CH159b; Maths part 2: Matrices—eigenvalues and eigenvectors

Problems

1 find the eigenvalues of the following matrices

(a) $\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & ¼ \\ 7 & 4 \end{pmatrix}$ (e) $\begin{pmatrix} -1 & ¼ \\ 7 & 4 \end{pmatrix}$ (f) $\begin{pmatrix} 1 & ½ \\ 7 & 4 \end{pmatrix}$

(g) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ (h) $\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ (i) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

2 Hence find the eigenvectors of the following matrices in parts (a), (b), (c), (d), (i) (then the rest?)

Answers

1 the eigenvalues:

(a) $\lambda = 2$ and $\lambda = 3$ (b) $\lambda = 2$ and $\lambda = 0$ (c) $\lambda = +1$ and $\lambda = -1$

(d) $\lambda = ½$ and $\lambda = 4½$ (e) $\lambda = \frac{3 \pm \sqrt{32}}{2}$ (f) no real roots (see later)

(g) $\lambda = 2$ and $\lambda = \frac{5 \pm \sqrt{5}}{2}$ (h) $\lambda = 2$ and $\lambda = \frac{5 \pm \sqrt{5}}{2}$ (i) $\lambda = 2$ and $\lambda = 2 \pm \sqrt{2}$

2 the eigenvectors:

(a) for $\lambda = 2$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
for $\lambda = 3$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) for $\lambda = 2$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
for $\lambda = 0$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(c) for $\lambda = -1$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
for $\lambda = 1$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(d) for $\lambda = ½$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
for $\lambda = 4½$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 14 \end{pmatrix}$

(i) for $\lambda = 2$: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
for $\lambda = 2 + \sqrt{2}$: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$
for $\lambda = 2 - \sqrt{2}$: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$