What brakes can tell you

Andrew Aylwin & Pravin Madhavan

Mathematics and Statistics Centre for Doctoral Training
University of Warwick

21st November 2012

Outline

Contents

MASDOC

Motivation for the Model

- Movement from old drum brake system to new disc brake system with improved stopping distances.
- Can still be seen to this day when comparing the American and European HGV industries where improving regulations require the disc brake system.
- However, disc brakes more prone to contamination and squeal.
- Thus, we would like to model the brake system and deformation at a small level.

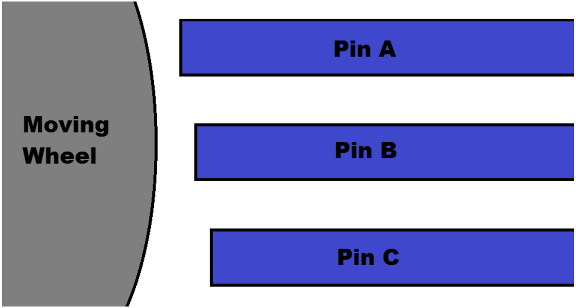
Contents

MAS DOC

J. R. Barber's Experiment

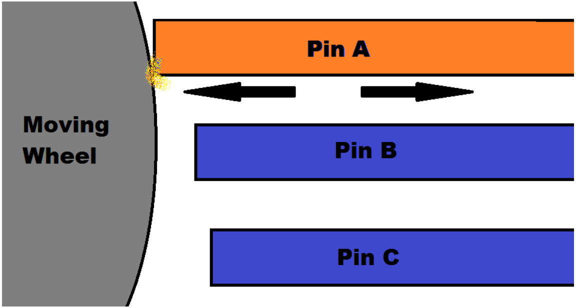
- Initial Experiment with full brake.
  - Unstable resonant vibration due to only small areas of contact.
  - In turn due to microscopic rough nature of surface accentuated by thermal deformation.
- To model this Barber devised a simple experiment:
  - 3 pins attached to a loading arm to act as a brake.
  - Pins could then be pushed against a fast moving wheel to simulate braking.
  - Each pin implanted with thermocouple to measure the temperature.

Barber's Observations and Explanation



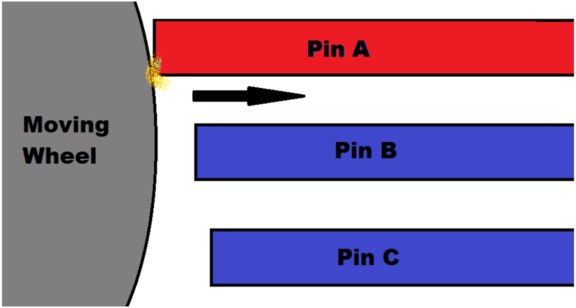
- Three cold pins at slightly different heights to mimic the microscopic jaggedness of the brake pad.

Stage 1



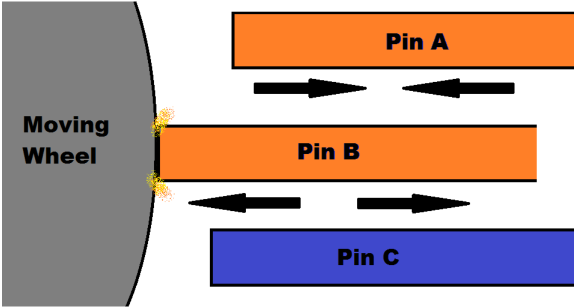
- The wheel makes contact with pin A.
- Temperature observed from the thermocouple associated with pin A rises.
- Pin A expands.
    - Temperature still relatively low - thermal expansion outweighs the wear.
    - Initial irregularity of surface of brake exaggerated.

Stage 2



- Temperature of pin reaches equilibrium - heat generated due to friction balanced by heat lost to surroundings. At this point the thermal expansion stops.
- Rate of wear on pin increases with temperature.
    - a time will be reached at which the increase in height of pin A due to thermal expansion is superseded by the decrease in height of the pin due to wear.

Stage 3



- Increase in wear means a change in contact point to pin B is inevitable.
- The pin has a lower wear rate and a higher rate of expansion - cycle is started again.
- Pin A cools and shrinks making it the smallest of the three pins.

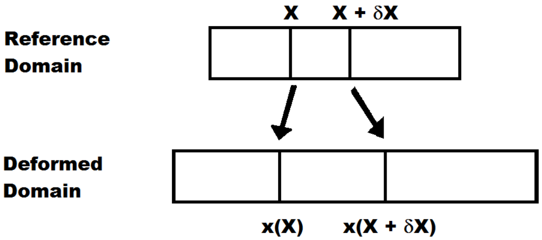
Contents

Model Derivation: Setting

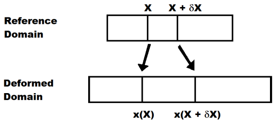
- Pin is modelled as 1D elastic material and is subject to physical laws of elasticity.
- Define displacement $u(\mathbf{X})$ of a material point $\mathbf{X}$ to be given by

$$u(\mathbf{X}) = x(\mathbf{X}) - \mathbf{X}$$

where $x = x(\mathbf{X})$ is spatial location of material point $\mathbf{X}$ once the pin has been deformed.



Model Derivation: Hooke's law



- Define $l_0 := (\mathbf{X} + \delta\mathbf{X}) - \mathbf{X} = \delta\mathbf{X}$ and $l := x(\mathbf{X} + \delta\mathbf{X}) - x(\mathbf{X})$.
- In isothermal case,

$$\tau = k\frac{l - l_0}{l_0} \ (\text{“stress”} \propto \text{“strain”}).$$

Model Derivation: Hooke's law

- Localising Hooke's law, we have

$$\tau(\mathbf{X}) = k \lim_{\delta\mathbf{X}\to 0} \frac{x(\mathbf{X}+\delta\mathbf{X}) - x(\mathbf{X}) - (\mathbf{X}+\delta\mathbf{X}-\mathbf{X})}{\delta\mathbf{X}} = k\frac{\partial u}{\partial \mathbf{X}}$$
$$= k\left(\frac{\partial x}{\partial \mathbf{X}} - 1\right).$$

- Under small strain assumption, we have

$$\left|\frac{\partial u}{\partial \mathbf{X}}\right| \ll 1 \Rightarrow \frac{\partial x}{\partial \mathbf{X}} \approx 1 \Rightarrow \frac{\partial u}{\partial \mathbf{X}} = \frac{\partial u}{\partial \mathbf{x}}.$$

- Can derive all equations in terms of more familiar spatial (*Eulerian*) coordinates x rather than material (*Lagrangian*) coordinates $\mathbf{X}$.

MAS DOC

Model Derivation: Thermal Expansion

- Elastic materials subject to thermal conditions without stress satisfy

$$\frac{\partial u}{\partial x} = \alpha(T - T_0) \text{ (“strain” } \propto \text{ “change in temperature”).}$$

Model Derivation: Linear Model

- Hooke's law (Isothermal model):

$$\tau = k\frac{\partial u}{\partial x} \text{ (“stress” } \propto \text{ “strain”).}$$

- Thermal expansion law (no stress model):

$$\frac{\partial u}{\partial x} = \alpha(T - T_0) \text{ (“strain” } \propto \text{ “change in temperature”).}$$

- Have both thermal forcing and stress in pin so model strain as being a *linear combination of both stress and the change in temperature*:

$$\frac{\partial u}{\partial x} = \frac{1}{k}\tau + \alpha(T - T_0).$$

Model Derivation: Thermoelasticity equation

- Assume domain of pin is $\Omega = [0, 1]$, friction with wheel occuring at $x = 0$.
- At $x = 1$, $\tau = -F_0$ where F_0 is the force resulting from applying the brakes.
- Conservation of momentum $\Rightarrow \frac{\partial \tau}{\partial x} = 0 \Rightarrow \tau = -F_0$ for all $x \in [0, 1]$.
- Re-scaled so that $T_0 = 0$, we obtain *thermoelasticity equation*

$$\frac{\partial u}{\partial x} = \alpha T - \frac{1}{k} F_0.$$

MAS DOC

Model Derivation: Heat conduction model

- Also pose a heat conduction model on pin which models diffusion of heat into pin due to contact with wheel at $x = 0$.
- Pointwise conservation of energy yields

$$\frac{\partial T}{\partial t} + a\frac{\partial^2 u}{\partial x \partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

  where $a > 0$ is small. The term $a\frac{\partial^2 u}{\partial x \partial t}$ takes into account expansion of elastic material as heat gets diffused through the material.
- Very similar model considered in Copetti & Elliott (1993).

Model Derivation: Heat Conduction IC/BC

- Need to prescribe physically appropriate initial/boundary conditions for thermoelasticity equation and heat conduction model satisfied by u and T at $x = 0, 1$.
- $\kappa \frac{\partial T}{\partial x} = 0 \quad \text{at } x = 1$ (no heat flow at opposite end).
- Frictional heating $\propto$ stress $\times$ velocity:

$$\kappa \frac{\partial T}{\partial x} = \gamma \tau = -\gamma F_0 \quad \text{at } x = 0.$$

- The initial condition for the temperature is chosen to be

$$T(x, 0) = 0 \quad \text{for all } x \in [0, 1].$$

Model Derivation: Thermoelasticity BC

- Only need one boundary condition for well-posedness of thermoelastic equation.
- Wear of break pad due to the friction with wheel $\propto$ stress $\times$ velocity:

$$u(0,t) = -\beta F_0 t \quad \text{at } x = 0.$$

Decoupled Problem

- Recall
$$\frac{\partial u}{\partial x} = \alpha T - \frac{1}{k} F_0. \tag{1}$$
- Differentiating with respect to t, we get
$$\frac{\partial^2 u}{\partial x \partial t} = \alpha \frac{\partial T}{\partial t}.$$
- Plugging into heat conduction model, we obtain
$$\frac{\partial T}{\partial t} = \frac{\kappa}{(1 + a\alpha)} \frac{\partial^2 T}{\partial x^2}. \tag{2}$$
- Solution algorithm for the one pin problem can be summarised as follows:
    - Find temperature T satisfying (2) along with B.C./I.C.
    - Construct right-hand side of (1).
    - Find displacement u satisfying (1) along with B.C.

MAS DOC

Contents

MASDOC

Two Pin Problem - Setup

- We now apply the model to two pins of differing lengths.
- Assume pin 1 is longer than pin 2 and only the former is in contact with the plate at time $t = 0$.
- Wearing will cause pin 1 to shrink in length.

At some time $t = t^*$ pin 1 will have shrunk to point where lengths of two pins equal.

Two Pin Problem - Setup

- We now apply the model to two pins of differing lengths.
- Assume pin 1 is longer than pin 2 and only the former is in contact with the plate at time $t = 0$.
- Wearing will cause pin 1 to shrink in length.

At some time $t = t^*$ pin 1 will have shrunk to point where lengths of two pins equal.

- Switch off the heat flux for pin 1 ($\kappa \frac{\partial T_1}{\partial x} = 0$ at $x = 0$).

Two Pin Problem - Setup

- We now apply the model to two pins of differing lengths.
- Assume pin 1 is longer than pin 2 and only the former is in contact with the plate at time $t = 0$.
- Wearing will cause pin 1 to shrink in length.

At some time $t = t^*$ pin 1 will have shrunk to point where lengths of two pins equal.

- Switch off the heat flux for pin 1 ($\kappa \frac{\partial T_1}{\partial x} = 0$ at $x = 0$).
- Simultaneously introduce heat flux into pin 2 ($\kappa \frac{\partial T_2}{\partial x} = -\gamma F_0$ at $x = 0$).

Two Pin Problem - Setup

- We now apply the model to two pins of differing lengths.
- Assume pin 1 is longer than pin 2 and only the former is in contact with the plate at time $t = 0$.
- Wearing will cause pin 1 to shrink in length.

At some time $t = t^*$ pin 1 will have shrunk to point where lengths of two pins equal.

- Switch off the heat flux for pin 1 ($\kappa \frac{\partial T_1}{\partial x} = 0$ at $x = 0$).
- Simultaneously introduce heat flux into pin 2 ($\kappa \frac{\partial T_2}{\partial x} = -\gamma F_0$ at $x = 0$).

Time for a little simulation...

Acknowledgements

We would like to thank John Ockendon for introducing us to this problem and Charlie Elliott for organising this workshop. We would also like to thank Bjorn Stinner, Colm Connaughton and Gideon Simpson for helping us tackle this problem.

Bibliography

J.R. Barber. Thermoelastic instabilities in the sliding of conforming solids. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 312(1510):381-394, 1969.

M.I.M. Copetti and C.M. Elliott. A one-dimensional quasi-static contact problem in linear thermoelasticity. *European J. Appl. Math*, 4:151-174, 1993.