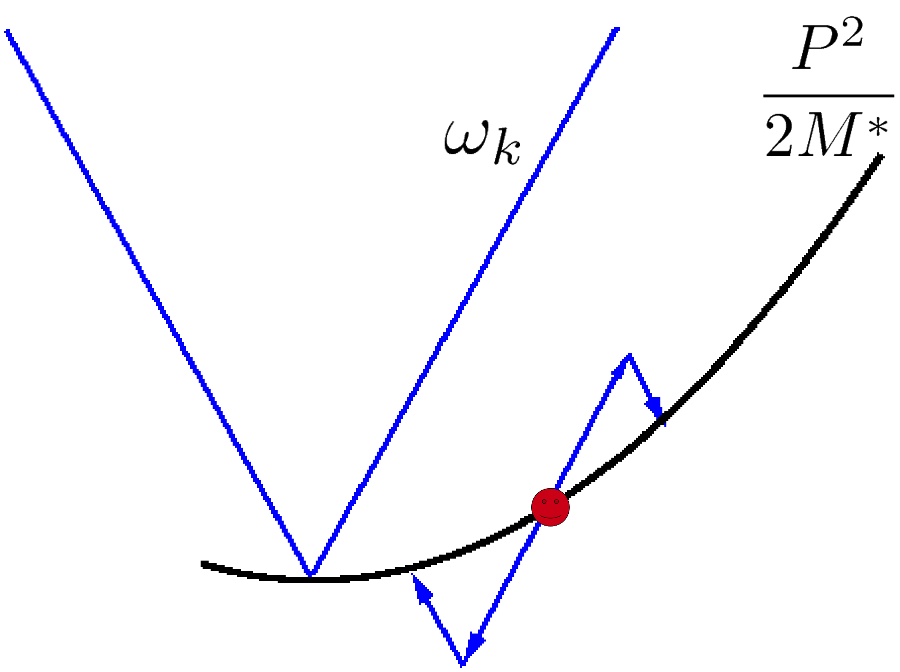
One phonon processes

$$\Delta E \sim c\Delta P - V(P)\Delta P$$

$$V(P) = P/M^* \ll c$$

- Landau criterion

Superfluid at $T = 0$



$T \neq 0$ two phonon processes

Viscosity coefficient

$$\dot{P} = -\kappa V$$

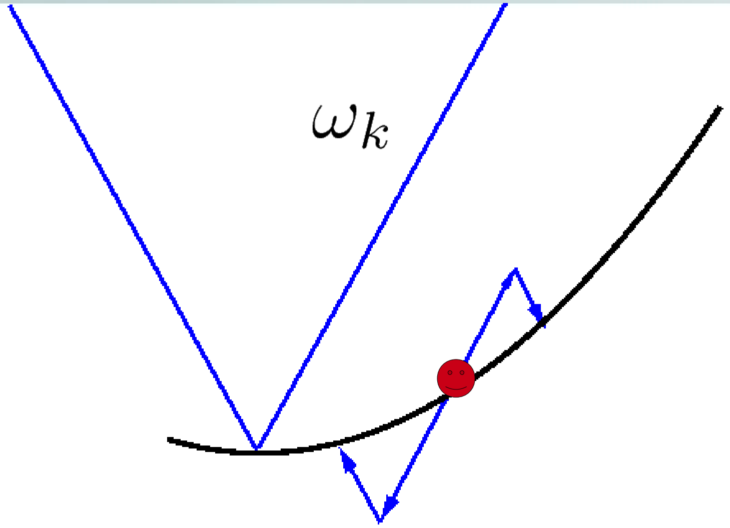
$$\omega_{\max} = c q_{\max} \sim T$$

$$\kappa \sim \int_0^T dq\; q\; q\; q^{2d-1} \sim T^{2d+2}$$

In 3 dimensions $\kappa \sim T^8$

depends crucially on coupling with phonons

In 1 dimension $\kappa \sim (\ldots) \dfrac{T^4}{c^4 \hbar^3 n^2}$



He^3 - He^4

Landau & Khalatnikov, 1949
Baym & Ebner, 1967

Castro Neto & M.P.A Fisher, 1996

Impurity – phonon coupling

Coupling to density

$$E(P, n+\rho(X,t)) = \varepsilon_0(n) + \frac{P^2}{2M} + \frac{\partial \varepsilon_0}{\partial n}\rho + \frac{1}{2}\frac{\partial^2 \varepsilon_0}{\partial n^2}\rho^2$$

Coupling to (super) velocity $v = \frac{\hbar}{m}\partial_x \phi$

Galilean transformation: $P' = P - Mv$

$$E' = E(P, n) - Pv + \frac{Mv^2}{2} =$$

$$= \varepsilon_0(n) + \frac{P'^2}{2M^*} - \left(\frac{M^* - M}{M^*}\right) P'v + \left(\frac{M^* - M}{M^*}\right)\frac{Mv^2}{2}$$

Gauge transformation

$$\rho(x,t) \to \rho(x,t) - R(t)\,\delta(x-X)$$

$$\phi(x,t) \to \phi(x,t) - F(t)\,\theta(x-X)$$

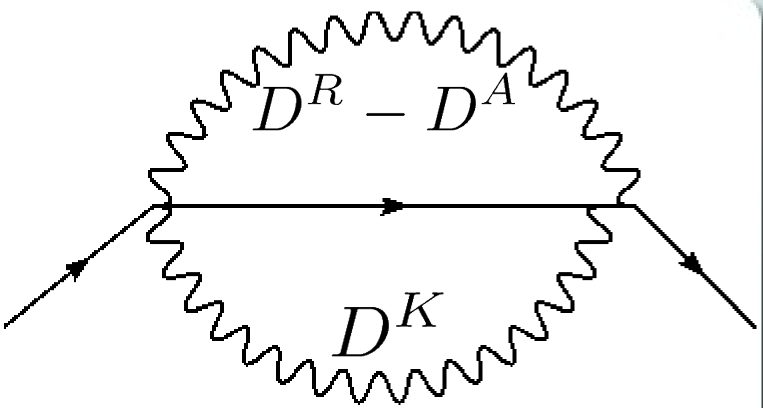
$$P \to P + mRu(X,t) + F\rho(X,t)$$

Choose $R(t), F(t)$ to eliminate linear couplings:

$$H_{\mathrm{imp-ph}} = \frac{1}{2}\Gamma_\rho\left[\rho(X)\right]^2 + \frac{1}{2}\Gamma_\phi\left[\partial_x\phi(X)\right]^2$$

Integrating out phonons

$$\dot{P} = -\kappa V$$



$$\kappa = \mathrm{Tr}\left[\Gamma\left(D^A - D^R\right)\Gamma D^K\right]$$

$$= \frac{1}{4}\left(\Gamma_\rho - \frac{m^2c^2}{n^2}\Gamma_\vartheta\right)^2 \int \frac{dq}{2\pi}\, q\,\Pi(q, qV)$$

Quantum interference – possible cancellation

$$\Pi(q,\omega) = \frac{n^2}{4m^2c^3}\left(q^2 - \frac{\omega^2}{c^2}\right)\left(\coth\frac{cq-\omega}{4T} - \coth\frac{cq+\omega}{4T}\right)$$

Effective coupling

$$\Gamma_\rho - \frac{m^2c^2}{n^2}\Gamma_\phi \sim \left(N_s - \frac{1}{g}\frac{\partial E_s}{\partial n_0}\right) + \left(N_s - \frac{M_s^*}{m}\right)$$

Number of particles expelled by the impurity

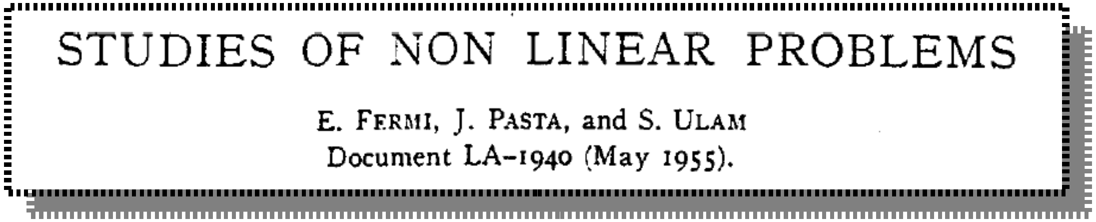
$$R = N_s = -\int (|\psi|^2 - n_0)dx$$

Integrable cases $\frac{\partial E_s}{\partial \mu} = N_s \;\; M_s^* = mN_s$

Integrability ?

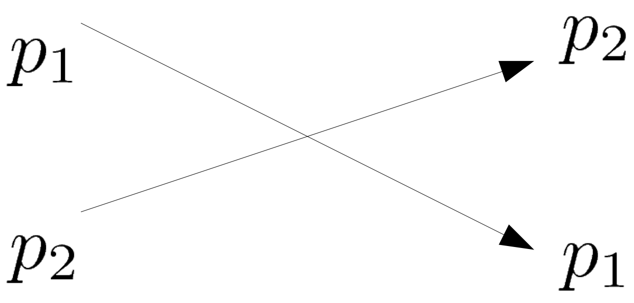
Mechanism of relaxation?

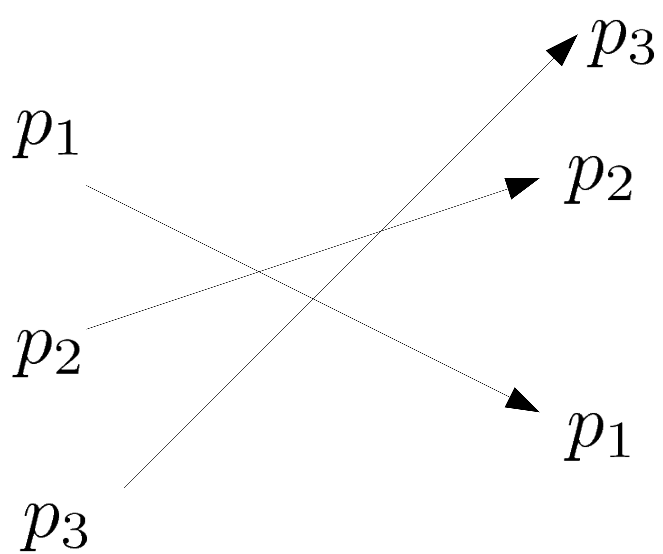
~~3-body collisions~~



STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM
Document LA-1940 (May 1955).





No dissipation

No decay of supercurrents

Even at $T \neq 0$

Dark Soliton

Phase drop θ_s:

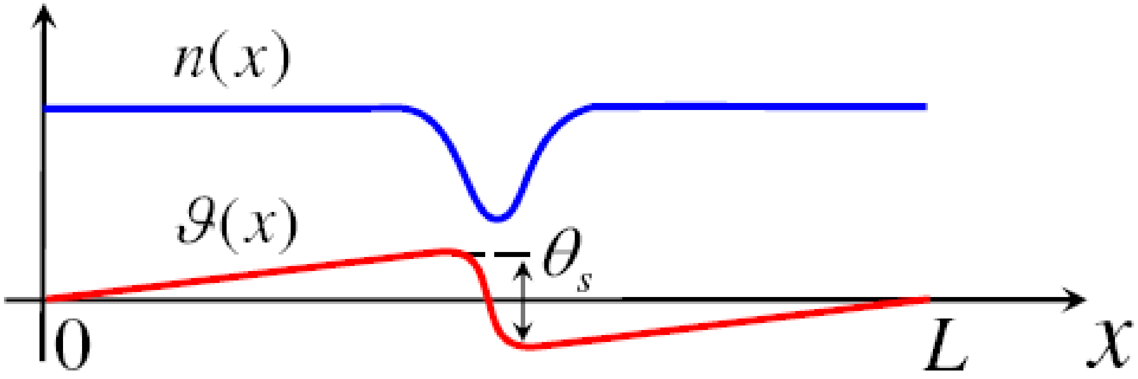
$$\cos\frac{\theta_s}{2} = \frac{V}{c}$$

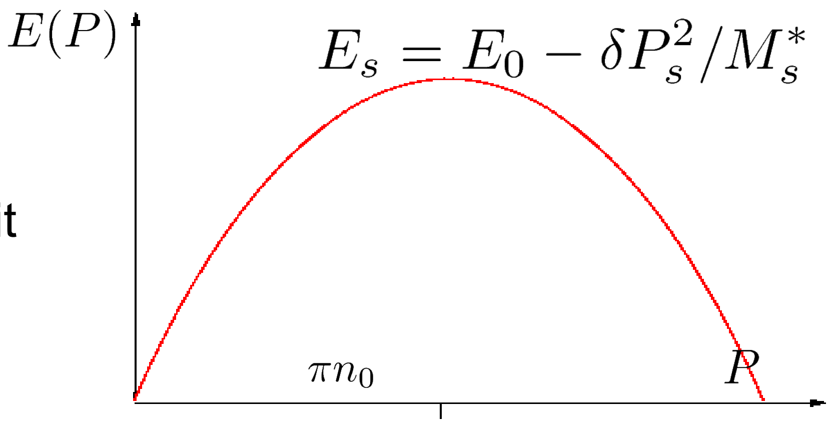
Number of part. expelled

$$N_s = \frac{2n}{mc}\sin\frac{\theta_s}{2}$$

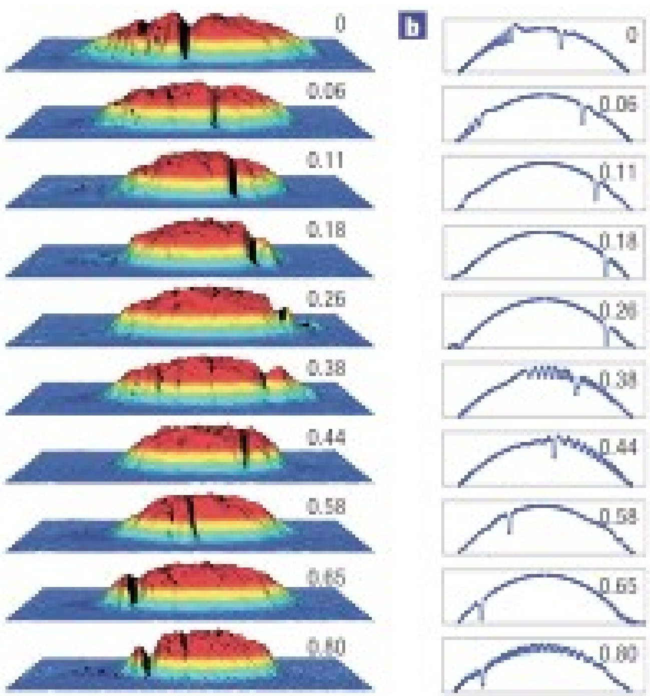
$E(P,n)$ can be obtained in the limit

$$G \to 0, M \to 0$$

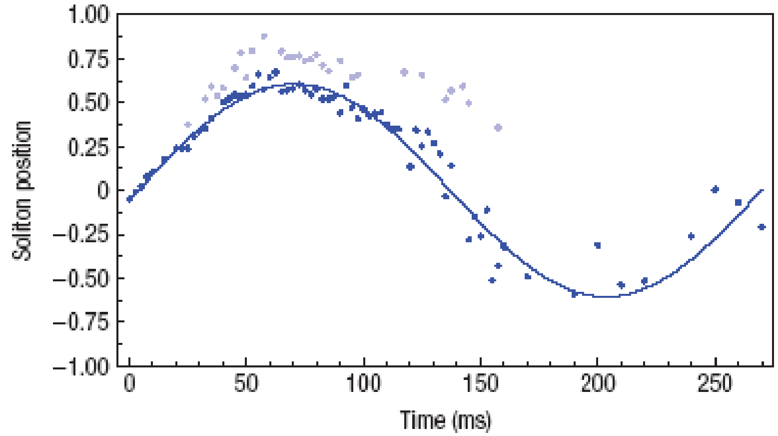




Experiments. Life-Time



C. Becker *et al.*, Nature Physics **4**, 496 (2008)



Lifetime ~300ms

- Finite lifetime due to background fluctuations (phonons)
- Absent at T=0 (Landau criterion)
- Absent for integrable case

1D fermions with spin ½

(Gaudin '67, Young '67)

$$H_{\text{int}} = u \int dx\, \psi^\dagger_\uparrow \psi^\dagger_\downarrow \psi_\downarrow \psi_\uparrow$$

Spin -1 excitation above a (metastable) ferromagnetic state

$$\kappa = 0 \quad \Rightarrow \quad \frac{m}{m^*} = \frac{\pi p_F}{m} \frac{1}{\frac{\partial}{\partial n}(\mu - \mu_d)} + \frac{p_F^2}{m} \frac{\frac{\partial^2}{\partial n^2}(\mu - \mu_d)}{\left(\frac{\partial}{\partial n}(\mu - \mu_d)\right)^2}$$

$$\mu - \mu_d = \frac{p_F u}{2\pi} \left[\left(\frac{2p_F}{mu} + \frac{mu}{2p_F} \right) \arctan \frac{2p_F}{mu} - 1 \right]$$

Castella, Zotos '93

$$\frac{m}{m^*} = \frac{\pi}{2 \arctan^2 \frac{2p_F}{mu}} \left(\arctan \frac{2p_F}{mu} - \frac{\frac{2p_F}{mu}}{1 + \left(\frac{2p_F}{mu}\right)^2} \right)$$

Perturbation around integrability

Weakly interacting bosons $\mu = gn \qquad \mu_d = Gn$

$$\kappa = \frac{16\pi^3}{15} \frac{T^4}{c^4\hbar^3 n^2} \left(\frac{G}{g}\right)^2 \left(\frac{mG}{Mg} - 1\right)^2 \qquad M^* = M$$

$$\mathrm{Rb}^{87} \quad G/g \sim 1.05$$

Strong interactions $m \neq M \qquad g, G \to \infty$

$$\kappa = \frac{16\pi^3}{15} \frac{T^4}{c^4\hbar^3 n^2} \left[\left(\frac{m}{M}\right)^{1/3} - \frac{M}{m}\right]^2$$

$$m = M \qquad g, G \gg \hbar^2 n/m$$

$$\kappa \sim \frac{\pi^3 T^4}{c^4\hbar^3 n^2} \left(\frac{\hbar^2 n}{gm}\right)^4 \left(1 - \frac{g}{G}\right)^2$$

Concluding remarks

- Impurity in 1D – model to study nonequilibrium many body quantum dynamics.
- Universal coupling to phonons
- Dissipation is important. Mechanism of dissipation
- Sensitive to parameters. Absence of dissipation for integrable systems even at nonzero temperature
- Can be applied to study excitation dynamics in nearly integrable systems. Lifetime of quasiparticles
- Experiments....