Title: Weak solutions to the Navier–Stokes inequality with internal singularities

Date: 28th November 2016,

Abstract: The 3D incompressible Navier–Stokes equations are the central model of fluid mechanics and, despite a significant effort since the celebrated work of Leray (1934), the fundamental question of existence of global in time strong solutions remains unsolved. Although we do not know whether singularities occur, we are able to estimate the size of the possible set $S$ of singular points, i.e. points $(x,t)$ at which the velocity field $u$ blows up. This is possible thanks to the partial regularity theory developed by Caffarelli, Kohn & Nirenberg (1982). This theory is concerned with velocity fields satisfying a localised form of the energy inequality and it provides sufficient conditions for boundedness of $|u|$ in small cylinders in space-time. As a result, one can bound the dimension of the singular set $S$, namely one can bound the box-counting dimension $d_B(S)$ by $5/3$ and the Hausdorff dimension $d_H(S)$ by $1$. The bound $d_B(S) \leq 5/3$ can be improved, with the most recent result $d_B(S) \leq 180/131 \approx 1.37$. However, the bound $d_H(S) \leq 1$ is sharp, and in the talk we will discuss examples of velocity fields $u$ satisfying the local energy inequality and admitting internal blow-ups. Such examples were first studied by Scheffer (1985 & 1987). The blow-up occurs on a set $S = (S', T_0)$, where $S' \subset \mathbb{R}^3$ is a uniform Cantor set with $d_H(S') = \xi$ for any preassigned $\xi \in (0, 1)$. We will present a method of sharpening the construction of such examples, which gives a blow-up on the set $S$ with $d_H(S) = 1$.

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