Bayes Factors and Brain Imaging

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1 Hypothesis Testing
2 Bayes Factors
3 Image Segmentation
Suppose we have a null hypothesis:

\[ H_0 : \theta \in \Theta_0 \subset \Theta \]

which we want to test against an alternative hypothesis:

\[ H_1 : \theta \in \Theta \setminus \Theta_0 \]

where \( \Theta \) is the parameter space.
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where \( \Theta \) is the parameter space.

The usual method of hypothesis testing involves a Likelihood Ratio Test Statistic, given by:

\[
S_{LR}(X) = \frac{\sup_{\Theta_0} L(\theta; X)}{\sup_{\Theta} L(\theta; X)}
\]
Under the Bayesian paradigm, we would like to modify this method to take into account our prior beliefs about the behaviour of the model. This gives rise to Bayes factors [Jeffreys (1935)].
Under the Bayesian paradigm, we would like to modify this method to take into account our prior beliefs about the behaviour of the model. This gives rise to Bayes factors \[\text{[Jeffreys (1935)]}\]. Bayes’ Theorem says:

\[
P(H_k|X) = \frac{P(X|H_k)P(H_k)}{P(X|H_0)P(H_0) + P(X|H_1)P(H_1)}
\]

with \(k = 0, 1\).
We then get:

$$P(H_0 | X) = P(H_1 | X) = P(X | H_0) P(H_0) / P(X | H_1) P(H_1)$$

where:

$$P(X | H_k) = \int P(X | \theta_k, H_k) \pi(\theta_k | H_k) d\theta_k$$

with $\theta_k$ the parameter under $H_k$ with prior $\pi(\theta_k | H_k)$. The highlighted term is the Bayes factor.
We then get:

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\frac{P(H_0|X)}{P(H_1|X)} = \frac{P(X|H_0)P(H_0)}{P(X|H_1)P(H_1)}
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Bayes Factors (II)

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One problem with images obtained by MRI, PET etc. is trying to determine boundaries in a noisy image. Specifically, we are interested in determining the number of gray levels to be used in an image.

Figure: PET image of a dog’s lung [Stanford & Raftery, 2002]
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Case Study - Image Segmentation (I)

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- We analyse the original image to generate a histogram of the distribution of the grayscale. This will form the basis of our updating mechanism.
- Our prior is a multivariate normal centred at the maximum likelihood estimator under the selected hypothesis.
- We use an approximation to estimate the value of the terms in the Bayes factor.
Case Study - Image Segmentation (III)

The result:

Figure: PET image of a dog's lung after final segmentation [Stanford & Raftery, 2002]
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Figure: PET image of a dog’s lung after final segmentation [Stanford & Raftery, 2002]
References


References


Thank you for listening!