Shear bands in dense granular flow: Towards a local rheology
Effects of friction, softness, cohesion

Kuniyasu Saitoh, Abhinendra Singh,
Vanessa Magnanimo, and Stefan Luding

Multi Scale Mechanics (MSM), Faculty of Engineering Technology, MESA+, University of Twente, The Netherlands

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Geophysics, engineering, and science

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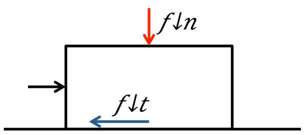
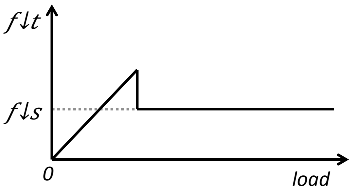
Rigid body

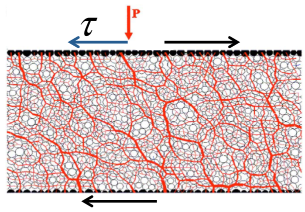


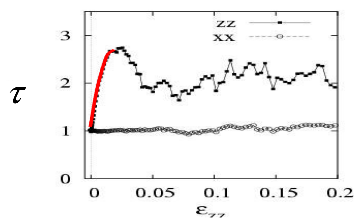


Sliding friction

$$f_s = \mu f_n$$

Dense granular flow





Shear stress

$$\tau = \mu(I)P$$

macroscopic friction

Motivation

Constitutive relation

$$\tau = \mu(I)P$$

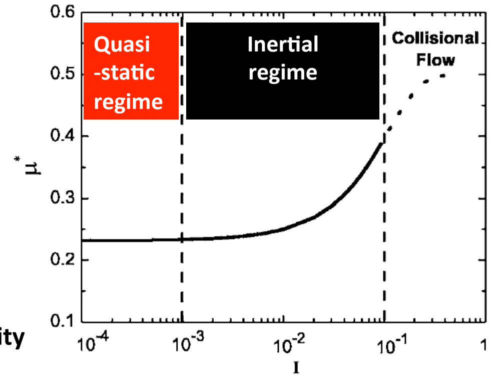
Inertial number $I \equiv \dot{\gamma}\tau_P$

$$\tau_P \equiv \sqrt{m/Pd}$$

shear rate	$\dot{\gamma}$
particle diameter	d
mass density	ρ

… for rigid particles & constant gravity

[F. da Cruz, et. al, PRE 72 (2005) 021309]



Aim of this study

Effects of particle **stiffness/softness & gravity**
as well as **particles' friction & cohesion**

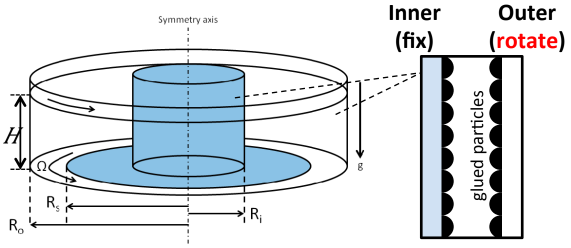
Method

Scaling units

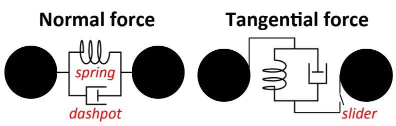
Mass & Length: The mean particle mass ($\bar{m}$) & diameter ($\bar{d}$)
Time: $t_0 \equiv \overline{m}/\eta_n$)

Cyclic shear geometry

split bottom	$R_s = 40\bar{d}$
inner wall	$R_i = 0.2R_s$
outer wall	$R_o = 1.3R_s$
filling height	$H = 0.4R_s$



Discrete element method (DEM)

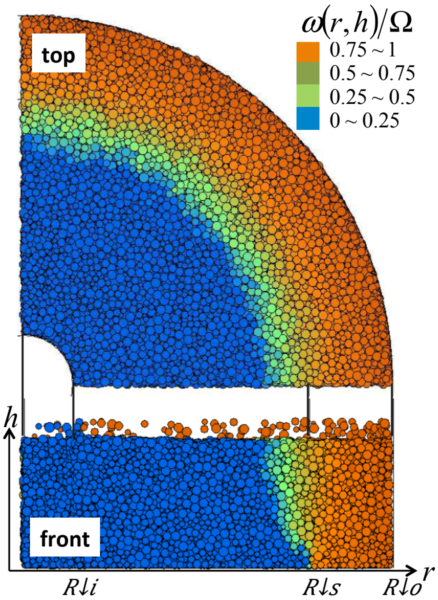


stiffness	$k_n = 10 \sim 10^4$	$k_t = 2k_n/7$
viscosity	$\eta_n = 1$	$\eta_t = \eta_n/4$
friction	$f_s = -\mu_p\lvert f_n\rvert$	$\mu_p = 0.01$
	$\lvert f_t\rvert > \mu_p\lvert f_n\rvert$	※in scaling units

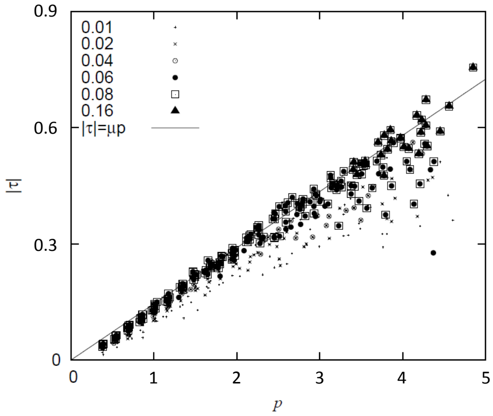
$N = 3.7 \times 10^4$ poly-dispersed particles
(homogeneous size distribution, $d_{min}/d_{max} = 1/2$)

Shear band

Scaled angular speed



Constitutive relation



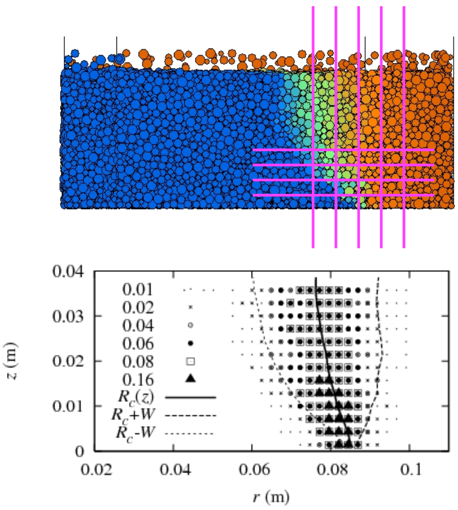
Local quantities in a cylindrical coordinate

shear rate $\dot{\gamma} = \dot{\varepsilon}_{r\theta}$

shear stress $\tau = \sigma_{r\theta}$

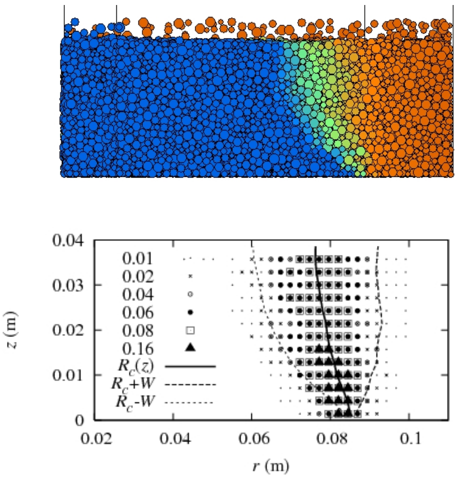
pressure $p = (\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{hh})/3$

Constitutive relations – shear rate $\dot{\gamma}$

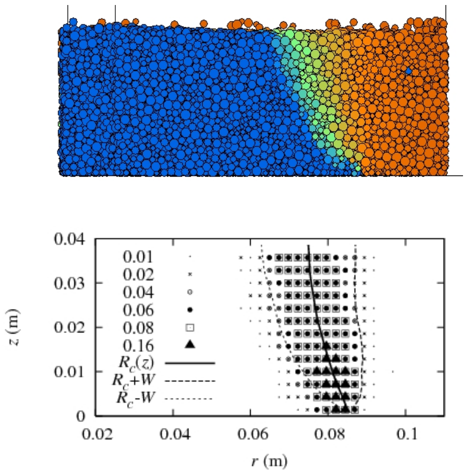


no friction

Constitutive relations – shear rate $\dot{\gamma}$



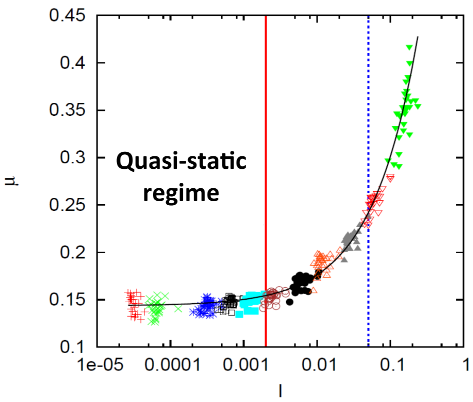
no friction



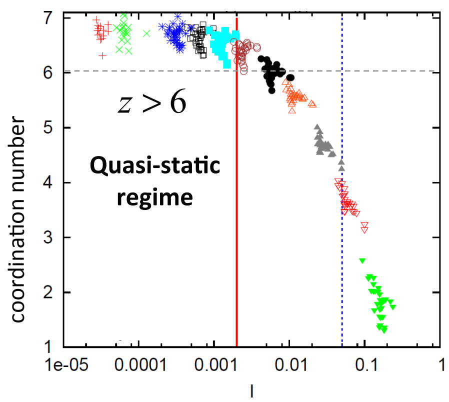
friction

Rigid particles

Friction coefficient



Coordination number

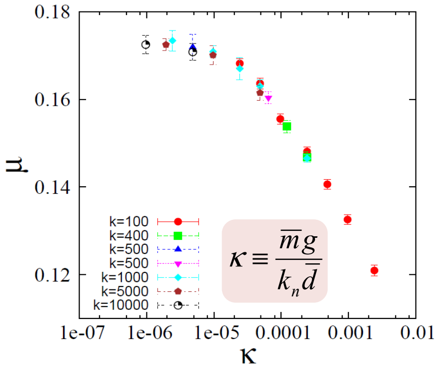


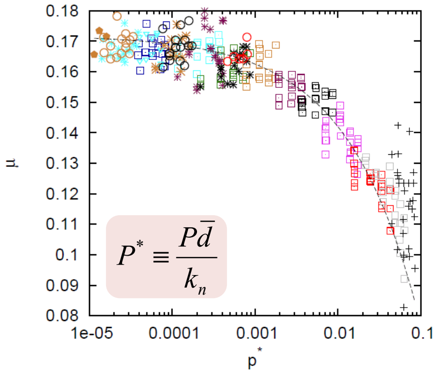
Friction coefficient

Fitting $\mu(I) = \mu_0 + aI^{\alpha}$ $\alpha \approx 1$

Dependence on stiffness and gravity

In quasi-static regime, $\mu(I) \approx \mu_0$



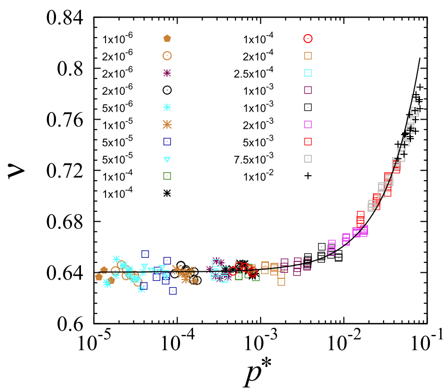


Friction coefficient **in quasi-static regime**

Fitting $\mu(P^*) = \mu_0 - bP^{*\beta}$ $\beta \approx 0.50$

Density dependence on stiffness and gravity

In quasi-static regime, $\nu(I) \approx \nu_0$



Volume fraction **in quasi-static regime**

Fitting $\nu(P^*) = \nu_0 + bP^{*\beta}$ $\beta \approx 0.50$

Macroscopic friction - rheology

Time scales

$$\tau_s = 1/\dot{\gamma} \qquad \tau_c = \sqrt{m/k_n}$$

$$\tau_g = \sqrt{\bar{d}/g} \qquad \tau_P = \sqrt{m/P\bar{d}} \qquad \kappa = \tau_P/\tau_g$$

Dimensionless numbers

Inertial number $\quad I = \tau_P/\tau_s$

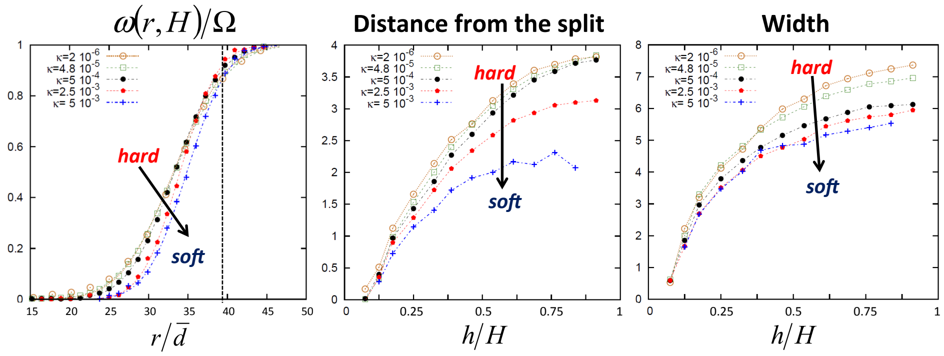
"Softness" parameter $\quad P^* = (\tau_c/\tau_P)^2$

Friction coefficient

$$\mu(I) = \mu_0 + aI + \ldots$$

$$\mu(P^*) = \mu_0 - b\left(P^*\right)^{1/2}$$ in quasi-static regime

Effect on shear band



Angular velocity profile

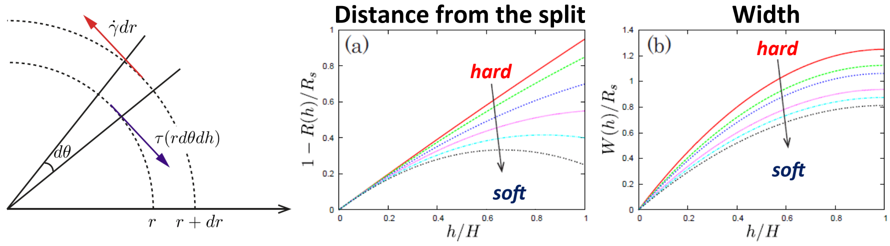
$$\omega(r,h) = \frac{\Omega}{2}\left[1 + \mathrm{erf}\left(\frac{r - R(h)}{W(h)}\right)\right]$$

Position $R(h)$ *Width* $W(h)$

Distance from the split bottom $R_S - R(h)$

[D. Fenistein & M. van Hecke, Nature 425 (2003) 256]

The least dissipation principle



Shear stress

$$\tau = \mu(I, P^*)P$$

Pressure

$$P = \rho g(H - h)$$

Friction coefficient

$$\mu(I, P^*) = \mu_0 + aI^\alpha - bP^{*\beta}$$

Dissipation function density

$$\phi = \frac{\tau\dot{\gamma}}{2} = \frac{\tau r}{2}\frac{\partial}{\partial r}\omega(r,h)$$

error-function

Energy dissipation rate

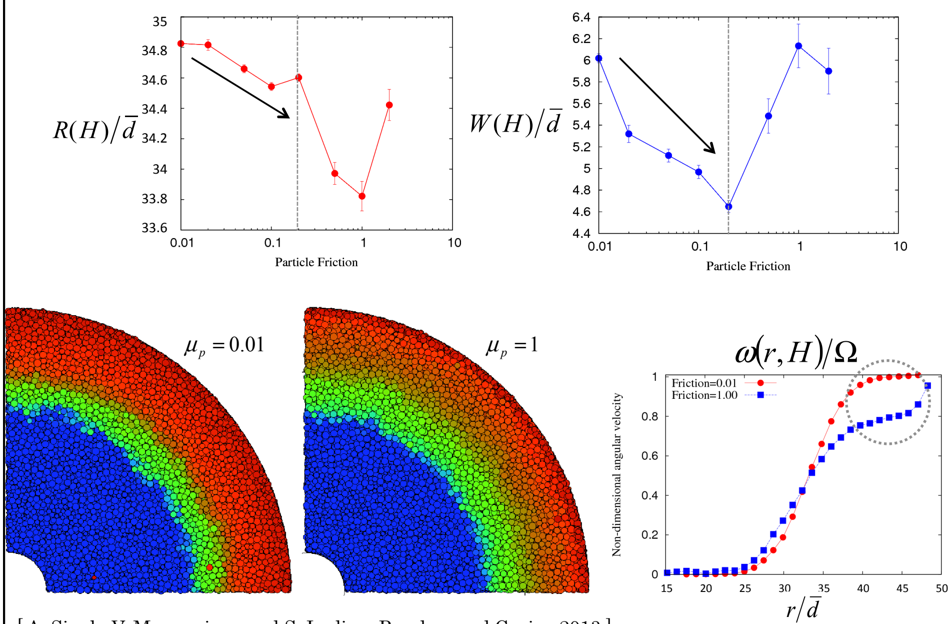
$$\dot{E} = -2\int_V \phi \, dV$$

The least dissipation

$$\frac{\delta \dot{E}}{\delta R(h)} = \frac{\delta \dot{E}}{\delta W(h)} = 0$$

Numerically solved by the ***Ritz-Galerkin method***

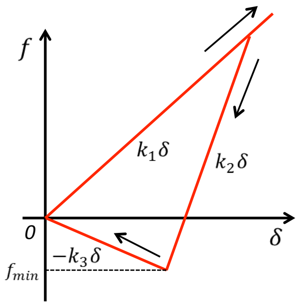
Effect of particle's friction



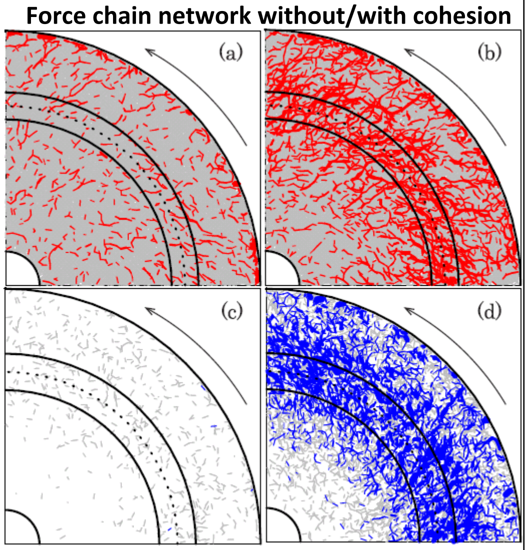
[A. Singh, V. Magnanimo, and S. Luding, Powders and Grains 2013]

Effect of particle's cohesion

Cohesive contact model



Force chain network without/with cohesion



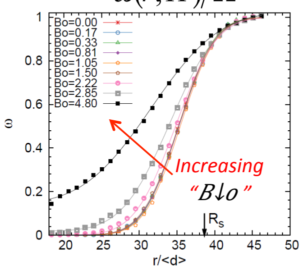
$B_O = 0.33$ $B_O = 2.85$

The Bond number

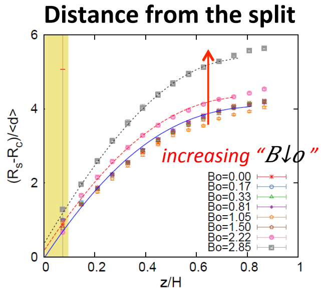
$$B_O = \frac{f_{\min}}{\langle f \rangle}$$

Effect of particle's cohesion

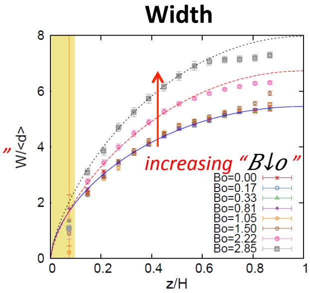
$\omega(r,H)/\Omega$



Distance from the split



Width

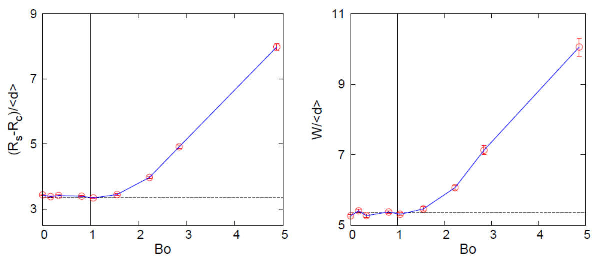


$B_O < 1$

Effect of cohesion is suppressed.

$B_O > 1$

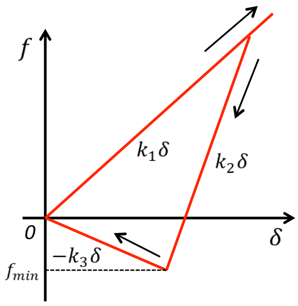
The difference increases with Bo.



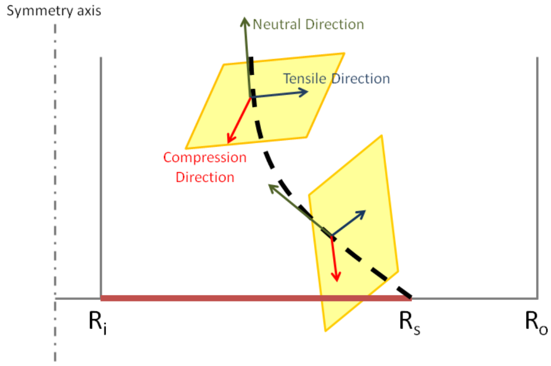
[A. Singh, V. Magnanimo, K. Saitoh, and S. Luding, PRE, 2014]

Effect of particle's cohesion

Cohesive contact model



Force network anisotropy with cohesion

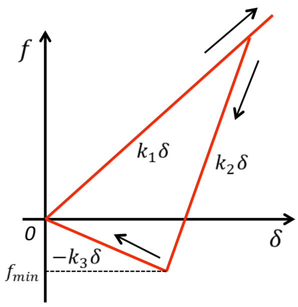


The Bond number

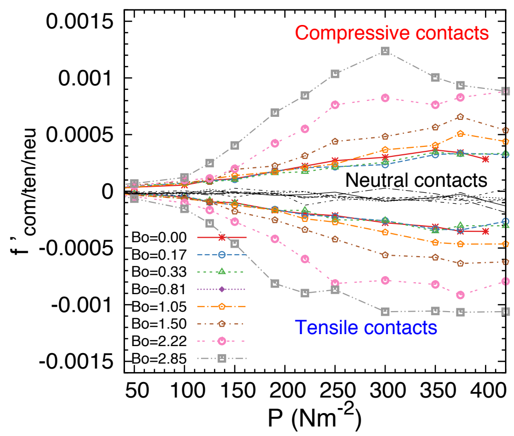
$$B_O = \frac{f_{\min}}{\langle f \rangle}$$

Effect of particle's cohesion

Cohesive contact model



Force network anisotropy with cohesion



The Bond number

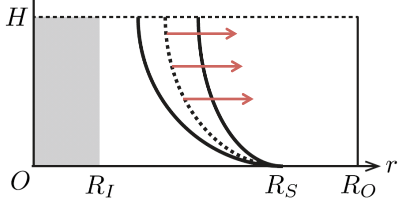
$$B_O = \frac{f_{\min}}{\langle f \rangle}$$

Summary

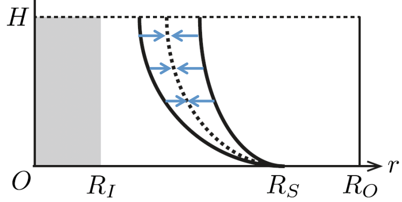
Effect of stiffness/softness and gravity

Friction coefficient in quasi-static regime $\mu(P^*) \approx \mu_0 - b\sqrt{P^*}$ $P^* = P\bar{d}/k_n$

SB approaches the split (outward)



The width of SB decreases



with softness, which was also confirmed by the least dissipation principle.

Effects of particles' friction

SB moves inward and the width *decreases* with friction

Effects of cohesion

SB moves inward and the width *increases* with cohesion

Conclusion

Dependence on stiffness (gravity) in inertial flow states

$$\mu(I, P^*) = \mu_0 + aI^\alpha - bP^{*\beta} \ \ ?$$

Effects of particle's friction and cohesion on μ

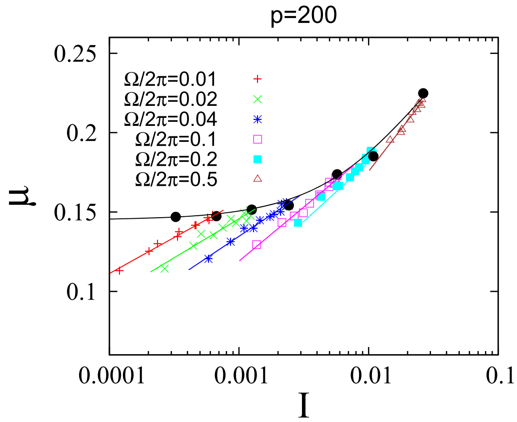
i.e. $\mu(I, \mu_p)$ $\mu(I, Bo)$

Conclusion

Dependence on stiffness (gravity) in inertial flow states

$$\mu(I,P^*) = \mu_0 + aI^\alpha - bP^{*\beta} \ \ ?$$

Very small strain rate?



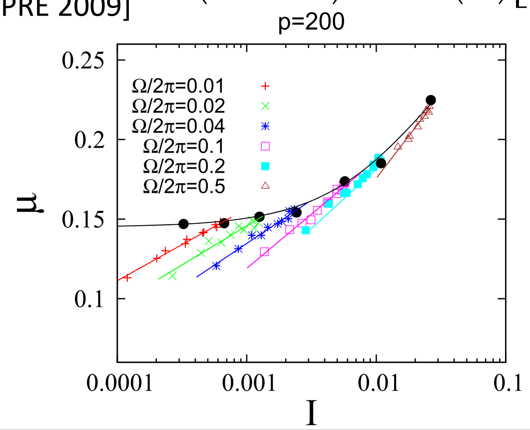
Conclusion

Dependence on stiffness (gravity) in inertial flow states

$$\mu(I,P^*) = \mu_0 + aI^\alpha - bP^{*\beta} \ \ ?$$

Very small strain rate:
[Koval et al. PRE 2009]

$$\mu\left(I < I^*, p^*\right) = \mu_0^{\mathrm{local}}\left(p^*\right)\left[1 - \alpha \ln\left(I/I^*\right)\right]$$

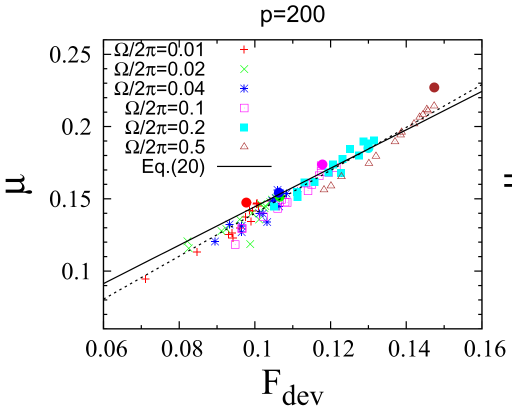


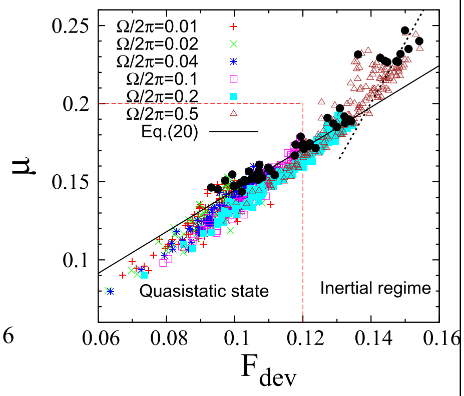
Conclusion

Dependence on stiffness (gravity) in inertial flow states

$$\mu(I,P^*) = \mu_0 + aI^{\alpha} - bP^{*\beta} \quad ?$$

Relation of μ to microstructure?





Conclusion

Dependence on stiffness (gravity) in inertial flow states

$$\mu(I,P^*) = \mu_0 + aI^{\alpha} - bP^{*\beta} \quad ?$$

Effects of particle's friction and cohesion on μ

i.e. $\mu(I,\mu_p) \quad \mu(I,Bo)$

Outlook

Non-local constitutive relations?

fluidity $f \equiv \dot{\gamma}/\mu \qquad \nabla^2 f = \frac{1}{\xi^2}(f - f_{\text{loc}})$

[D.L. Henann & K. Kamrin, PNAS 110 (2013) 6730]

Conclusion

Dependence on stiffness (gravity) in inertial flow states

$$\mu(I, P^*) = \mu_0 + aI^\alpha - bP^{*\beta} \ ?$$

Effects of particle's friction and cohesion on μ

i.e. $\mu(I, \mu_p)$ $\mu(I, Bo)$

Outlook

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[D.L. Henann & K. Kamrin, PNAS 110 (2013) 6730]

Thank you!

[1] "*Does gravity have an effect on the slow shear rheology of granular matter?*", A. Singh, V. Magnanimo, K. Saitoh, and S. Luding, N.JP 17, 043028, 2015

[2] "*Effect of cohesion on shear banding in quasi-static granular material*", A. Singh, V. Magnanimo, K. Saitoh, and S. Luding, Phys. Rev. E 90, 022202, 2014