CH159 – Part b Test
Answer sheet for practice tests

The time allowed for the test is 40 minutes.

Attempt every question giving your answers clearly in the space provided in **black** or **blue** ink.

Graphical calculators are not permitted.

The formula for solving a quadratic equation is: $\left(x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \right)$

Standard Derivatives

$$\frac{d}{dx}e^x = e^x \quad \frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}\sin(ax) = a\cos(ax)$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}\cos(ax) = -a\sin(ax)$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} \quad \frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \qquad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

Standard Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \quad (n \neq -1) \qquad \int x^{-1} dx = \ln x + c$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c \qquad \int \ln x \, dx = x\ln x - x + c$$

Taylor Series

$$f(x) = f(a) + (x-a)f'(a) + \frac{1}{2!}(x-a)^2 f''(a) + \frac{1}{3!}(x-a)^3 f'''(a) + \ldots$$

Practice Test 1

1. Evaluate the following, given the matrices

$$\mathbf{A}=\begin{pmatrix}0 & 1 & -1\\ 2 & 2 & 1\\ 1 & 3 & -1\end{pmatrix},\quad \mathbf{B}=\begin{pmatrix}1 & 2 & 0\\ 0 & -3 & 1\\ 1 & 3 & 1\end{pmatrix},\quad \mathbf{C}=\begin{pmatrix}1 & 2\\ 1 & 1\\ 1 & 3\end{pmatrix},\quad \mathbf{D}=\begin{pmatrix}1 & -4\\ 2 & 3\end{pmatrix}$$

(a) $\mathbf{A}+\mathbf{B}$ [2]

(b) $\mathbf{AC}$ [3]

(c) $|\mathbf{D}|$ [2]

(d) $\mathbf{B}^{\mathrm{T}}$ [1]

2. All parts of this question relate to the following simultaneous equations: $\begin{matrix}2x-3y=1\\ x+2y=3\end{matrix}$.

(a) write these simultaneous equations in matrix form [2]

(b) Calculate the inverse of the matrix of coefficients [3]

(c) Use the inverse matrix to solve the simultaneous equations

[3]

3. Given the matrix $\mathbf{M} = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$:

(a) calculate the eigenvalues of **M** [4]

(b) hence find the two eigenvectors of **M** [4]

6. All parts of this question refer to the complex numbers $c = 2 + 3i$; $d = 1 - i$; $k = 2i$; $m = 4$

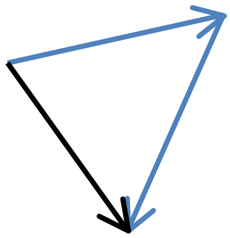
(a) plot *c*, *d* and *m* on an Argand diagram [3]

(b) evaluate $c + 2d$ [1]

(c) evaluate $k \times d$ [2]

(d) evaluate $c \times d$ [2]

(e) evaluate $c \times c^{*}$ [2]



6. All parts of this question refer to the complex numbers $c = 2 + 3i$; $d = 1 - i$; $k = 2i$; $m = 4$

(a) plot *c*, *d* and *m* on an Argand diagram [3]

(b) evaluate $c + 2d$ [1]

$4 + i$

(c) evaluate $k \times d$ [2]

$2 + 2i$

(d) evaluate $c \times d$ [2]

5+i

(e) evaluate $c \times c^*$ [2]

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Practice Test 2

1. Evaluate the following, given the matrices

$$\mathbf{A}=\begin{pmatrix}1 & 2 & -1\\ 2 & 0 & 1\\ 4 & 3 & -1\end{pmatrix},\quad \mathbf{B}=\begin{pmatrix}3 & 1 & 2\\ 1 & -3 & 0\\ 2 & 2 & 1\end{pmatrix},\quad \mathbf{C}=\begin{pmatrix}0 & 2 & 1\\ 1 & 1 & -1\end{pmatrix},\quad \mathbf{D}=\begin{pmatrix}1 & -2\\ -1 & 2\end{pmatrix}$$

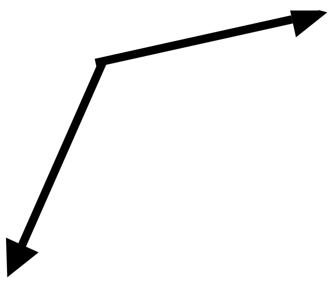
(a) $\mathbf{A}-\mathbf{B}$ [2]
(b) $\mathbf{CB}$ [3]
(c) $|\mathbf{D}|$ [2]
(d) $\mathbf{B}^{\mathrm{T}}$ [1]

2. All parts of this question relate to the following simultaneous equations: $\begin{matrix}2x-2y=-4\\ x+3y=2\end{matrix}$.

(a) write these simultaneous equations in matrix form [2]
(b) Calculate the inverse of the matrix of coefficients [3]
(c) Use the inverse matrix to solve the simultaneous equations [3]

3. Given the matrix $\mathbf{M}=\begin{pmatrix}4 & 2\\ -1 & 1\end{pmatrix}$:

(a) calculate the eigenvalues of **M** [4]
(b) hence find the two eigenvectors of **M** [4]



1. Evaluate the following, given the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 4 & 3 & -1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 & 1 & 2 \\ 1 & -3 & 0 \\ 2 & 2 & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & -1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 & -2 \\ -1 & 2 \end{pmatrix}$$

(a)	$\mathbf{A} - \mathbf{B}$	[2]
(b)	$\mathbf{CB}$	[3]
(c)	$\lvert\mathbf{D}\rvert$	[2]
(d)	$\mathbf{B}^T$	[1]

2. All parts of this question relate to the following simultaneous equations: $\begin{matrix} 2x - 2y = -4 \\ x + 3y = 2 \end{matrix}$.

(a)	write these simultaneous equations in matrix form	[2]
(b)	Calculate the inverse of the matrix of coefficients	[3]
(c)	Use the inverse matrix to solve the simultaneous equations	[3]

3. Given the matrix $\mathbf{M} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$:

(a) calculate the eigenvalues of **M** [4]

(b) hence find the two eigenvectors of **M** [4]

6. All parts of this question refer to the complex numbers $c = 1 + 4i$; $d = 2 - 3i$; $k = i$; $m = 2$

(a)	plot c, d and m on an Argand diagram	[3]

(b)	evaluate $3c - d$	[1]
(c)	evaluate $k \times c$	[2]
(d)	evaluate $c \times d$	[2]
(e)	evaluate $d \times d^*$	[2]

Practice Test 3

1. Evaluate the following, given the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & 10 \\ 3 & 2 & -2 \\ 2 & 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -2 & 3 & 1 \\ 0 & 0 & 2 \\ 2 & -1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 0 & -2 \\ 4 & 1 & -1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

(a) $\mathbf{A} - \mathbf{B}$ [2]

(b) $\mathbf{DC}$ [3]

(c) $|\mathbf{D}|$ [2]

(d) $\mathbf{B}^{\mathrm{T}}$ [1]

2. All parts of this question relate to the following simultaneous equations: $\begin{array}{r} x - 3y = 8 \\ 2x + y = 2 \end{array}$.

(a) write these simultaneous equations in matrix form [2]

(b) Calculate the inverse of the matrix of coefficients [3]

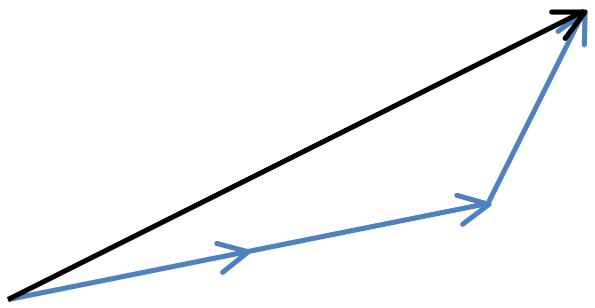
(c) Use the inverse matrix to solve the simultaneous equations [3]

3. Given the matrix $\mathbf{M} = \begin{pmatrix} 3 & 2 \\ 3 & 4 \end{pmatrix}$:

(a) calculate the eigenvalues of **M** [4]

(b) hence find the two eigenvectors of **M** [4]

(b)



5. (a) Write out the Taylor series expansion for $x \ln x$ about the point x=1, giving the first four terms. [5]

(b) Hence evaluate $\ln(1.1)$ to two decimal places **without using your calculator**. [3]

function and derivatives | evaluate at $x = 1$

$f(x) = x \ln x$ | $f(x) = 0$

$\frac{df}{dx} = \ln x + 1$ | $\frac{df}{dx} = 1$

$\frac{d^2 f}{dx^2} = \frac{1}{x}$ | $\frac{d^2 f}{dx^2} = 1$

$\frac{d^3 f}{dx^3} = \frac{-1}{x^2}$ | $\frac{d^3 f}{dx^3} = -1$

So the Taylor series is

$$f(x) = 0 + (x-1)1 + \frac{1}{2}(x-1)^2(1) + \frac{1}{6}(x-1)^3(-1)$$

$$= (x-1) + \frac{1}{2}(x-1)^2 - \frac{1}{6}(x-1)^3$$

(b) Hence evaluate ln(0.9) to four decimal places **without using your calculator**. [3]

use Taylor series; if $x = 1.1$, then $(x-1) = 0.1$

$$f(x) = 0.1000 + 0.0050 - 0.00017$$

$$= 0.1033$$

6. All parts of this question refer to the complex numbers $c = 2 + 3i$; $d = 1 + 2i$; $k = 5i$; $m = -2$

(a) plot c, d and m on an Argand diagram [3]

(b) evaluate $2c - d$ [1]

(c) evaluate $k \times c$ [2]

(d) evaluate $c \times d$ [2]

(e) evaluate $d \times d^*$ [2]