1. Tropical Operations

In Tropical Mathematics the fundamental operations are the minimum (the tropical +) of two numbers and addition (the tropical X).

\[ 2 + 3 = 2 \quad 2 \times 3 = 5 \]

2. Why tropical?

The adjective tropical was given in honour of the Brazilian mathematician Imre Simon who was one of the founders of this new theory.

3. Tropical Algebraic Geometry

Algebraic Geometry is the study the solutions of systems of polynomial equations in many variables, called varieties. These solutions form geometric objects such as lines, curves or surfaces. It is possible to tropicalize the equations and hence the set of solutions to obtain tropical varieties so tropical lines, tropical curves, tropical surfaces etc. These are the objects studied in Tropical (Algebraic) Geometry.

\[ \text{Classical line} \quad \text{Tropicalization} \quad \text{Tropical line} \]

4. My research

The tropicalization of a classical variety is easier to study than the variety itself. It is always given by a collection of linear pieces such as half lines or half planes. Nonetheless this simplified version of the variety carries a lot of information of the variety itself hence we can try to use tropical geometry to solve problems in algebraic geometry. This is the focus of my research.

I am studying geometric objects called Fano Schemes which give information on the number of lines contained in a variety. The goal is to find the relation between the tropical lines inside the tropicalization and the classical lines which are in the starting classical variety. This has potential application to some problems in complexity theory from computer science. For example proving that computing the permanent of a matrix cannot be reduced to compute the determinant.

5. Applications

Phylogenetics is a branch of Biology which studies the evolution of species. The key tool is the phylogenetic tree which records the distance between species. Speyer and Sturmfels proved that the properties that characterise phylogenetic trees are the same that define tropical lines. Hence understanding all the possible kinds of tropical lines gives us a complete collection of all possible phylogenetic trees.

Baldwin and Klemperer used tropical geometry to understand product-mix auctions in Economics. They represented the different agents in the auction by tropical curves and find the equilibrium by looking at the points of intersection.