Bayes Factors and Brain Imaging

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Hypothesis Testing & Likelihood

Suppose we have a null hypothesis:

$$H_0 : \theta \in \Theta_0 \subset \Theta$$

which we want to test against an alternative hypothesis:

$$H_1 : \theta \in \Theta \backslash \Theta_0$$

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where Θ is the parameter space.
The usual method of hypothesis testing involves a Likelihood Ratio Test Statistic, given by:

$$S_{LR}(\mathbf{X}) = \frac{sup_{\Theta_0} L(\theta; \mathbf{X})}{sup_{\Theta} L(\theta; \mathbf{X})}$$

Bayes Factors (I)

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Bayes' Theorem says:

$$\mathbb{P}(H_k|\mathbf{X}) = \frac{\mathbb{P}(\mathbf{X}|H_k)\mathbb{P}(H_k)}{\mathbb{P}(\mathbf{X}|H_0)\mathbb{P}(H_0) + \mathbb{P}(\mathbf{X}|H_1)\mathbb{P}(H_1)}$$

with $k = 0, 1$.

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where:

$$\mathbb{P}(\mathbf{X}|H_k) = \int \mathbb{P}(\mathbf{X}|\theta_k, H_k)\pi(\theta_k|H_k)d\theta_k$$

with θ_k the parameter under H_k with prior $\pi(\theta_k|H_k)$.

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The highlighted term is the Bayes factor.

Case Study - Image Segmentation (I)

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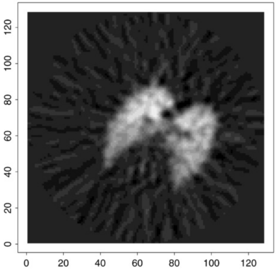

Figure: PET image of a dog's lung [Stanford & Raftery, 2002]

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- We analyse the original image to generate a histogram of the distribution of the grayscale. This will form the basis of our updating mechanism.
- Our prior is a multivariate normal centred at the maximum likelihood estimator under the selected hypothesis.
- We use an approximation to estimate the value of the terms in the Bayes factor.

Case Study - Image Segmentation (III)

The result:

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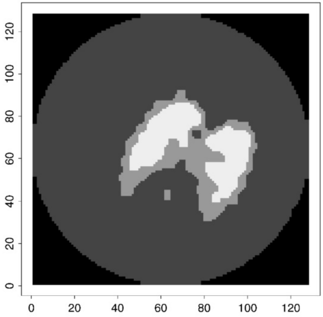

Figure: PET image of a dog's lung after final segmentation [Stanford & Raftery, 2002]

References

- [Jeffreys (1935)] - Some Tests of Significance, Treated by the Theory of Probability - Proceedings of the Cambridge Philosophy Society, Vol 31, 1935
- [Stanford & Raftery, 2002] - Approximate Bayes Factors for Image Segmentation: The Pseudolikelihood Information Criterion - IEEE Transactions on Pattern Analysis & Machine Intelligence, Vol 24, No 11, November 2002

References

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Thank you for listening!