# To split or not to split? An investigation using computational complexity and game theory

**Haris Aziz**

(joint work with Mike Paterson)

An important aspect of mechanism design in social choice protocols and multi-agent systems is to discourage insincere behaviour. Manipulative behaviour has received increased attention since the famous Gibbard-Satterthwaite theorem. In recent years, theoretical computer scientists have utilized computational complexity as a barrier to manipulation. We examine the computational complexity of manipulation in weighted voting games which are ubiquitous mathematical models used in economics, political science, neuroscience, threshold logic, reliability theory and distributed systems. They model situations where agents with variable voting weight vote in favour of or against a decision. A coalition of agents is winning if and only if the sum of weights of the coalition exceeds or equals a specified quota.

Splitting of a player into sub-players can be seen as a false name manipulation by an agent where it splits itself into more agents so that the sum of the utilities of the split-up players is more than the utility of the original player. We examine situations when a player splitting up into smaller players may be advantageous or disadvantageous in the context of weighted voting games and Banzhaf indices. We provide tight limits to how much the Banzhaf index of a player can increase or decrease if it splits up into sub-players. Surprisingly, it is found that the limits are much different from the previously examined limits for the Shapley-Shubik index.

From a computational perspective, it is #P-hard for a manipulator to find the ideal splitting to maximize his payoff. A prospective manipulator could still be interested in enabling a beneficial split even if the improvement in payoff is not high. We prove that it is NP-hard to even decide whether a split is beneficial or not. In the end a pseudo-polynomial algorithm is proposed which returns 'no' if no beneficial split is available and the returns the optimal split otherwise.

Haris Aziz & Mike Paterson

Computer Science Department, University of Warwick

{Haris.Aziz, M.S.Paterson}@warwick.ac.uk