Hybrid games for controller synthesis

Michał Rutkowski

June 2, 2008

Hybrid systems and automata. Systems that exhibit both discrete and continuous behaviour are referred to as *hybrid systems*. Continuous changes to the system's state are interleaved with discrete ones, which may alter the constraints for future continuous behaviours. *Hybrid automata* are a formalism for modeling hybrid systems. Hybrid automata are finite automata augmented with continuous real-valued variables. The discrete states can be seen as modes of execution, and the continuous changes of the variables as the evolution of the system's state over time. The mode specifies the continuous dynamics of the system, and mode changes are triggered by the changes in variable's values.

Controller synthesis. The designer of the system often lacks full control over its operation. The behaviour of the system is a result of an interaction between a controller and the environment. This gives rise to the *controller synthesis* problem, where the goal is to design a program such that, regardless of the the environment's behaviour, the system behaves correctly and optimally. Game-based approach to the controller synthesis problem was first proposed by Church, and we apply it to hybrid automata. There are two players, the *controller* and the *environment*, and they are playing a zero-sum game on the hybrid automaton.

In order to model the performance of the system, the hybrid automaton is augmented with a price function. The price function assigns a price to the change of systems state. The prices of individual changes contribute to the payoff of the game. The goal of controller is to first guarantee correctness, and then to minimise the payoff. Conversely, the goal of the environment is to cause the system to behave incorrectly, and if that is not possible, to maximise the payoff. This problem is further referred to as *reachability-price*.

When correctness is less important, and we are interested purely in the performance criterion, then *average-price* games are considered. In these games, the players play indefinitely, and the payoff of the game, is the average price per change of state.

Contributions. We are considering a subclass of hybrid automata: hybrid automata with strong resets. The term "strong resets" comes from the property of the system that all the continuous variables are non-deterministically reset after each mode change.

For controller synthesis, only *reachability-price* games were studied so far, and only restricted price functions were considered. We extend the previous results to arbitrary price functions. Moreover, we show decidability of solving average-price games.