FDRC, Warwick 11 February 2015

Quantum Turbulence

Robert M. Kerr, Warwick University

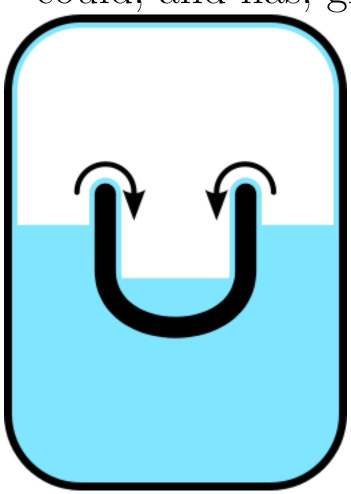
Abstract: If one cools any gas sufficiently it will become a liquid, and with sufficient pressure will become a solid. Helium can also become an inviscid superfluid if the temperature is sufficiently low and the pressures are not very high. And for larger velocities, can become "turbulent".

What is a superfluid?

It is a very low temperature quantum state with zero viscosity whose underlying equation is a quantum nonlinear Schrodinger equation, not the Navier-Stokes or Euler equations. A state similar to that in a resistance-free superconductor. That is, at least a low velocities, a superfluid will flow without resistance.

Why would engineers be interested in this bizarre medium?

Because despite being inviscid, once the velocity exceeds a threshold, a superfluid resists flow in exactly same way as a classical turbulent fluid obeying the viscous Navier-Stokes equations. Why? Just as we dont know exactly why a classical laminar fluid becomes turbulent, we dont know why a superfluid becomes turbulent. But in both cases, vortices seem to be the key and in the quantum case these are easier to model than classical vortices. So if we understand the quantum case, it could, and has, give us clues for the origin of classical turbulence.



Early superfluid experiments 1937-38

Superfluid Helium II will "creep" along surfaces in order to find its own level - after a short while, the levels in the two containers will equalize. The Rollin film also covers the interior of the larger container; if it were not sealed, the helium II would creep out and escape.

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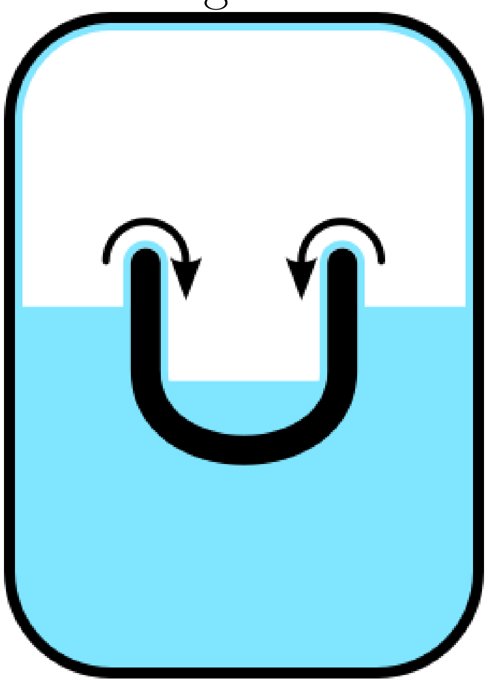
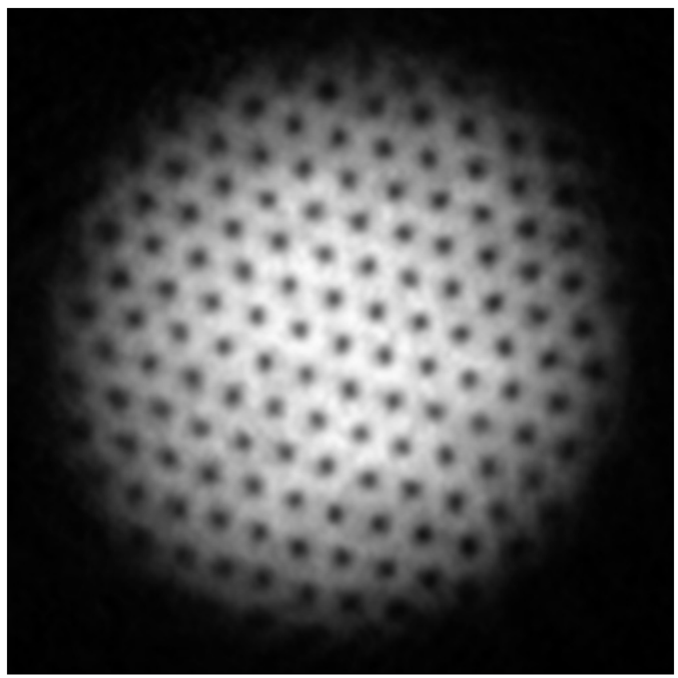
What is a superfluid? Examples

Left: Early superfluid experiments 1937-38 Superfluid Helium II (^{4}He) $T < 2.7°K$ will "creep" along surfaces in order to find its own level - after a short while, the levels in the two containers will equalize. The Rollin film also covers the interior of the larger container; if it were not sealed, the helium II would creep out and escape.





Ultra-cold ($T \approx 0°K$) quantum gases in magnetic traps. Vortices are density $\rho = 0$ black holes.

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- Why?

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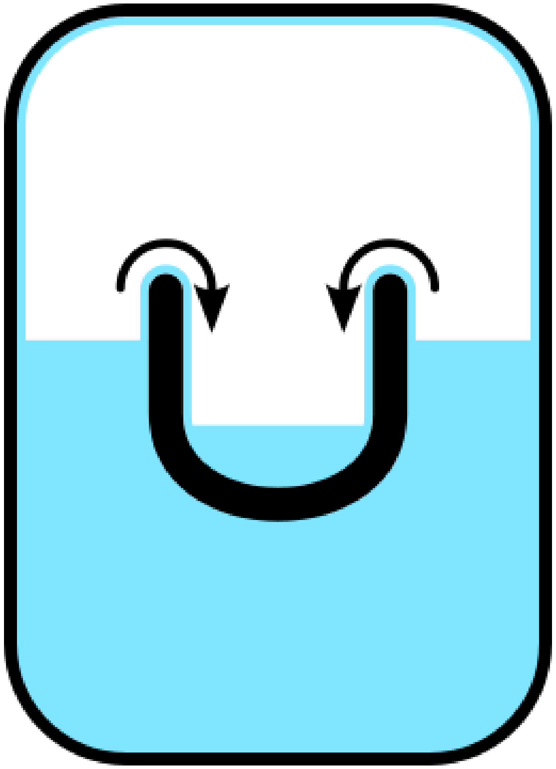

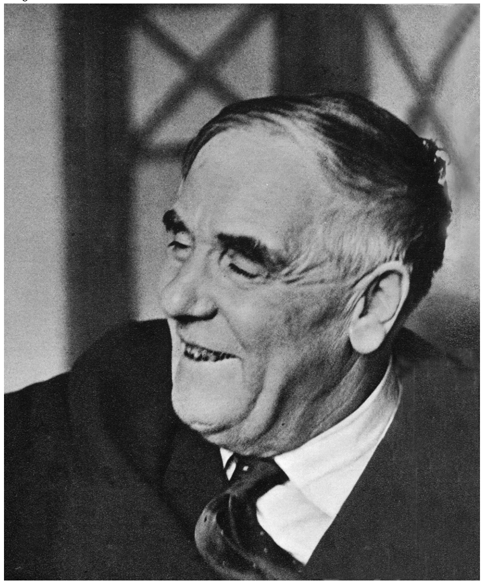

Cambridge

Pyotr Leonidovich Kapitsa

In 1934 he developed new and original apparatus (based on the adiabatic principle) for making significant quantities of liquid helium.

Then, on a visit to the USSR, his passport was confiscated. He stayed in Moscow and Rutherford allowed his Cambridge equipment to be bought.

This led to a new series of experiments, eventually in 1937 discovering **superfluidity**.

Early superfluid experiments 1937-38

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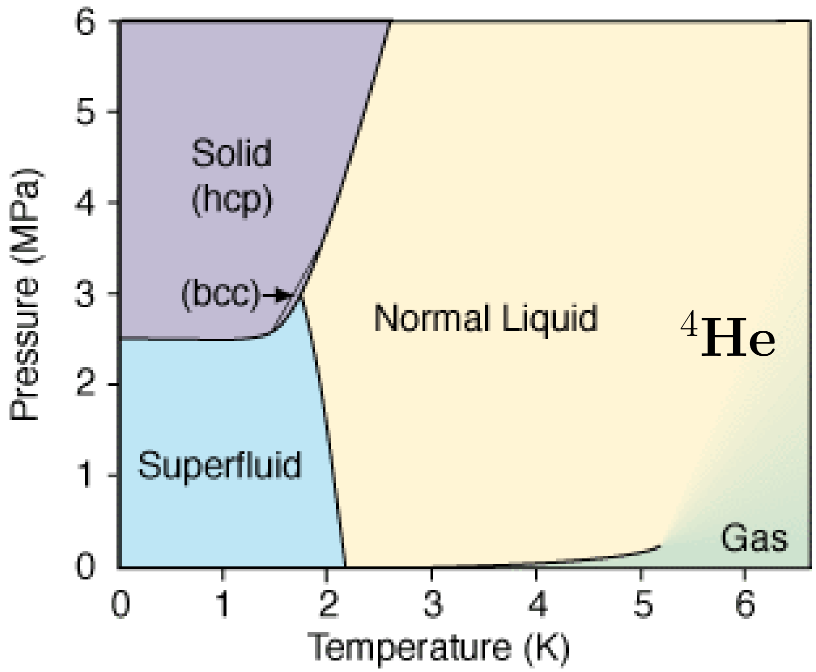
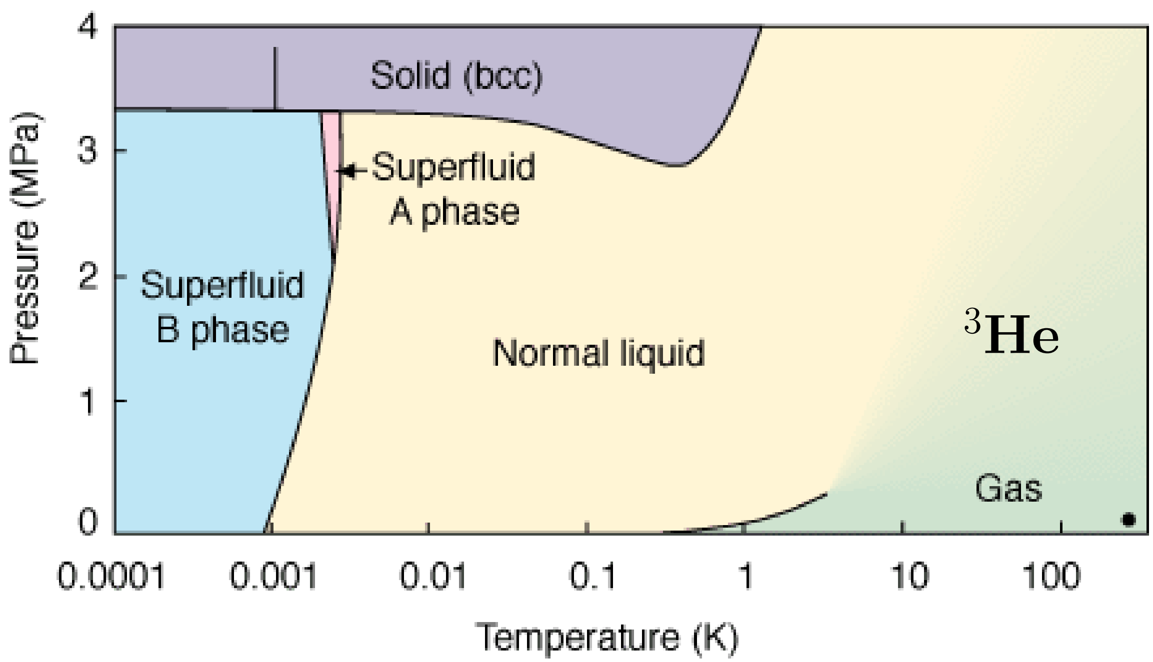
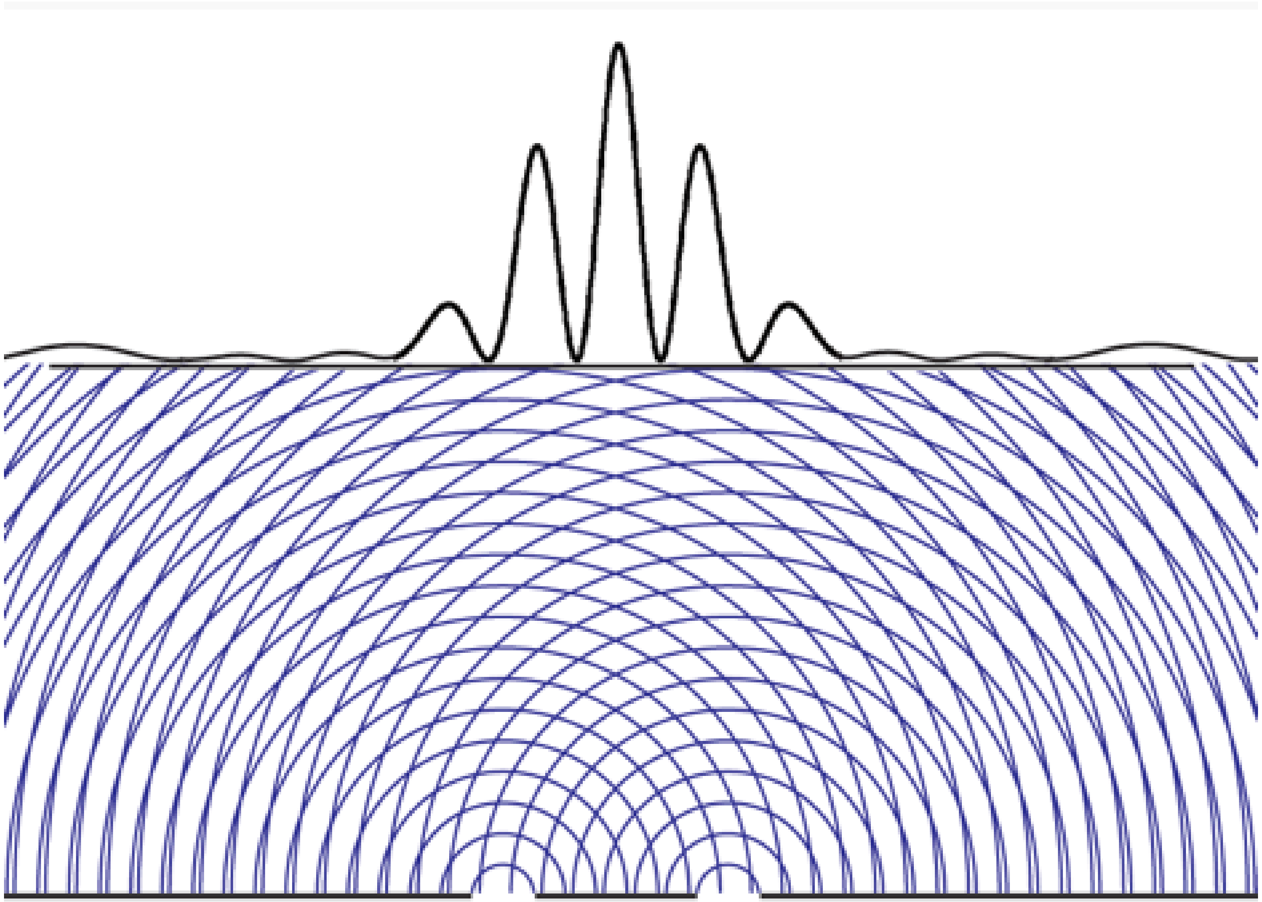


^{4}He is the more common isotope of helium. The figure shows the phase diagram of ^{4}He at low temperatures. ^{4}He remains liquid at zero temperature if the pressure is below 2.5 MPa (approximately 25 atmospheres). The liquid has a phase transition to a superfluid phase, also known as He-II, at the temperature of 2.17 K (at vapor pressure). The solid phase has either hexagonal close packed (hcp) or body centered cubic (bcc) symmetry.



The phase diagram of ^{3}He is shown in the figure. Note the logarithmic temperature scale. The dot in the lower right hand corner denotes room temperature and pressure. There are two superfluid phases of ^{3}He, A and B. The line within the solid phase indicates a transition between spin-ordered and spin disordered structures (at low and high temperatures, respectively).

Two-slot experiment. Either phonons (light) or electrons.



Schrödinger equation: $i\hbar\dfrac{\partial}{\partial t}\Psi = \hat{H}\Psi.$

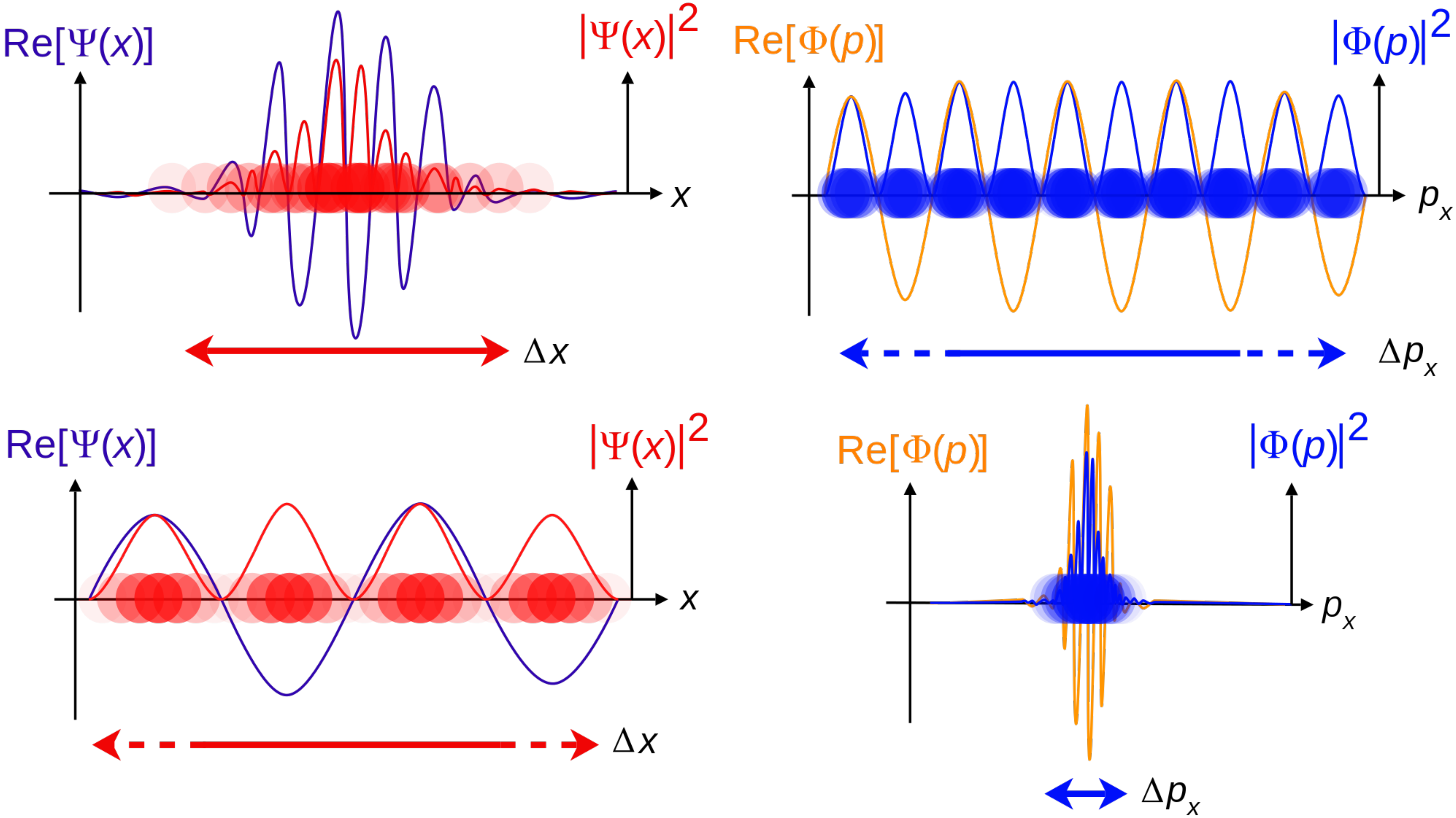
More generally:

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{x},t) = \left[\frac{-\hbar^2}{2\mu}\Delta + V(\boldsymbol{x},t)\right]\Psi(\boldsymbol{x},t)\,.$$

Heisenberg uncertainty: $\Delta x \Delta p = \hbar$**.** There is a limit to how well one can know the combined position and momentum of a particle or wave.

Upper left Wave packet localised in space.

Upper right Momentum $p = mv$ periodically arranged.

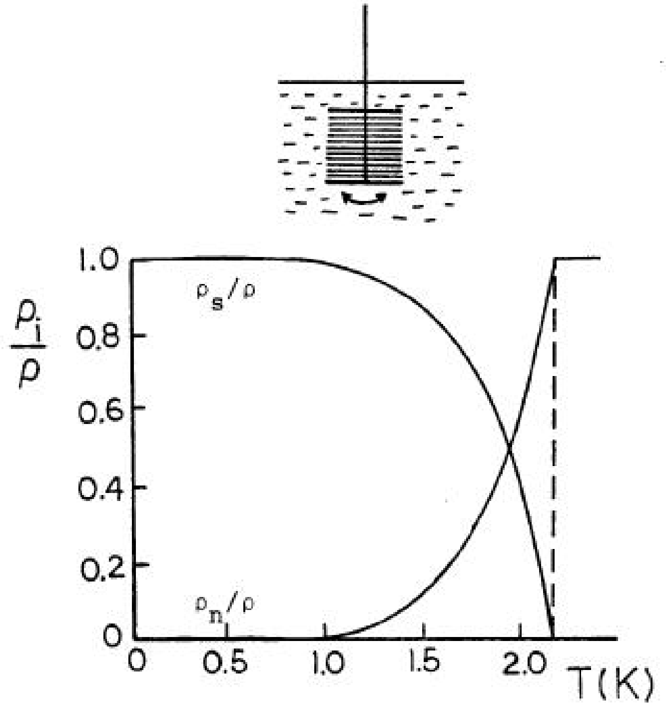


Lower left Coherence in space.

This is what a quantum fluid is like.

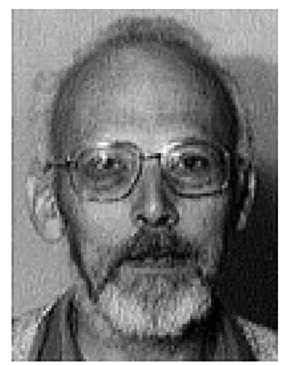
Lower right Momentum localised.

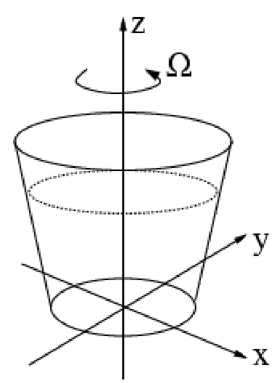
Temperature very small.

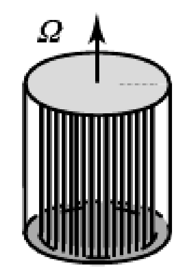


The original experiment by Andronikashvilli for the densities of normal and superfluid based on Landau's two-fluid model. Andronikashvilli was the first to notice that above certain flow rates (through openings) that the superfluid resisted motion in a manner analogous to a classical, viscous fluid.

- **Heat currents and mutual friction: 1950s**
- Through a series of experiments measuring heat currents, Henry Hall (left) and Joe (W.F.) Vinen (right) established the idea of **mutual friction between the quantum and normal fluids along vortex cores**.
- First use of second-sound: waves carrying oscillations between the normal and quantum parts of a superfluid.

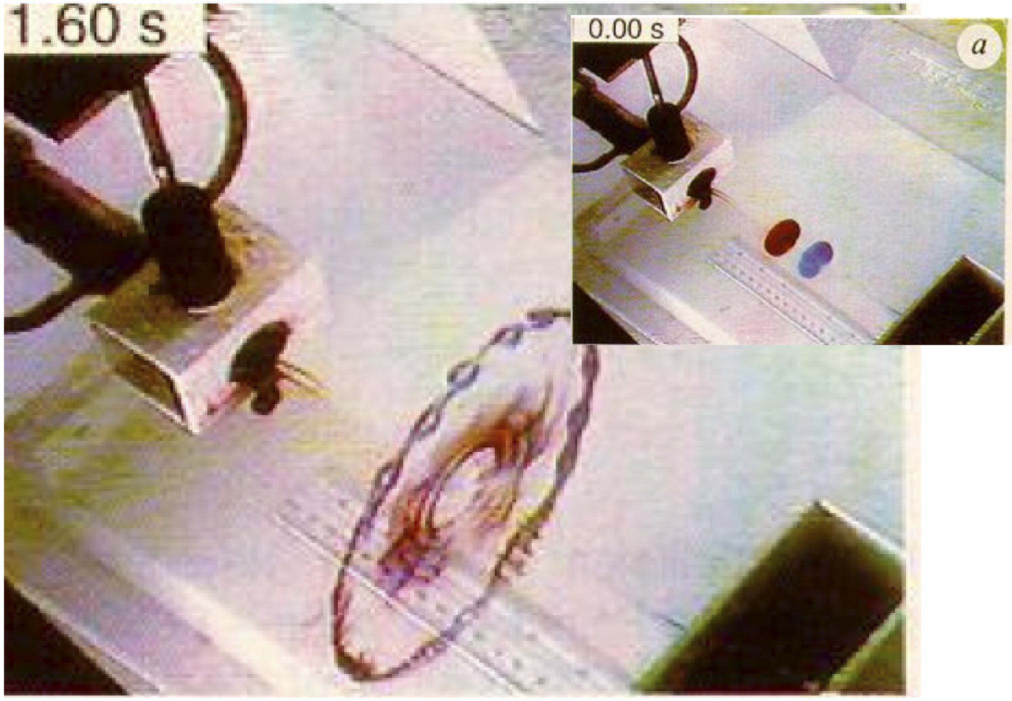




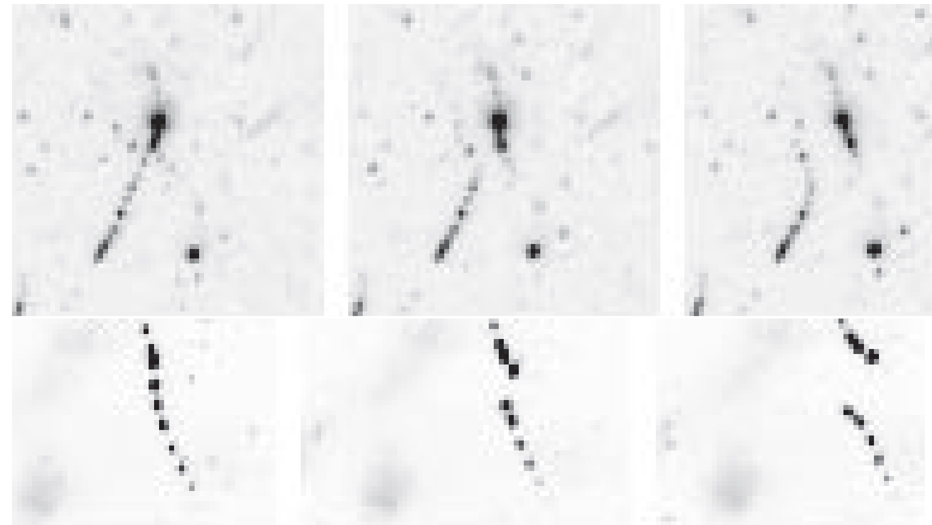


Experimental vortices: Can we represent them numerically?

Lim/Nickels Colliding classical rings

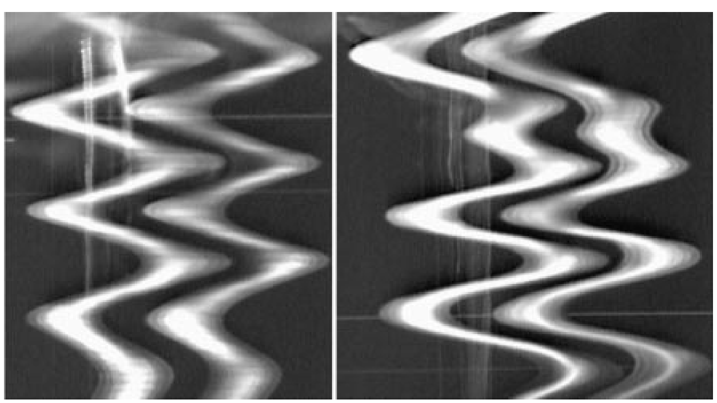


Reconnecting Superfluid Vortices
Bewley et al, PNAS 2008

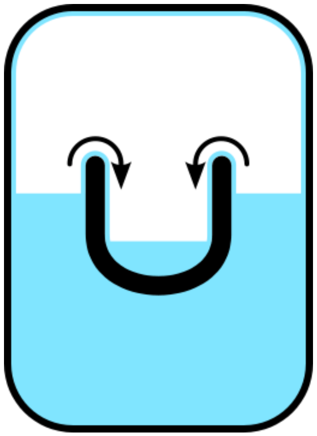


Anti-parallel vortices

Stratified: Billant & Chomaz



Contrails



Early superfluid experiments 1937-38

Quantum turbulence: Described by a tangle of **quantum vortices** in a superfluid or Bose-Einstein condensate (BEC).

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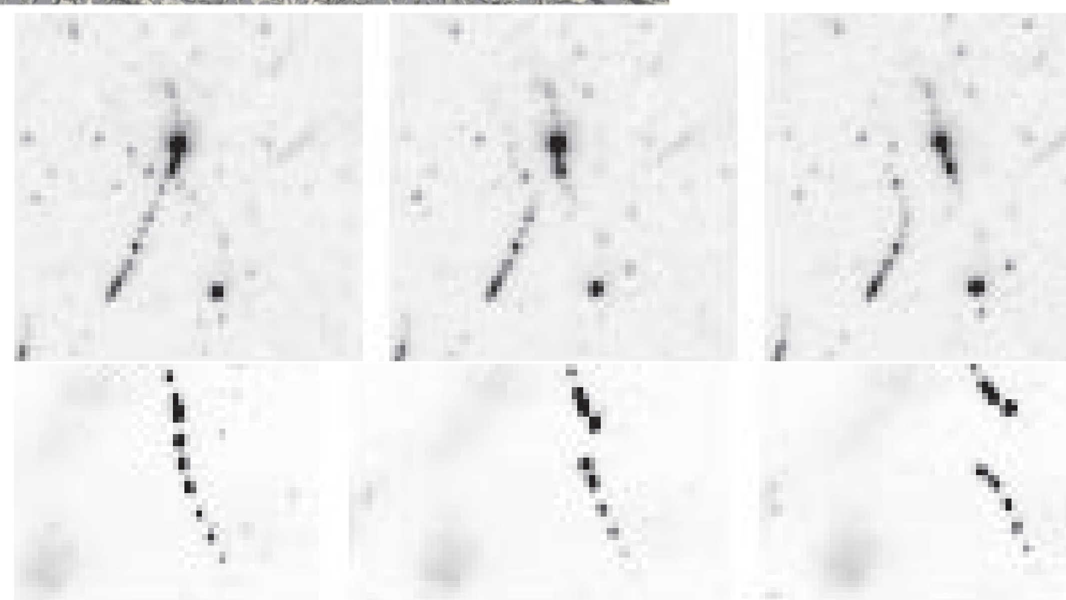
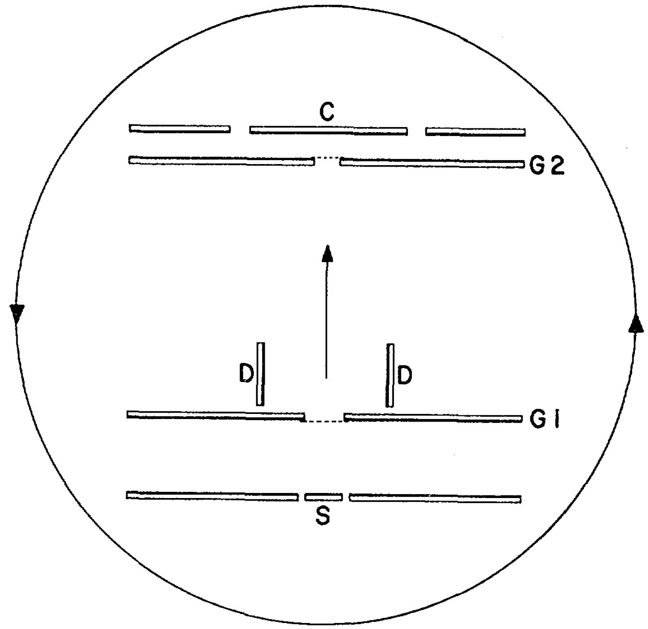
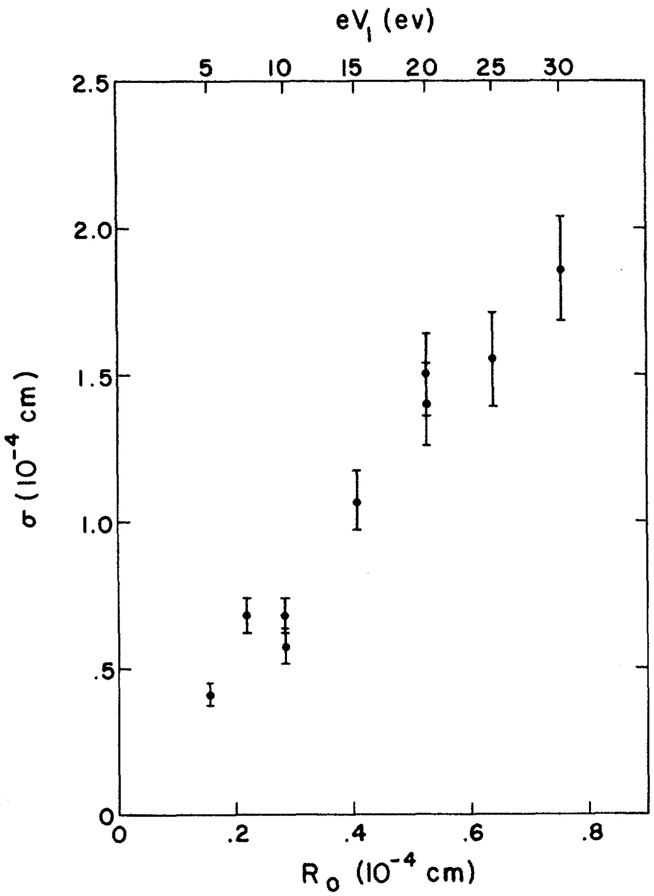
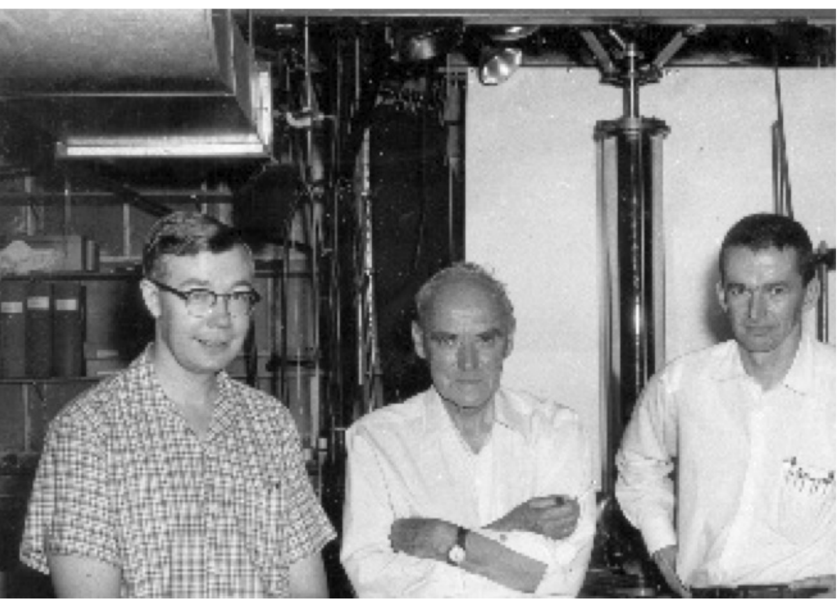


But I am getting ahead of my-self.

- **Rotating buckets and ions: 1960s/70s**
- Donnelly/Schwarz at Chicago created one, then fired ions at it.
- The ions have vortex rings attached, so these are really vortex ring scattering experiments.
- Ions from source S were fired through the superfluid vat between the electrodes G1 and G2.

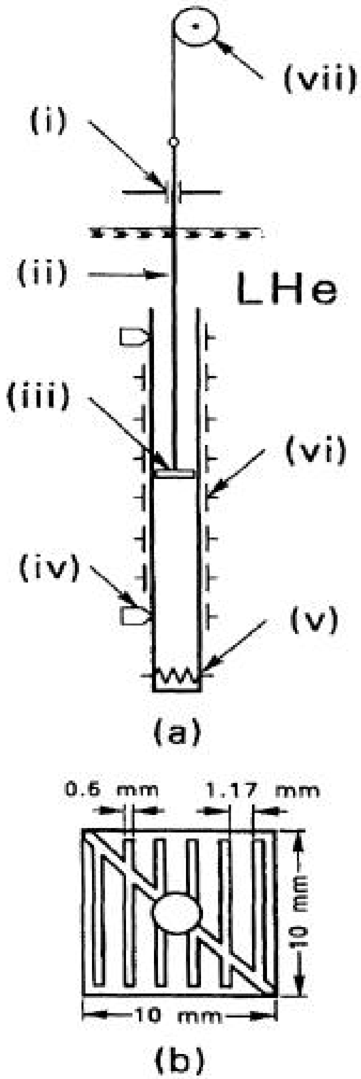




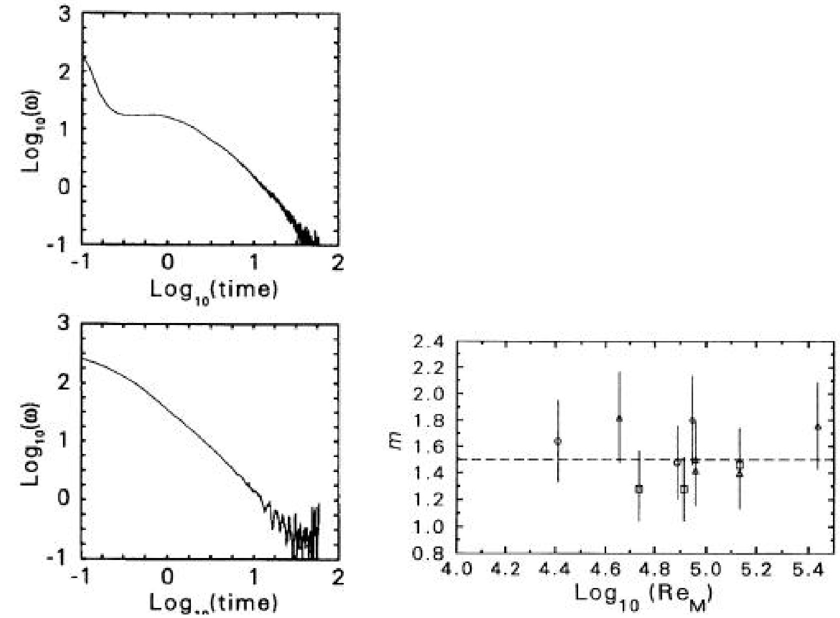


Russ Donnelly (Chicago), GI Taylor (Cambridge), Dave Fultz (Chicago)

- More voltage meant more energy and bigger, but slower, rings.
- **Experiment established the possibility of quantum vortices distinct from those due to rotation and that they were very thin:**
- That is there was no interaction unless the vortex rings hit a central vortex.

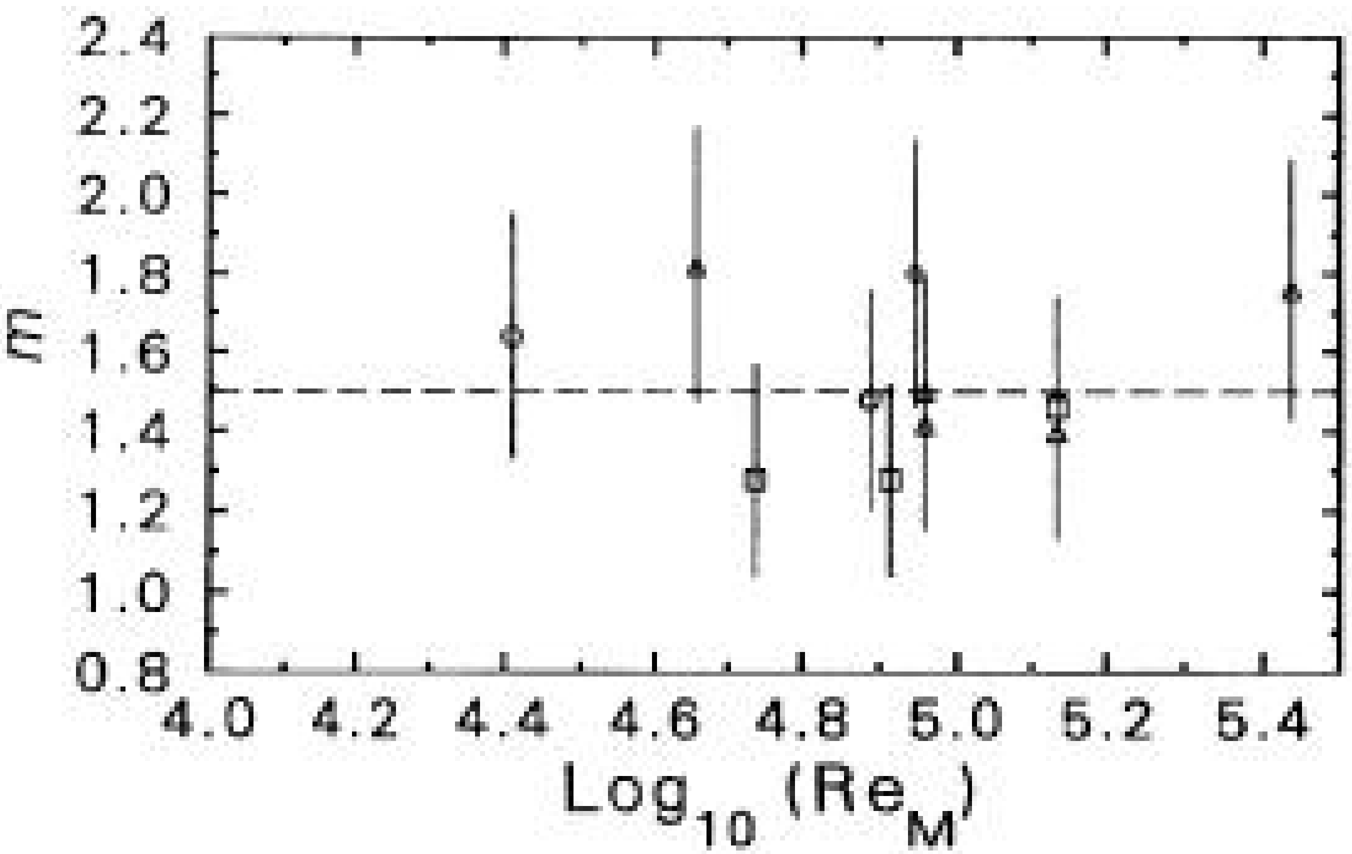


Smith, Donnelly, Goldenfeld, Vinen Phys. Rev. Lett. (1993)
(a) Layout of apparatus used to study grid turbulence. (i) Vacuum seal, (ii) 5/16 rod, (iii) grid, (iv) germanium thermometer, (v) counterflow heater, (vi) second sound transducer pair, and (vii) stepper motor. (b) Detail of grid construction.



- Two experiments (counterflow versus grid) show the same decay at long times. **This obeys $\ell \sim t^{-m}$ with $m = 1.5 \pm 0.2$.**
- This corresponds to as classical decay of enstrophy of $\Omega \sim t^{-3}$ **which corresponds to a classical kinetic energy decay of $KE \sim t^{-2}$**

However, despite serious flaws, the theoretical interpretation was still based on the existence of the normal fluid component.



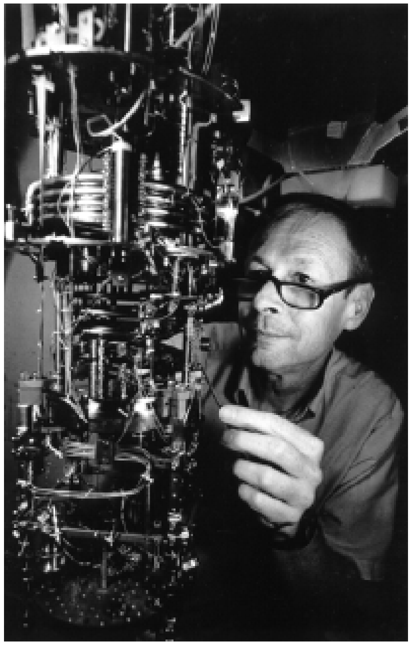
- **Quantum turbulence was not expected to be similar to classical turbulence.**

- Note: The $KE \sim t^{-2}$ law is the classical decay law only when there are periodic boundary conditions. (Reference: Kerr, 1981, PhD thesis, Cornell) This condition and decay rate are never realised in a physical system with real boundaries.

- Decay was based on the two-fluid model. **In addition to the ideal superfluid component, there was a classical (maybe not Navier-Stokes) normal fluid component, and mutual friction to couple the two. So maybe quantum is not so different than classical?**

1975: Why my Chicago professors said to go to Cornell

DAVID M. LEE, DOUGLAS D. OSHEROFF and BOB RICHARDSON



Bob Richardson at cryostat

Nobel toast of David Lee and Bob Richardson

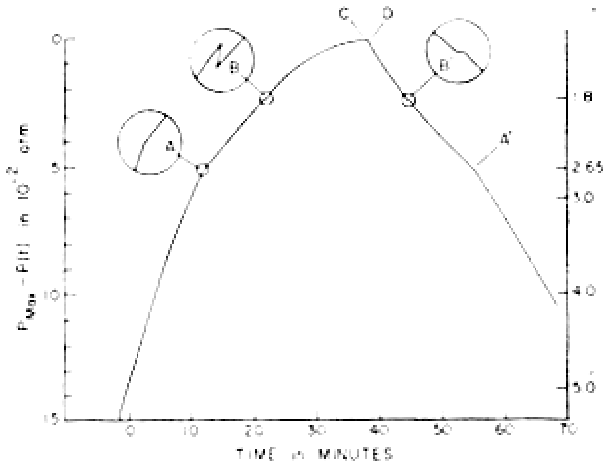


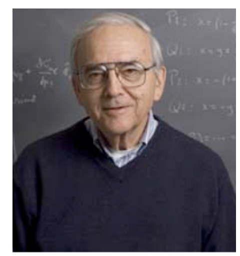
FIG. 2. Time evolution of the pressure in the Pomeranchuk cell during compression and subsequent decompression.

Phase diagram from 1972 PRL.

Doug Osheroff

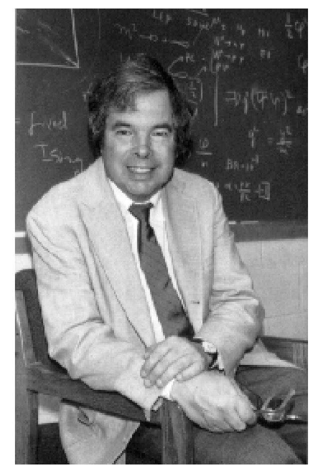
And also the Renormalization Group

Ben Widom brought Michael Fisher to Cornell, who shared 1980 Wolf Prize with Kadanoff and Wilson, who got the Nobel.



Ben Widom

Michael Fisher



Ken Wilson 1936-2013 Nobel 1982

Gross-Pitaevskii equations. The nonlinear Schrödinger equation integrated for: an ideal quantum fluid, superfluid or Bose-Einstein condensate.

$$\frac{1}{i}\frac{\partial}{\partial t}\psi = 0.5\triangle\psi + 0.5\psi(1-|\psi|^2)\,. \tag{1}$$

Fluid-like representation and defects that behave like vortices.

i.e. quantum vortices

If: $\psi=\sqrt{\rho}e^{i\phi}$, $\boldsymbol{v}=\nabla\phi$, **and** $\boldsymbol{\Sigma}$ **a quantum pressure, then**

$$\rho\frac{D\boldsymbol{v}}{Dt} = -\nabla p + \nabla\boldsymbol{\Sigma} \qquad \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0 \tag{2}$$

The Navier-Stokes equations or the NSE:

$$\frac{\partial\boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \underbrace{\nu\triangle\boldsymbol{u}}_{\textbf{dissipation}} \tag{3}$$

$$\rho = 1 \qquad \underbrace{\nabla\cdot\boldsymbol{u}=0}_{\text{incompressibility}}$$

What I integrate in time is vorticity: $\boldsymbol{\omega}=\nabla\times\boldsymbol{u}$

$$\frac{\partial\boldsymbol{\omega}}{\partial t} + \underbrace{(\boldsymbol{u}\cdot\nabla)\boldsymbol{\omega}}_{\text{advection}} = \underbrace{(\boldsymbol{\omega}\cdot\nabla)\boldsymbol{u}}_{\text{vortex stretching}} + \underbrace{\nu\triangle\boldsymbol{\omega}}_{\textbf{dissipation}} \tag{4}$$

Euler

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p \qquad (5)$$

$$\rho = 1 \qquad \underbrace{\nabla\cdot\boldsymbol{u} = 0}_{\text{incompressibility}}$$

What is integrated is vorticity: $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \underbrace{(\boldsymbol{u}\cdot\nabla)\boldsymbol{\omega}}_{\text{advection}} = \underbrace{(\boldsymbol{\omega}\cdot\nabla)\boldsymbol{u}}_{\text{vortex stretching}} \qquad (6)$$

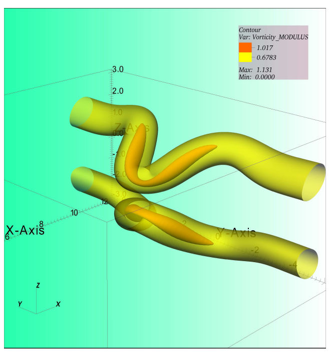
Gross-Pitaevskii

$$\rho\frac{D\boldsymbol{v}}{Dt} = -\nabla p + \nabla\boldsymbol{\Sigma} \qquad (7)$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0$$

Integrate: $\psi = \sqrt{\rho}e^{i\phi}$, $\boldsymbol{v} = \nabla\phi$

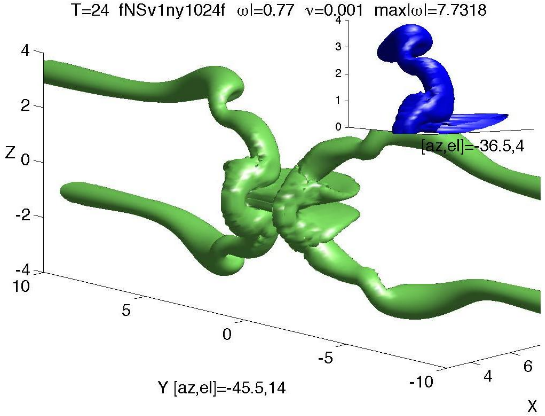
$$\frac{1}{i}\frac{\partial}{\partial t}\psi = 0.5\nabla^2\psi + 0.5\psi(1-|\psi|^2) \qquad (8)$$



3D anti-parallel

Full domain, Early times.

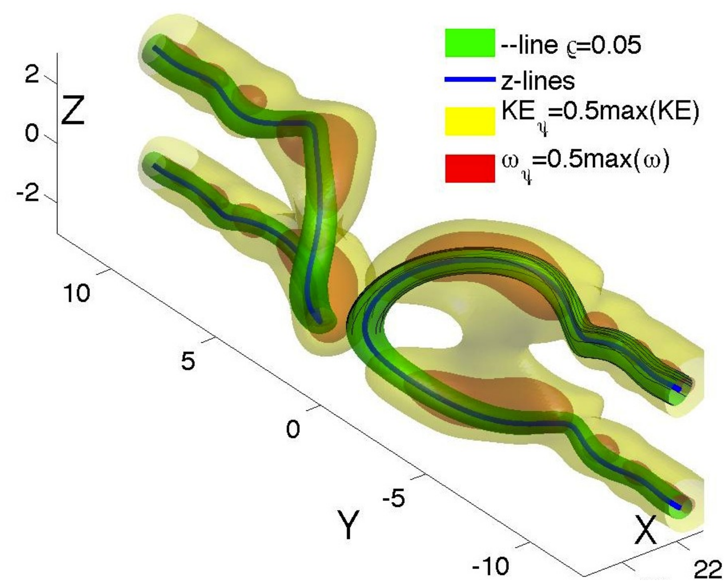
Kinks,
then reconnect



CLASSICAL - messy reconnection

QUANTUM - clean reconnection

antip-bbe t=4 ς, max(KE)=0.83 maxlωl=0.4 ω-lines



How can there be circulation?

- **A quantum fluid is irrotational except along infinitely thin defects, quantum vortices.**
  - If the wavefunction is $\psi = \sqrt{\rho} e^{i\phi}$, consider points around which ϕ changes by 2π.
  - Around these defects $\int \boldsymbol{v} \cdot d\boldsymbol{s} = 2\pi$ and the **quantum** circulation is defined as $\Gamma = \hbar/m = 2\pi$ where m is the pass of a single atom. **for all vortices.**
- In a classical fluid, vorticity is distributed uniformly in space and the circulation about vortex cores depends on the initial condition.
  - For an ideal classical fluid **(Euler equations)**, these values of the circulation are constant along Lagrangian trajectories.

    With viscosity **(Navier-Stokes)** vortices **reconnect**, the topology of circulation changes and **not** follow Lagrangian trajectories.
- **What is the analogy** for a quantum fluid for how in a viscous fluid the circulation changes in time?

  There will be **reconnection** associated with how the topology of the zero density lines changes.

Energy in a quantum fluid?

- **A quantum fluid has no dissipation.**
- **Its Hamiltonian includes components described as the kinetic, quantum and interaction energies**

  The total should not decay in time

  except through unknown physics or through interactions with the boundaries.
- However, it will be shown that **energy can be transferred between the components so that the kinetic energy can decay.**
- A quantum fluid is compressible, with a complex relation between the density and the pressure.

These observations suggest that despite different physics, classical and quantum turbulence share many properties. **Turbulence in both has notable similarities.**

These include:

- There is a fluid-like equation (Madelung transformation)
- Circulation
- Energy transfer
- Reconnection

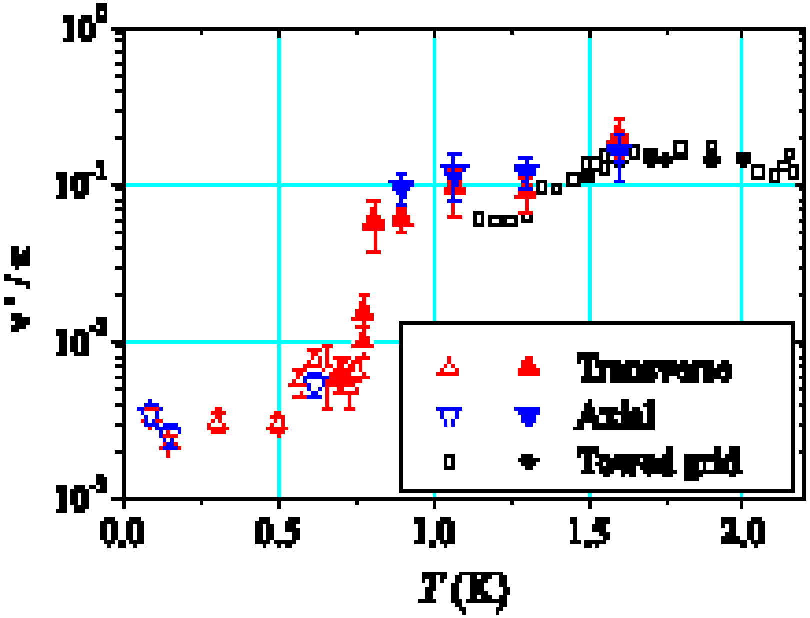
Outline:

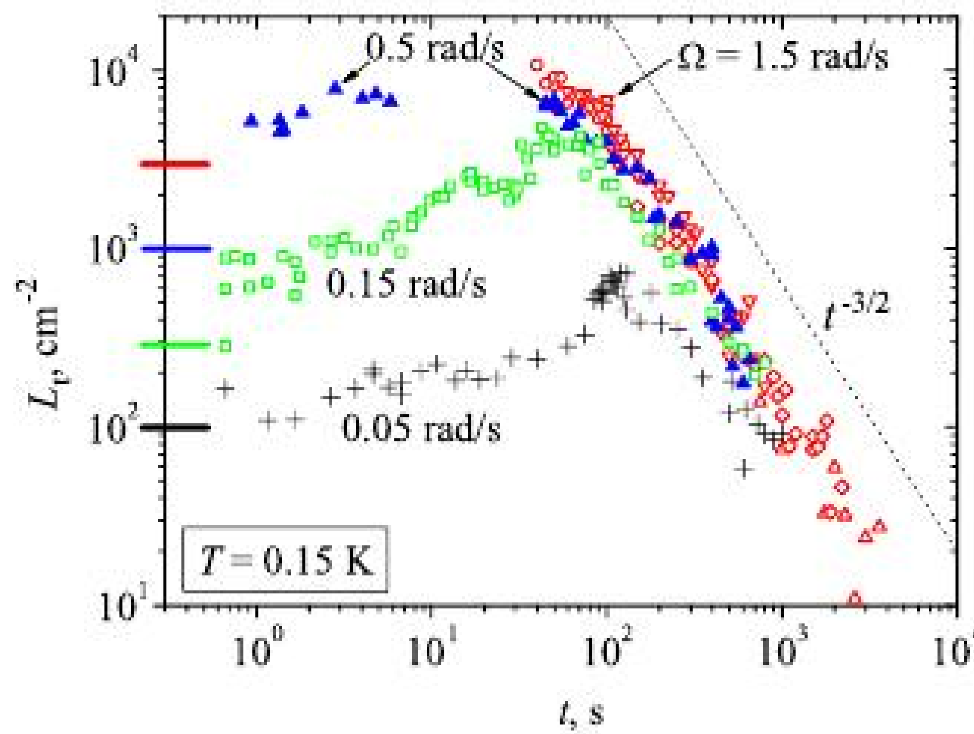
- **PUNCH-LINE: Anti-parallel quantum vortex reconnection suggests how to generate:**
- Depletion of kinetic energy by creation of interaction energy during vortex stretching in a quantum system.
- Non-local oscillations. Not really waves, and certainly not 1D Kelvin waves.
- Disconnected vortex rings that propagate out of the system.
- These rings also can evaporate into 3D phonons/Kelvin waves.
- A -5/3 spectrum. **The Mechanism:**
- Probably gradient/kinetic energy cascades to small scales.
- Is converted to interaction energy, which cascades to large scales.

What about an ENCORE?

- Classical vortex reconnection probably does many of the same things.
- Differences are many and similarities are proving more difficult to show.
- If this could be shown: Then these events could be the building blocks for all of classical turbulence, **with minimal viscosity**, including: **finite energy dissipation and a -5/3 energy spectrum.**

Ultra-cold Walmsley, Golov etc (2007).



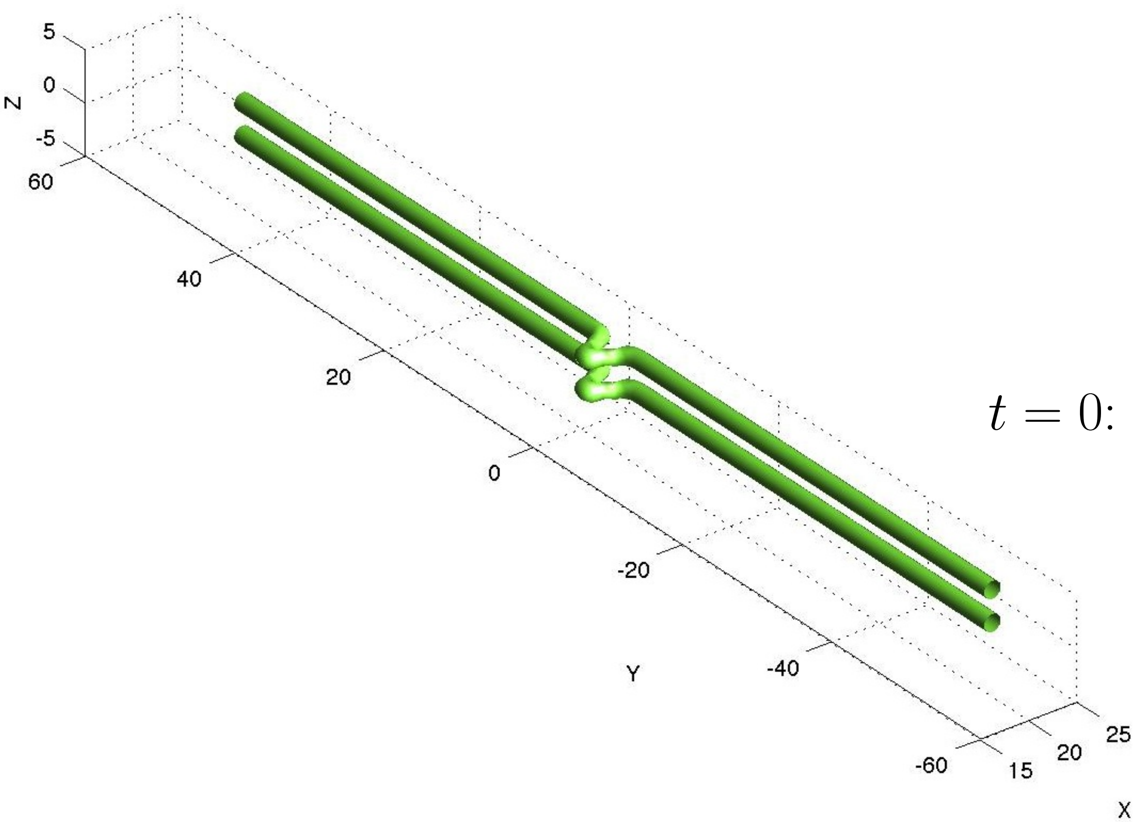


Left: FIG. 5 The effective kinematic ν' after a spin down from $\Omega = 1.5$ rad=s measured in the transverse (△) and axial (▽) directions. Closed (open) triangles cor- respond to measurements with free ions (charged vortex rings). Error bars specify the uncertainty of fitting. Squares and dia- monds: second sound measurements of grid turbulence [12,22].

Right: FIG. 2 $L_t(t)$ at $T = 0.15$ K for four values of Ω. Average electric fields used for Ω = 1.5 rad/s: 5 V/cm (◇), 10 V/cm (△), 20 V/cm (◯), 25 V/cm (▽). The dashed line shows the dependence $t^{-3/2}$. Horizontal bars indicate the equilibrium values of L at Ω=1.5, 0.5, 0.15. 0:05 rad/s (from top to bottom).

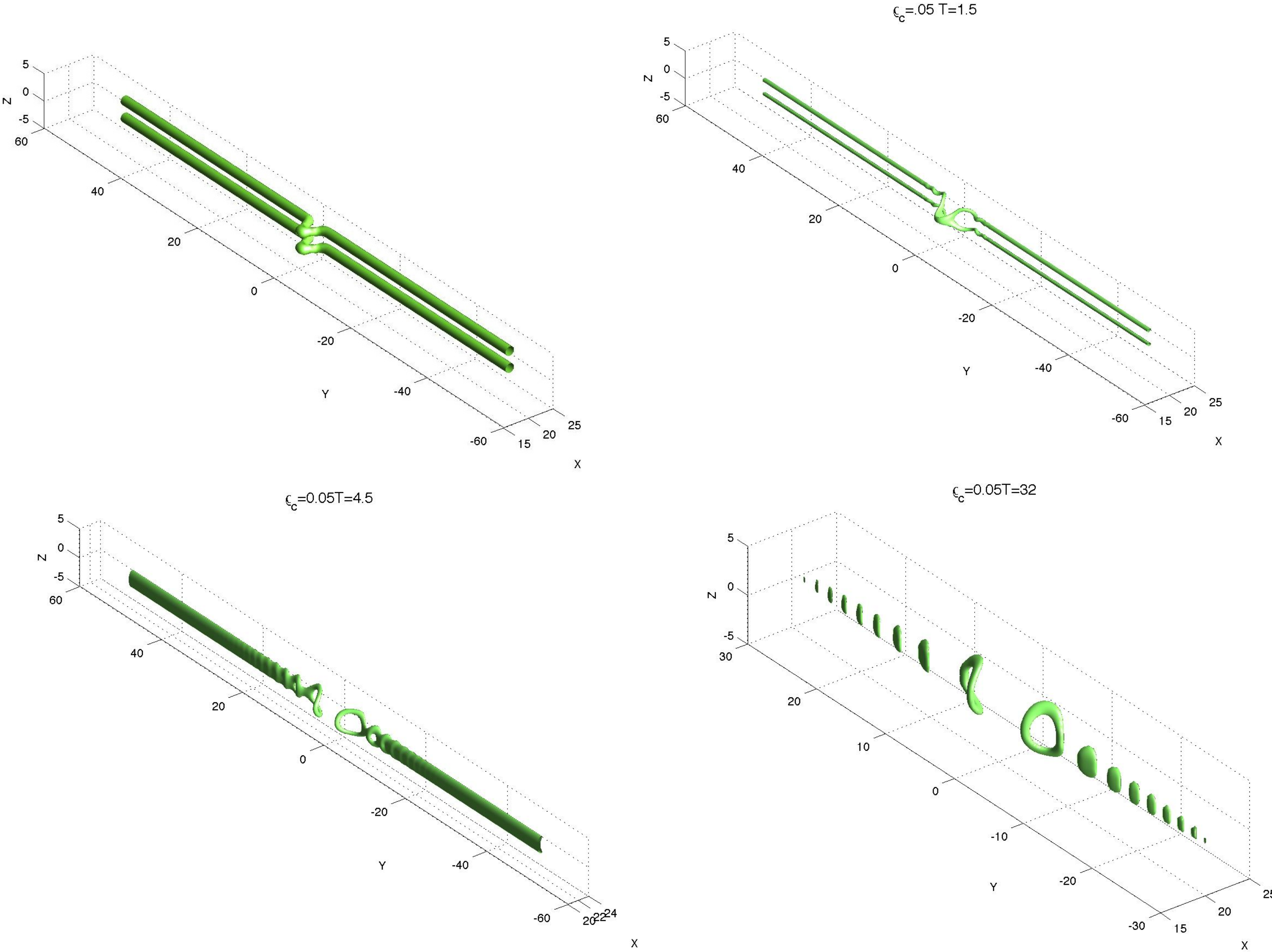
Now the normal fluid explanation cannot be used and the decay still looks classical. Where is the energy going?

- **My numerics:** $128 \times 512 \times 64$ for $8\pi \times 16\pi \times 4\pi$ domain.
  - Spectral,
  - 3rd-order Runge-Kutta on nonlinear term.
  - Integrating factor on linear term.
  - Timestep chosen by $\nabla\psi$.
  - Using symmetries, I simulation only 1/2 of 1 of 2 vortices.
- Still, to get smooth functions at the boundaries, I found that the initial condition needs to be a superposition of 24 image vortices.
- This suppresses anomalous waves generated by discontinuities in the I.C.

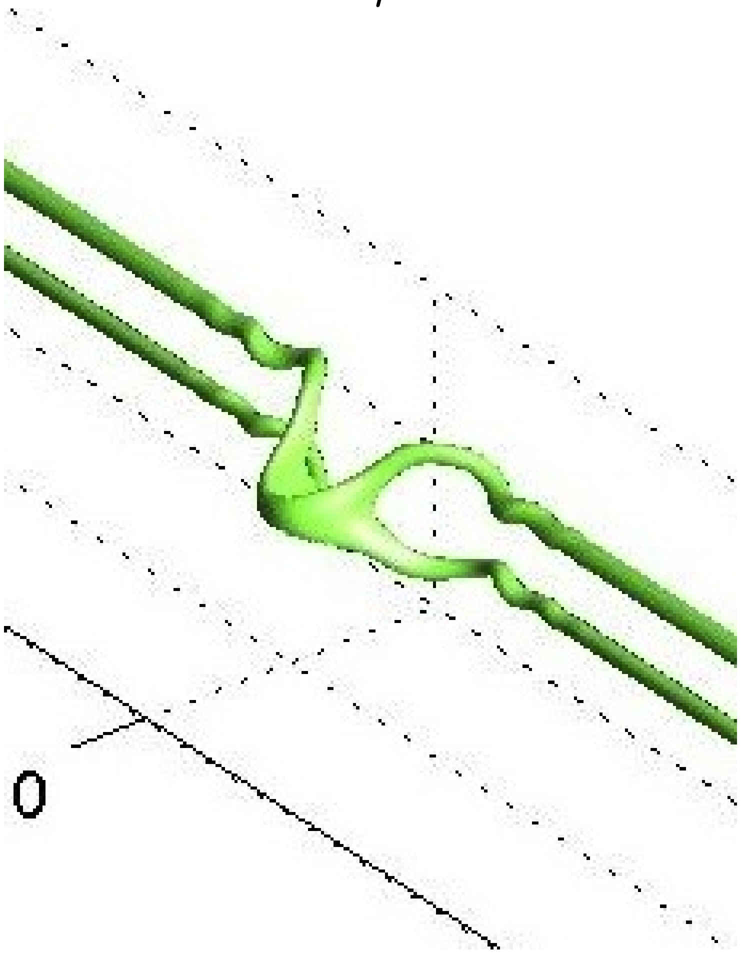


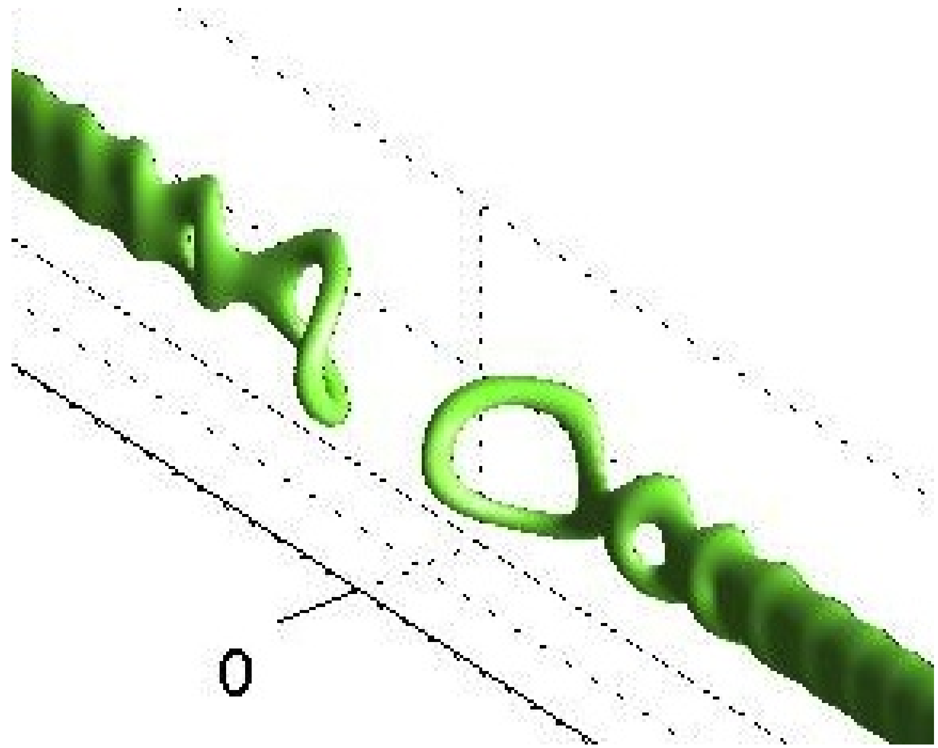
$t = 0$: $\rho = 0.05$ surfaces. Full domain

$t = 0,\ 1.5,\ 4.5,\ 32$: $\rho = 0.05$ surfaces. Full domain

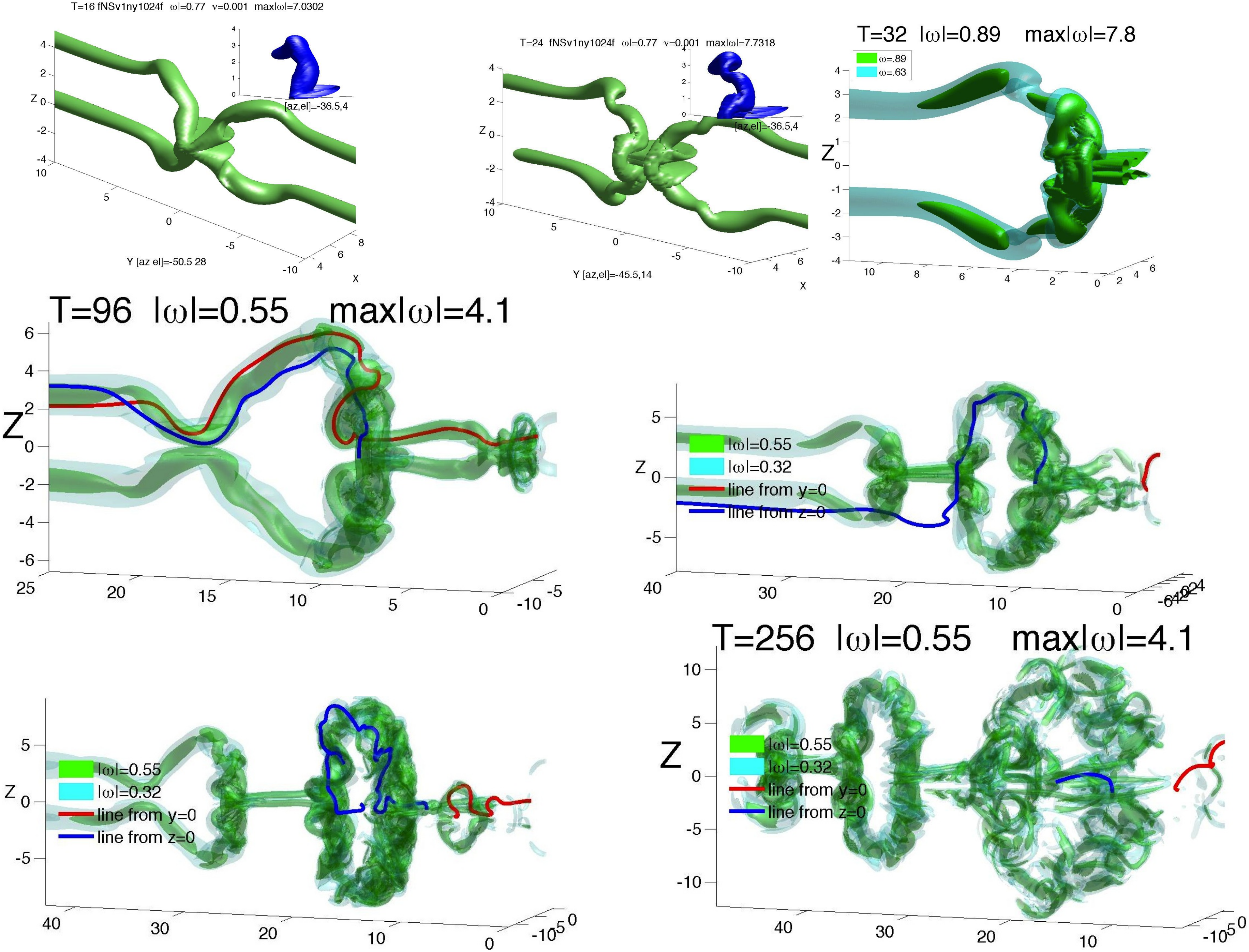


$t = 1.5$ and 4.5: $\rho = 0.05$ surfaces.





Navier-Stokes Overview: From reconnection to turbulence



But is this turbulence?

What makes a flow turbulent?

- An energy cascade? A $k^{-5/3}$ spectrum? Energy decay?

What is the energy?

- There are two parts to the Hamiltonian, which is conserved.
- There is $K_{\nabla\psi} = \int dV \frac{1}{2}|\nabla\psi|^2$, the gradient/kinetic energy.
- There is $E_I = \int dV \frac{1}{4}(1-\rho)^2 = \int dV \frac{1}{4}(1-|\psi|^2)^2$. This is the interaction energy.

$K_{\nabla\psi}$ and E_I can be expressed in Fourier space. Thus, their interactions and the direction of cascade in Fourier space can be calculated. **Work in progress**

Note that $K_{\nabla\psi} = \int dV \frac{1}{2}(\sqrt{\rho}\boldsymbol{v})^2 + \int dV \frac{1}{2}|\nabla\sqrt{\rho}|^2$, components sometimes called the velocity energy and the quantum energy. Their spectra are nonlinear quantities and are not well-defined, with high wavenumbers unphysically dominated by the singularities on the quantum vortex cores.

- **What would effective kinetic energy decay be like in this system?**

Either: **Depletion of $K_{\nabla\psi}$ within a region**.

Conversion of $K_{\nabla\psi}$ into **interaction energy E_I**.

Or conversions into low intensity fluctuations in $K_{\nabla\psi}$ and E_I, that is waves.

- **What decay property do the experiments actually measure?**

What decay property do the experiments actually measure?

- Experiments measure the scattering of either 2nd-sound or ions off the vacuums around the vortex cores. **Let $V_{\rho=0}$ be volume of vacuum around lines.**

  Then a number of largely unjustified assumptions are made to relate this to the kinetic energy.

1. It is claimed that $V_{\rho=0}$ can be converted into the length of the vortex lines L because their cross-sectional area A is fixed, therefore $L = V_{\rho=0}/A$.
2. Use this to generate an effective enstrophy Z_e (mean squared vorticity).
3. Assume the classical relation between enstrophy and kinetic energy: $\nu_e Z_e = \frac{d}{dt} K_{\nabla\psi}$, where ν_e is some effective viscous coefficient.

- If $L \int ds\ell = t^{-3/2} \quad \Rightarrow \quad Z_e = \int ds\ell^2 \sim t^{-3}$

  In classical, homogeneous isotropic turbulence in a periodic box the following is observed (Kerr, thesis, 1981; originally due to Patterson)

$$\text{Energy}: \quad K(t) \sim t^{-2} \quad \Rightarrow \quad \nu \overline{Z} \sim t^{-3}$$

- Note that this decay law is never seen experimentally, as all classical experiments have boundary layers with $K(t) \sim t^{-\gamma}$, $\gamma \sim 1.2 - 1.6$.
- **How is the observed decay of L explained?**

How is the observed decay of L explained?

- All explanations for how energy can be removed in a quantum fluid assume that the energy sink is the non-ideal boundaries.

  The question is how to get it there. Three mechanisms have been proposed:

i) Quantum vortex lines could reconnect to form vortex rings, which then propagate out (Feynman, 1955).

ii) Linear waves, or phonons could be generated internally and propagate out.

iii) Waves on vortices could cascade to small scales and their energy be radiated as phonons (Kozik/Svistunov, 2004; Laurie et al., 2010).

- **Do any work?**

iv) I will propose vortex stretching and the conversion of $K_{\nabla\psi}$ into E_I.

v) All vortex wave explanations assume that the local induction approximation is valid approximation for quantum vortex motion.

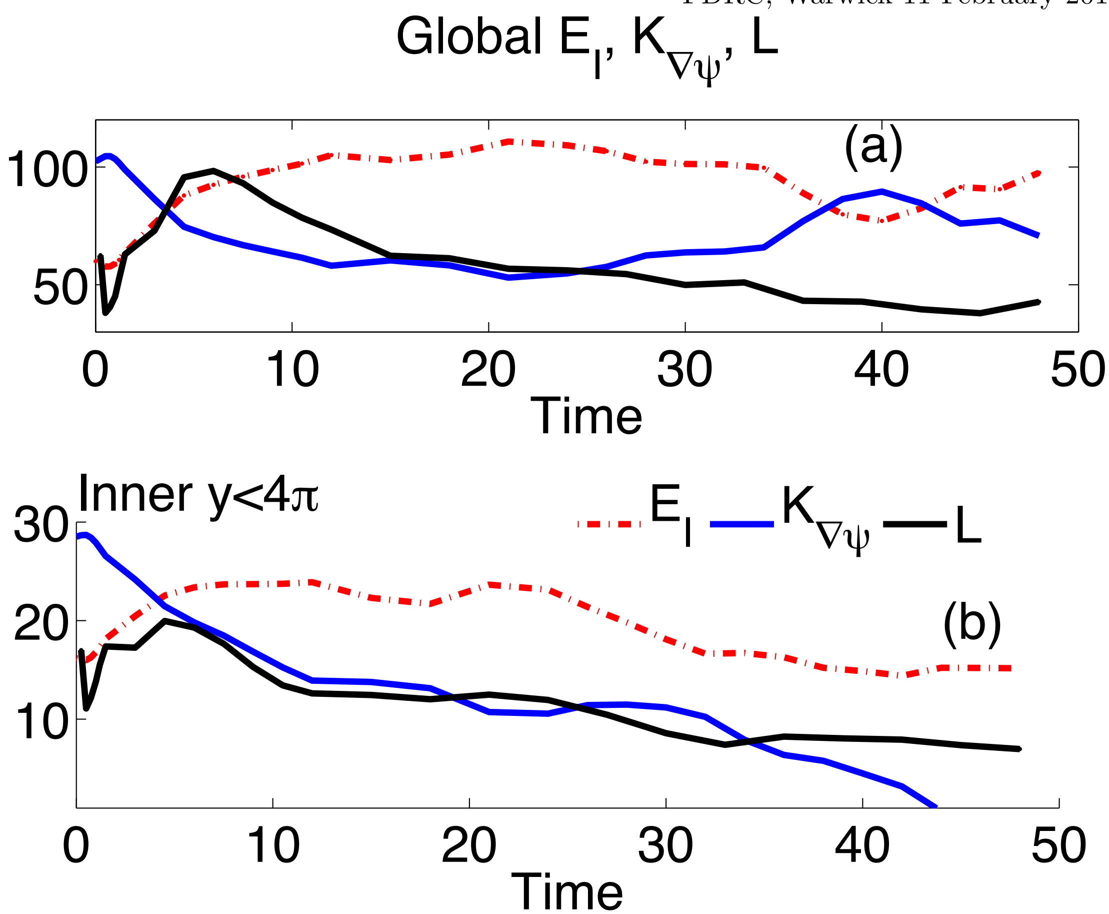
All two ring and two line calculations I have recently done, many reproducing existing results, **say no**.

Why did Klaus Schwarz suggest it? That is another talk.

vi) Phonon generation probably requires ring generation first.

vii) **First goal: Analysis of energy components and vortex line length,**

ix) then compare to i) and iv).



Estimates of the line length compared to changes in the interaction and kinetic energies.

a) Analysis over the full domain.

b) Only the first y-quadrant.

a) There is strong global E_I and vortex line L growth for $0.5 < t < 6$. For $6 < t < 25$ both $K_{\nabla\psi}$ and L decrease. For $T > 30$ the global kinetic energy $K_{\nabla\psi}$ grows again. This is associated with the accumulation of energy for $y > 4\pi$.

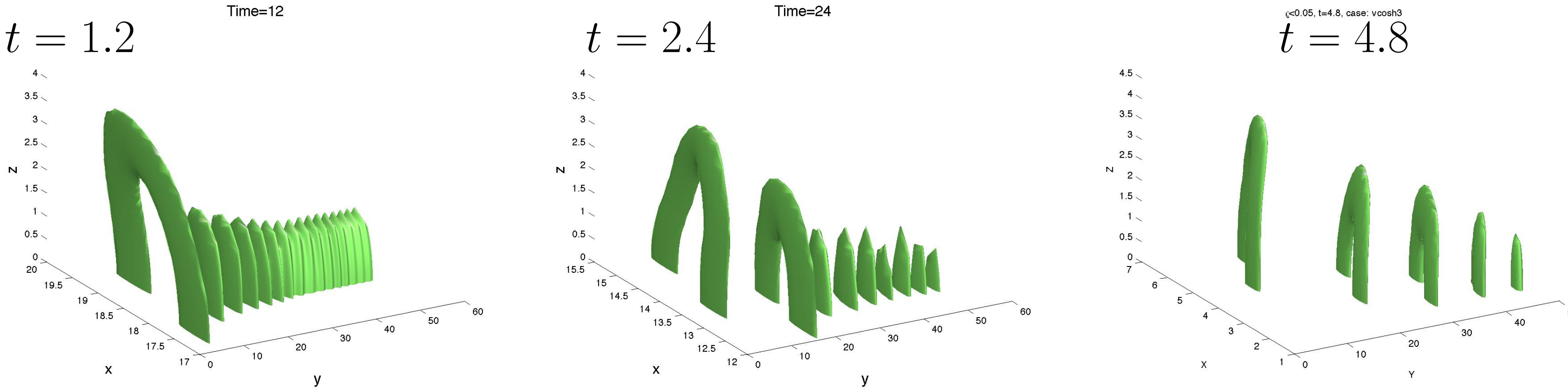
b) First y-quadrant. Shows that $K_{\nabla\psi}$ and L continue to decrease in the original interaction region.

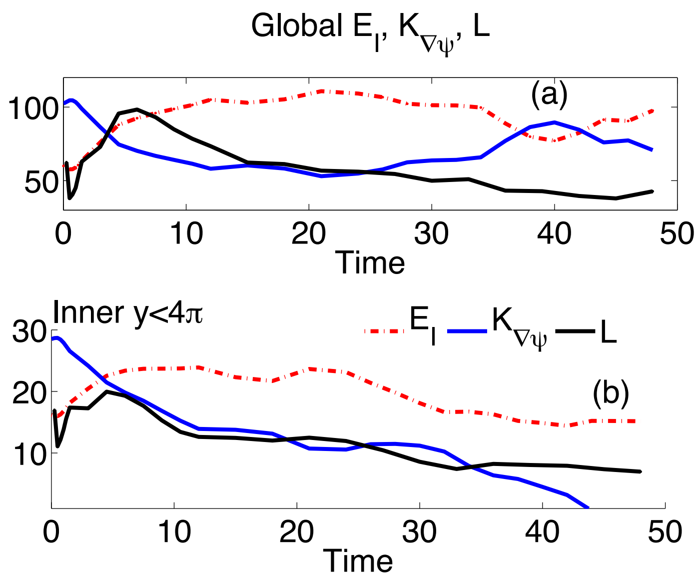
- Is experimental line length L a useful proxy for a pseudo-classical vorticity associated with kinetic energy? **Probably yes.**

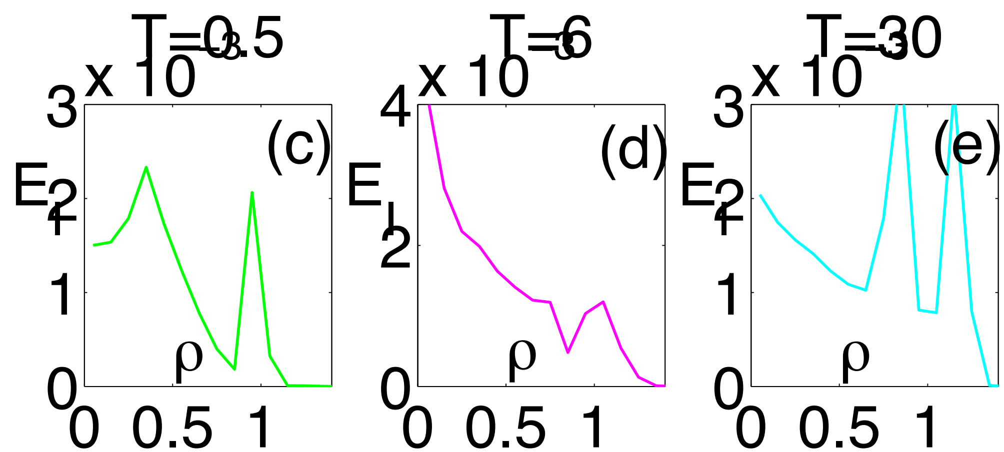
  In a physical cell, this energy would be absorbed by the outer wall.

- **Does this energy become rings or phonons in the present case?**

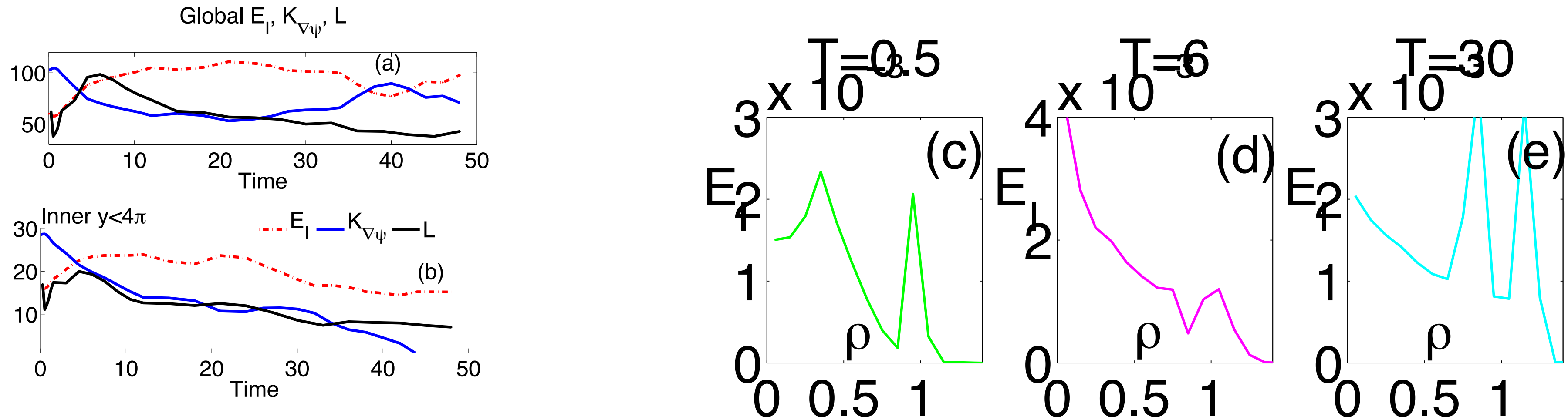
Rings propagate out







c-e) **Distributions** of the E_I with respect to density at $t = 0.5, 6, 30$ to show how energy appears to flow from $K_{\nabla\psi}$ to E_I to waves.



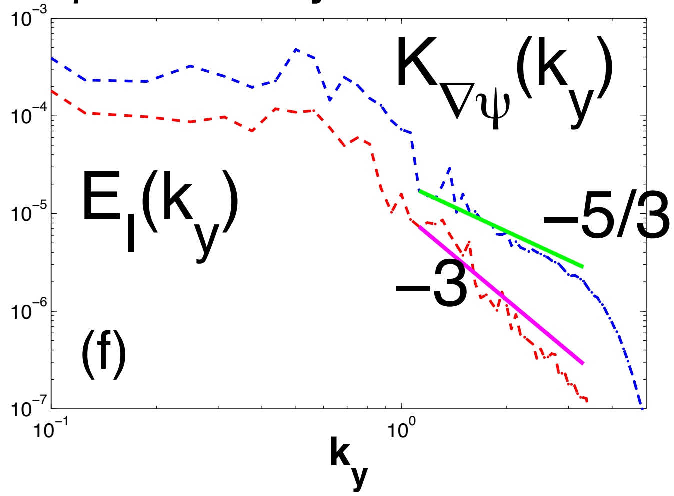
Distributions and line length To understand the different stages, subplots Fig. c-e show distributions of E_I with respect to density at three times. The $t = 0.5$ distribution in Fig. c demonstrates that initially E_I has a maximum near $\rho = 1$.

Fig. d shows that at $t = 6$, when stretching is greatest, there has been a dramatic growth in E_I, with most of the growth for $\rho \approx 0$. This implies a large growth in the number of points with $\rho \approx 0$. Note that the increases in E_I for $t \leq 20$ are compensated for by a strong decrease in the global kinetic energy in Fig. a.

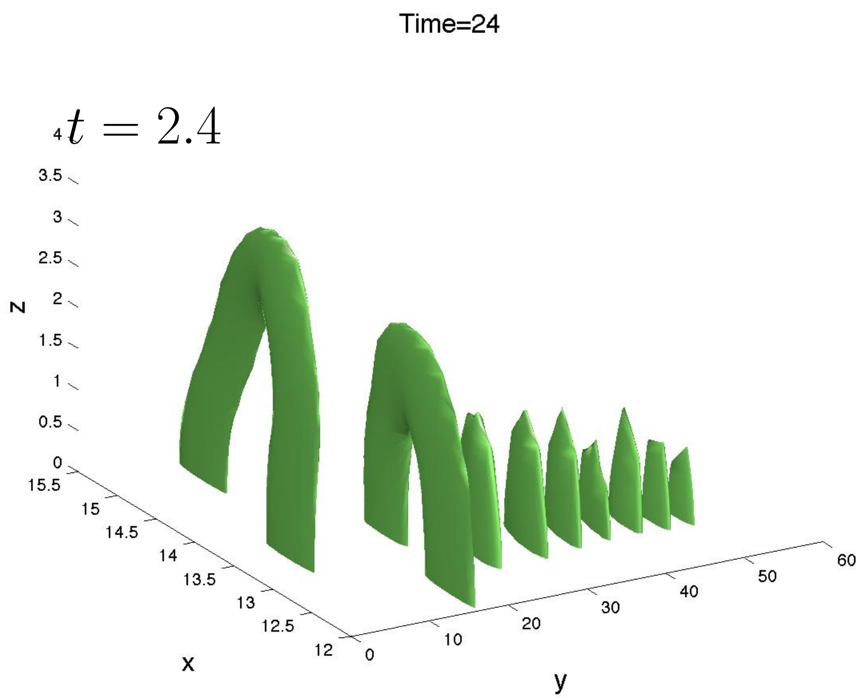
Immediately after $t = 6$, L begins to decrease dramatically while the kinetic energy $K_{\nabla\psi}$ continues to decay, which is compensated for by a continuing increase in the interaction energy E_I. At the end of this stage, there is a growth in large values of E_I on either side of $\rho = 1$, shown by the distribution at $t = 30$. Around, not at, because for $\rho = 1$, $E_I \equiv 0$, This would be consistent the development of waves and visualizations of waves being emitted from colliding vortices.

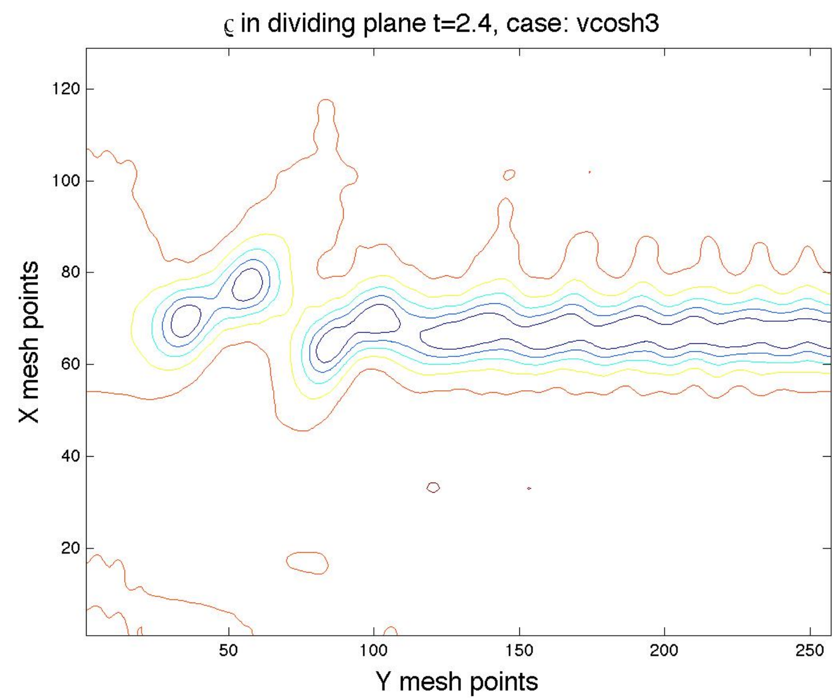
The decrease in the global kinetic energy does not persist. Eventually interaction energy is converted back into kinetic energy, possibly due to oscillations between $K_{\nabla\psi}$ and E_I in the released phonons. Similar oscillations were observed in GP calculations with a symmetric Taylor-Green initial condition (Nore et al., 1997). This would not persist in a real experimental device because the waves would be absorbed by the non-ideal boundaries.

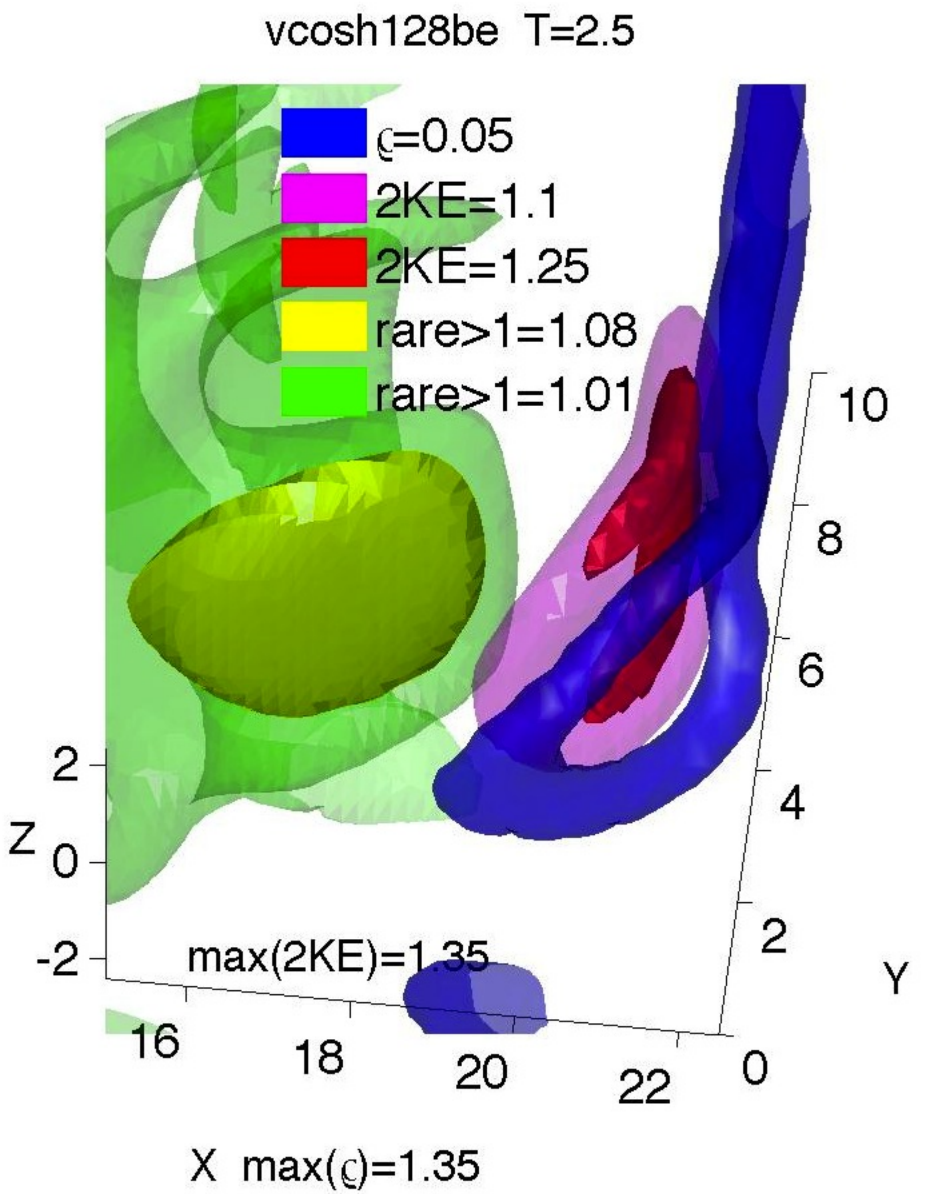
$K_{\nabla\psi}$ and $E_I(k_y)$ spectra. T=48

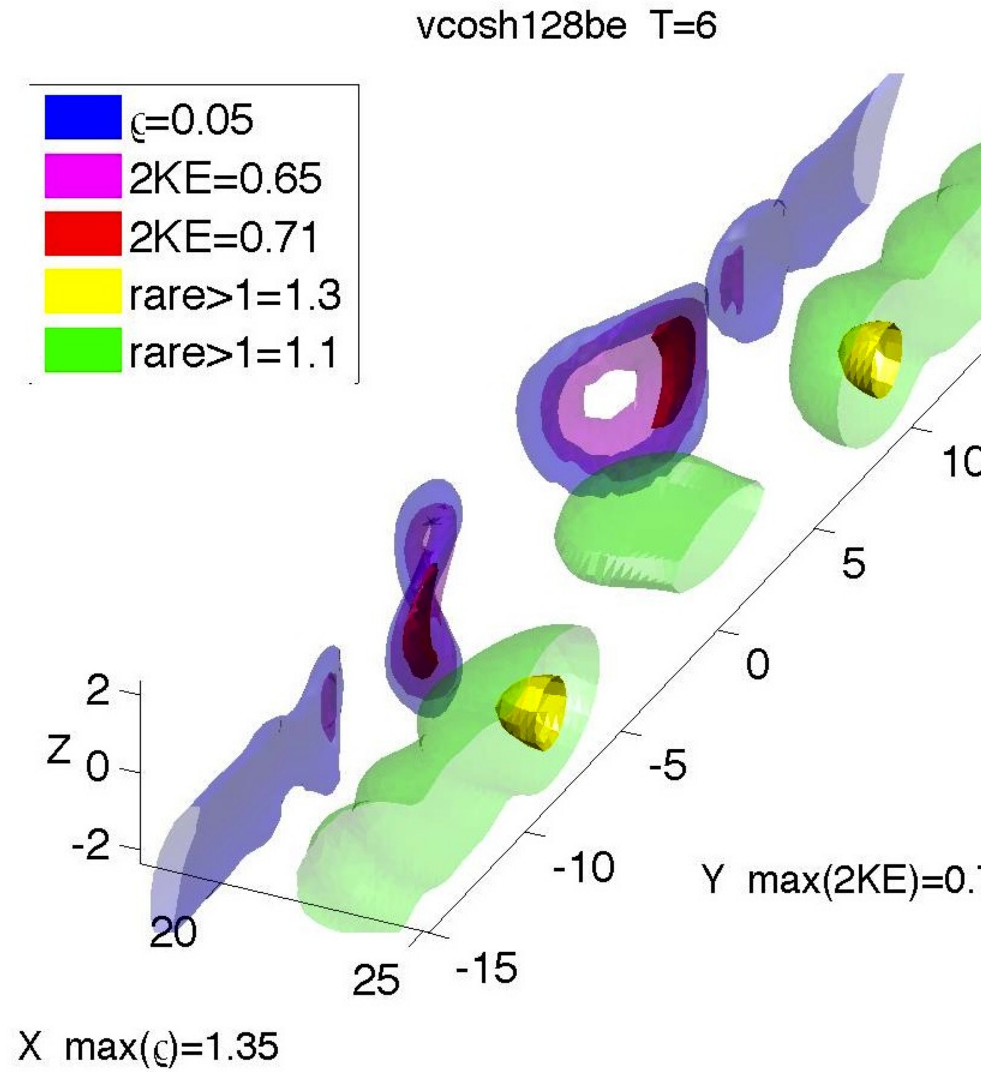


Spectra: f) By $t = 48$, there is a noticeable $K_{\nabla\psi}(k_y)$ the order of $k_y^{-5/3}$, while $E_I(k_y)$ is still dominated by a k_y^{-3} slope. Spectra in the other directions have similar trends but are less distinct.









t=2.5

Green: Ejected GP wave. $\rho = 0.05$**.**

$2K_{\nabla\psi} = 1.1$.

$2K_{\nabla\psi} = 1.25$**.**

Later time $\mathbf{t} = \mathbf{6}$.

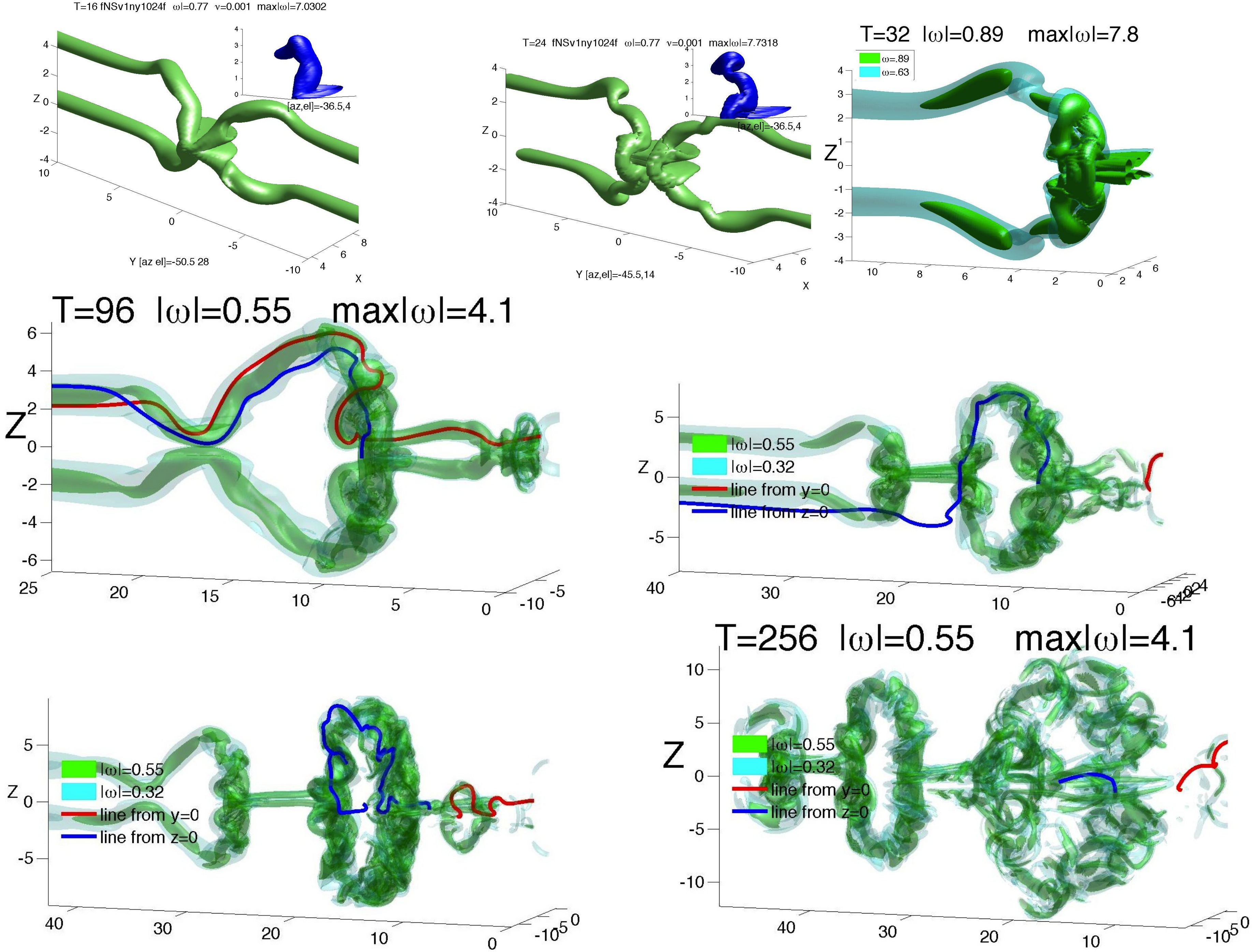
Intense $2K_{\nabla\psi}$ is inside vortex.

Note twist in nearer reconnected vortex.

Twisting, stretching: All the elements needed for a cascade.

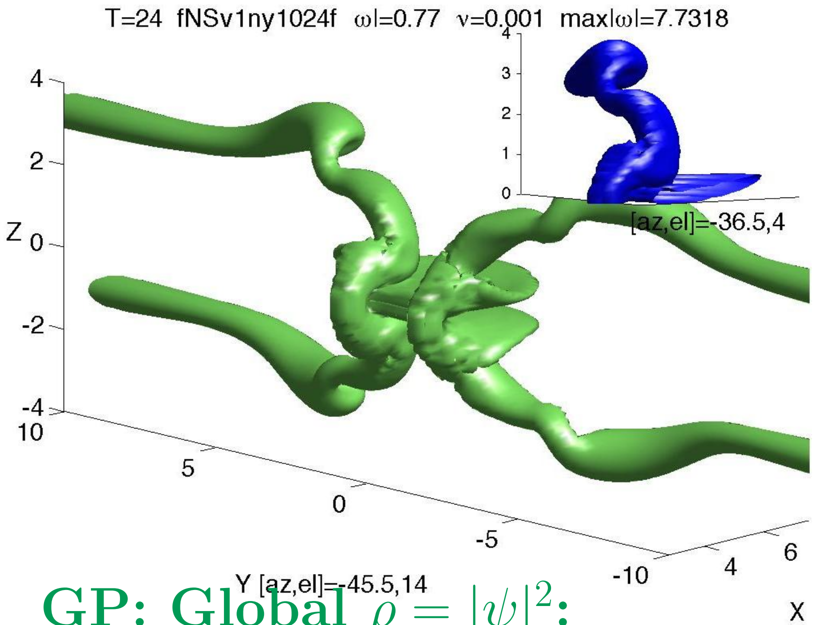
Green is pressure waves from the $x = 8\pi$ wall.

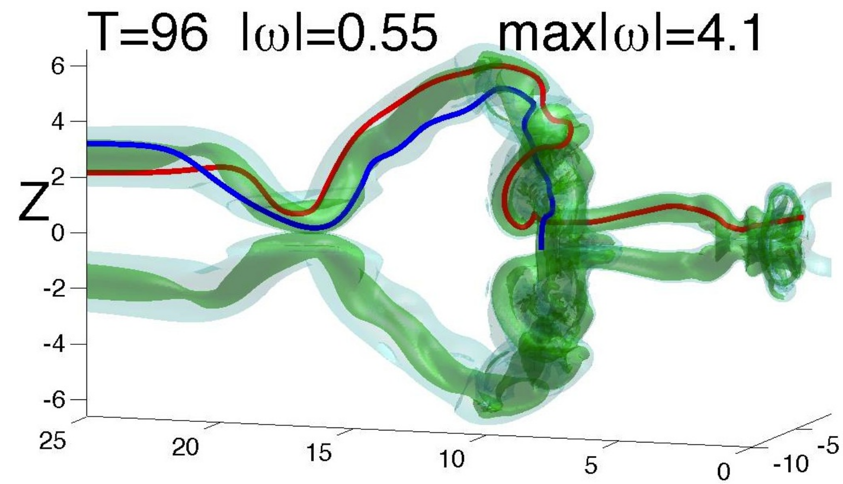
Navier-Stokes Overview: From reconnection to turbulence



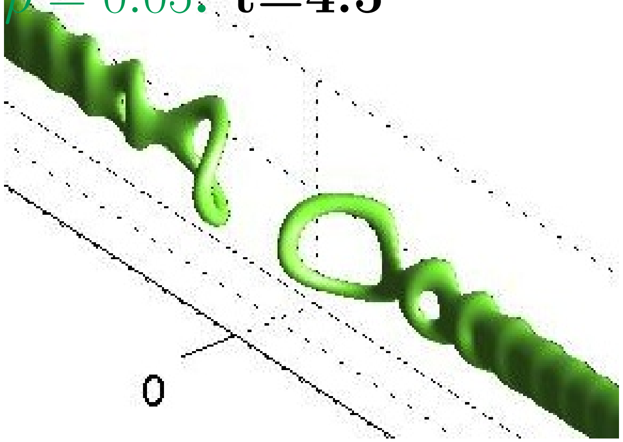
Reconnection and origins of cascade.

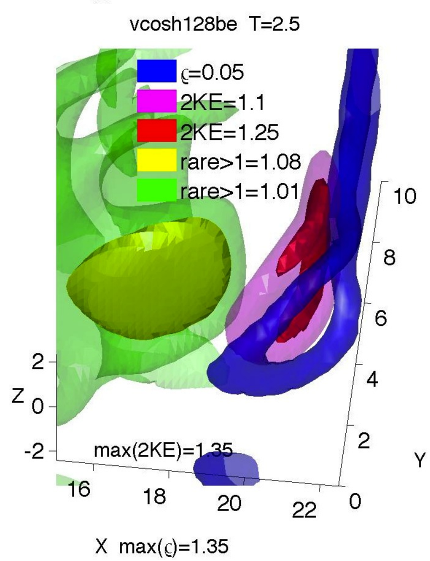
Navier-Stokes reconnection $t = 24$ leads to spirals and a ring with stretching $t = 96$.





GP: Global $\rho = |\psi|^2$:
$\rho = 0.05$. **t=4.5**





t=2.5

Green: Ejected GP wave.

$\rho = 0.05$.

$2K_{\nabla\psi} = 1.1$.

$2K_{\nabla\psi} = 1.25$.

Intense $K_{\nabla\psi}$ skirts edges of vortex at 2nd reconnection site.

Summary

- **Pre-reconnection:** twisted structure consistent with vortex dynamics.

  (Filament calculations, mine and de Waele/Aarts.) **No spirals.**

  Stretching increases line length which leads to the generation of interaction energy, and removal of kinetic energy.

- **After first reconnection: driven by the anti-parallel interaction, vortex oscillations appear.**

  These are NOT the Kelvin waves resulting from sharp vortex filament reconnections in LIA (Schwarz, mid-1980s).

- *Oscillations deepen further:*

  Second reconnection releases a vortex ring. Cascade of rings forms as well as a cascade of kinetic energy to small scales and $k^{-5/3}$ kinetic energy spectrum.

- **Local kinetic energy is depleted as if dissipated.**

  Mechanism appears to be a combination of emission of vortex rings and quantum waves.

- **Finally: Can classical reconnection do the same?**

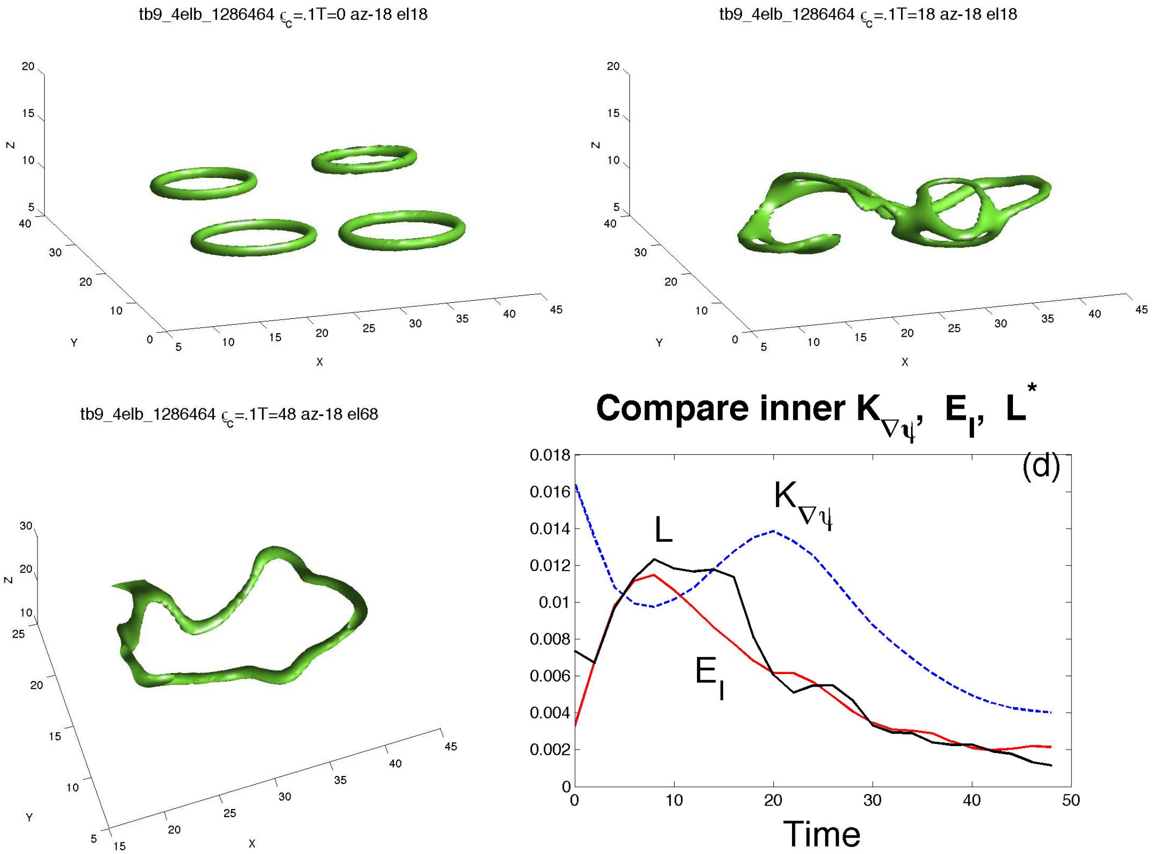
Acknowledge: Support of the Leverhulme Foundation. Discussions with Miguel Bustamante, Carlo Barenghi, Sergey Nazarenko, ME Fisher, Dan Lathrop.

Future work

- A long-standing question in classical turbulence is whether the energy cascade is mostly statistical, or originates with the interaction of fluid structures.
  - No matter how special or non-classical, even a single case that started with a simple vortical configuration and then generated a cascade could provide new insight. Such an initial condition could then be adapted to classical reconnection and turbulence calculations to determine whether similar dynamics and stages can form.
  - The results here suggest how to start a search for similar classical events that would begin with vortex stretching, then form a tangle followed by multiple reconnections, and finally lead to the creation of small scale dissipative structures.
- The other major point is the roles stretching and the creation of interaction energy play in decreasing the kinetic energy.
  - This provides the first step in allowing waves to be created and serve as an energy sink far from boundaries.
  - These properties seem to hold for a number of cases where rings and lines are allowed to interact. In anti-parallel, the release of one ring is followed by further reconnections and smaller rings, which is evidence for a physical space cascade.
- How much of this can be transferred over to classical fluids?
  - A preliminary calculation of reconnection in Navier-Stokes using the improved trajectory and initial profile suggested by these calculations does produce one ring after two reconnections. Further work is in progress.

Following pages: Further material not used on this presentation.

Four colliding vortices



Four colliding rings, the entangled state generated, the final relaxed state, and the time dependence of the kinetic and interaction energies plus a measure of line length in the inner region that contained the original vortices. In this case the measure of line length, the volume where $\rho < 0.1$, tracks the interaction energy E_I more closely than the kinetic energy.

Not quite standard 3D Gross-Pitaevski equations. (Extra 0.5)

$$\frac{1}{i}\frac{\partial}{\partial t}\psi = 0.5\nabla^2\psi + 0.5\psi(1-|\psi|^2) \quad \textbf{cubic nonlinearity}$$

- Conserves mass $M = \int dV |\psi|^2$
- Conserves Hamiltonian $H = \int dV \left[\frac{1}{2}\nabla\psi\cdot\nabla\psi^\dagger + \frac{1}{4}(1-|\psi|^2)^2\right]$
- Background density $\rho = |\psi|^2 = 1$.

  Neumann (free-slip) boundary conditions in all directions.

- A **semi-classical velocity** can be defined by the gradient of the phase of the wave function. $\boldsymbol{v} = \nabla\phi$. This gives potential flow.

  If $\psi = \sqrt{\rho}e^{i\phi}$, then by the Madelung transformation:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0, \qquad p = \frac{V_0}{2m^2}\rho^2, \qquad \Sigma_{jk} = \left(\frac{\hbar}{2m}\right)^2 \rho\frac{\partial^2 \log\rho}{\partial x_j \partial x_k}$$

$$\rho\frac{\mathbf{D}\boldsymbol{v}}{\mathbf{Dt}} = -\nabla\mathbf{p} + \nabla\boldsymbol{\Sigma} \quad \text{a strange type of barotropic Euler equation.}$$

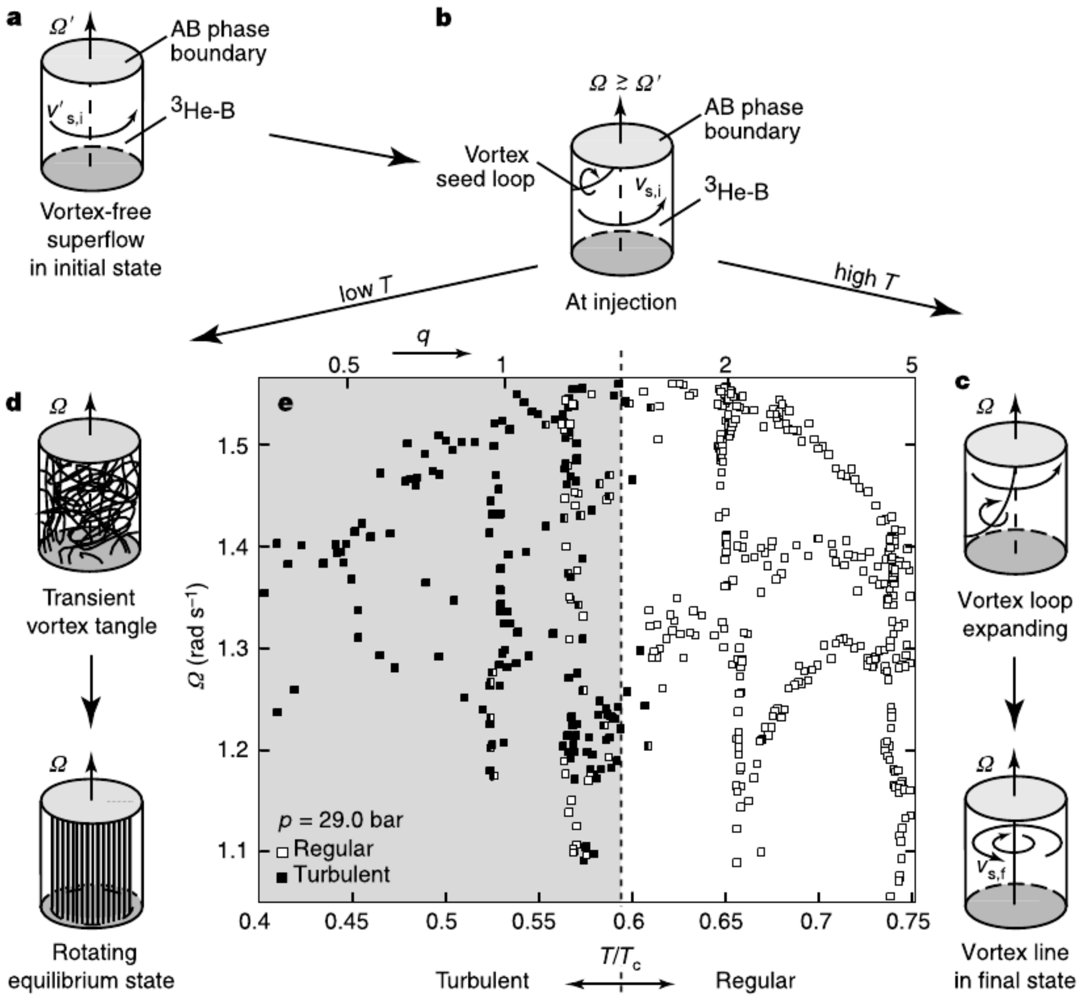
(I use $E_0 = 0.5$, $V_0 = 0.5$, $\hbar = m = 1$.)

To these equations, people usually add a normal fluid component whose details are still debated, but is certainly some type of barotropic fluid with a classical viscous term, i.e. not Hamiltonian and dissipates energy. Its role is discussed below.

Ultra-cold ^{3}He

An intrinsic velocity-independent criterion for superfluid turbulence. Finne et al (mostly **Helsinki)** Nature 242, 1022 **(2003)**.

Figure 3 Measurement and phase diagram of turbulent superflow in ^{3}He-B.



b , A few (ΔN) vortex loops are injected and, after a transient period of loop expansion, the number of rectilinear vortex lines N_f in the final steady state is measured. It is found to fall in one of two categories. **c** and **d**

d , $\Delta N << N_f \leq N_{\rm eq}$, turbulent loop expansion. This process leads to a total removal of the macroscopic vortex-free superflow as the superfluid component is forced into solid- body-like rotation

I learned of this result in 2007 from Dieter Vollhardt, Augsburg. I haven't quite figured out how they can claim this is equivalent to the -3/2 decay rate.