Solving Mean Payoff Parity Games With Strategy Improvement

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May 30, 2008

Two player games played on finite graphs have attracted much interest in the formal methods community. It has been shown that the problem of model checking the modal μ-calculus is equivalent to the problem of solving a two player parity game [2]. In these games, each vertex is assigned an integer priority and the two players attempt to ensure that the minimum priority occurring infinitely often is either odd or even. Much effort has been expended in an attempt to find an algorithm which solves parity games in polynomial time. However, no such algorithm has been found. One approach that has attracted much attention is strategy improvement [4]. Strategy improvement algorithms terminate in polynomial time for all known input instances. However, no one has yet been able to prove that they terminate in polynomial time for all parity games.

Mean payoff games are a second example where strategy improvement techniques can be applied. In a mean payoff game, each vertex is assigned a payoff and the two players attempt to maximize or minimize the limit average reward of an infinite play. In this case the optimality equations described by Puterman [3] can be adapted for a two player setting, and it can be shown that strategy improvement will terminate with a solution to the optimality equations. As with parity games, the precise complexity of strategy improvement for mean payoff games is unknown.

Mean payoff parity games are a combination of both mean payoff games and parity games. Each vertex in the graph is assigned both a priority and a payoff. The two players are referred to as the maximizer and the minimizer. The result of an infinite play can be determined as follows: if the play is parity losing for the maximizer then he loses an infinite amount and the minimizer wins an infinite amount, if the play is parity winning then the maximizer is awarded with the limit average reward of the infinite play.

Mean payoff parity games have previously been studied by Chatterjee, Henzinger and Jurdziński [1]. They showed that optimal strategies for the maximizing player may require infinite memory whereas finite memory strategies are sufficient to prove optimality for the minimizing player. They also provide an algorithm that solves the games in $O(n^d \cdot (m + \text{MP} + \text{Parity}))$, where d denotes the number of priorities in the parity game, MP is the complexity of solving a mean payoff game and Parity is the complexity of solving a parity game.

In this work, we apply the principles of strategy improvement in an attempt to solve mean payoff parity games. We will discuss the difficulties of forming optimality equations in games where both players do not posses memoryless optimal strategies. We will show how our methods could lead to proofs that the minimizing player always has a memoryless optimal strategy, and that the complexity of algorithms using our methods is likely to be expressible in terms of the complexity of strategy improvement algorithms on parity and mean payoff games.

References

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