CH159b; Maths Part 2; Matrices—simultaneous equations and determinants

Problems

1 evaluate the following matrix products

(a) $\begin{pmatrix} 2 & 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \end{pmatrix} =$ (b) $\begin{pmatrix} 2 & 1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} =$ (c) $\begin{pmatrix} 2 & 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} =$

(d) $\begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} =$ (e) $\begin{pmatrix} 2 & 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} =$ (f) $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 2 & 1 & 3 \end{pmatrix} =$

2 express the following sets of simultaneous equations in matrix notation

(a) $\begin{aligned} x - y &= 3 \\ 2x - y &= 4 \end{aligned}$ (b) $\begin{aligned} x + y &= -3 \\ 2x - y &= 4 \end{aligned}$ (c) $\begin{aligned} 3u + 2v &= 2 \\ 2u - y &= -2 \end{aligned}$

(d) $\begin{aligned} x + y + z &= 1 \\ 2x - y + 2z &= 2 \\ x - 2y - z &= 3 \end{aligned}$ (e) $\begin{aligned} 2x + y &= 1 \\ y - 2z &= 2 \\ x - z &= 1 \end{aligned}$

3 find the inverse of the following matrices, and verify that they satisfy $\underset{\approx}{\mathbf{A}}^{-1} \cdot \underset{\approx}{\mathbf{A}} = \underset{\approx}{\mathbf{1}}$

(a) $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ (c) $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

Hence solve the following sets of simultaneous equations

(a) $\begin{aligned} 2x + 3y &= 1 \\ 3x + 4y &= 2 \end{aligned}$ (b) $\begin{aligned} y - 2u &= 1 \\ 3y + 4u &= 5 \end{aligned}$

(c) $\begin{aligned} 0.8x + 0.6y &= 1 \\ -0.6x + 0.8y &= 2 \end{aligned}$ (d) $\begin{aligned} 2u + 3v &= 0 \\ 4u + 6v &= 0 \end{aligned}$

4 Evaluate the following determinants

(a) $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ (b) $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$ (c) $\begin{vmatrix} 2 & 3 \\ 5 & 10 \end{vmatrix}$

(d) $\begin{vmatrix} 2 & 4 \\ 5 & 10 \end{vmatrix}$ (e) $\begin{vmatrix} 1 & 2 & -1 \\ 1 & 3 & 2 \\ 1 & -2 & 1 \end{vmatrix}$ (f) $\begin{vmatrix} 1 & 2 & -2 \\ 1 & 3 & 4 \\ 1 & -2 & 2 \end{vmatrix}$

Answers

1 evaluate the following matrix products

(a) 4 (b) 5 (c) 0

(d) 3 (e) can't! (f) can't (more accurately: not in this course)

2 express the following sets of simultaneous equations in matrix notation

(a) $\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}=\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (e) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -2 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

3 find the inverse of the following matrices, and verify that they satisfy $\underset{\approx}{\mathbf{A}}^{-1}\cdot\underset{\approx}{\mathbf{A}}=\underset{\approx}{\mathbf{1}}$

(a) $\begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0.4 & 0.2 \\ -0.3 & 0.1 \end{pmatrix}$ (c) $\begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$ (d) undefined

Hence solve the following sets of simultaneous equations

(a) $\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} y \\ u \end{pmatrix}=\begin{pmatrix} 1.4 \\ .2 \end{pmatrix}$

(c) $\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -0.4 \\ 2.2 \end{pmatrix}$ (d) many solutions: $u=-1.5v$

4 Evaluate the following determinants

(a) 1 (b) −1 (c) 5

(d) 0 (e) 14 (f) 28