

1. Show how equation A can be rearranged to give Equation B. Show all your working.

[6]

Equation A:
$$a = \frac{b(\ln y - 2 \ln A)}{x^2 - 1}$$

Equation B:
$$y = A^2 e^{\left(\frac{ax^2 - a}{b}\right)}$$

2. (i) Solve the simultaneous equations:

[5]

$$x + \frac{y}{3} = 2.5$$

$$3x + 2y = 13.5$$

3. Combine the two functions $f(x)$ and $g(x)$ to give expressions for $f(g(x))$ and $g(f(x))$.

[5]

$$f(x) = x \sin^4(x^3)$$

$$g(x) = \sqrt{x}$$

4. (i) Determine the x -coordinates of the turning points of the following cubic equation by differentiating and factorising. Show all your working.

[7]

$$y = \frac{4}{3}x^3 - \frac{3}{2}x^2 - x + 4$$

(ii) Sketch the function below marking the points at which it crosses both axes **and** the coordinates of the maxima/minima.

[6]

$$y = 4x^2 - 3x - 1$$

5. For the function $f(x) = \left(\frac{1}{e^{(x-2)} + 3} \right)$ evaluate the following limits:

[3×4]

(i) $\lim_{x \rightarrow 0} \left(\frac{1}{e^{(x-2)} + 3} \right)$

(ii) $\lim_{x \rightarrow -\infty} \left(\frac{1}{e^{(x-2)} + 3} \right)$

(iii) $\lim_{x \rightarrow 2} \left(\frac{1}{e^{(x-2)} + 3} \right)$

6. (i) Change the base of the following logarithmic expression from 3 to base 10 and simplify as far as possible:

[5]

$$\log_3(10x^4)$$

(ii) Expand and simplify the following logarithmic expression:

[5]

$$\log_3 \left(\frac{27}{x^2} \right)$$

7. What is the derivative of y for each of the following functions?

[4×4]

(i) $y = \tan(x) + 2e^{5x} - \frac{1}{x^2}$

(ii) $y = x^2 \tan(3x)$

(iii) $y = \ln(x^3 - 1)\sqrt{x^2 - 2}$

(iv) $y = (e^{2x} + 2x)^2$