2007/08 Warwick EPSRC Symposium on Algebraic Geometry

Warwick
Mathematics Research Centre

Warwick EPSRC Symposium 2007 – 2008
Algebraic Geometry

Autumn School

Introduction to explicit methods in algebraic geometry (XPL)
Monday 3rd – Friday 7th September 2007

Introductory workshop on Higher Dimensional Minimal Model Program (HD MMP)
Monday 10th – Saturday 15th September 2007

Research workshop: Toric degenerations, tropical geometry and mirror symmetry
Monday 17th – Wednesday 19th September 2007

EPSRC
Engineering and Physical Sciences Research Council

THE UNIVERSITY OF WARWICK

Introduction to explicit methods in algebraic geometry (XPL)

Monday 3rd – Friday 7th September 2007

An introductory workshop run in collaboration with the long-running Polish September algebraic geometry school.

Organisers: Buczynski, Langer, Reid, Wisniewski

Topic: Explicit construction of moduli and parameter spaces.

Lecturers: Alastair Craw (Glasgow) and Diane Maclagan (Warwick)

Overview:

These lectures are intended as an introduction to some of the ways that moduli problems arise in algebraic geometry, with strong emphasis on explicit descriptions of the resulting spaces. One place where such problems arise is when one is interested in a family of geometric objects, such as subschemes of a projective variety. By restricting to objects satisfying a numerical condition, like Hilbert polynomial of a subscheme, one may obtain a moduli space of such objects. One can then expect to improve one's understanding of the objects by studying geometric or topological properties of the moduli space itself, including dimension, connectedness and the structure of its irreducible components.

For a second situation, imagine that one is interested in geometric or topological properties of a given scheme. In this case, it may be possible to introduce a moduli problem for which the underlying moduli space is the scheme in question and, moreover, where the tautological family on the moduli space gives insight into properties of the space. For instance, if the tautological family defines a collection of vector bundles on the scheme, do the classes of these bundles freely generate the Grothendieck group of vector bundles, say, or even the bounded derived category of coherent sheaves?

Background: These lectures should be accessible to a beginning graduate student in algebraic geometry.

Outline

Maclagan: Hilbert schemes

Lecture 1: Introduction/Hilbert scheme of subschemes of $\mathbf{P}^n$;
Lecture 2: Details of construction;
Lecture 3: Connectedness of the Hilbert scheme and pathologies;
Lecture 4: Hilbert schemes of points on surfaces;
Lecture 5: Multigraded Hilbert schemes.

Craw: Quiver representations in toric geometry

Lecture 1: Introduction/GIT for torus actions;
Lecture 2: Toric varieties following Cox, toric quiver varieties;
Lecture 3: Quivers of sections and multilinear series;
Lecture 4: Bound quivers, and toric varieties as fine moduli of algebras;
Lecture 5: Bound McKay quiver and the coherent component, open questions.

Topics in HD MMP - an instructional workshop

Monday 10th – Saturday 15th September 2007

The themes will include

1. Explicit Mori theory of 3-folds
2. Criteria for graded and multigraded algebra to be finitely generated
3. The biregular and birational geometry of Fano varieties
4. Introductory topics in Shokurov's approach to HD MMP and the recent breakthrough of [BCHM]
5. Other topics

Overview:

The workshop aims to give a first presentation of some of the ideas behind the minimal model program for 3-folds and higher dimensional varieties, in a way designed to be accessible to starting graduate students. The intention is to set up a base camp by introducing the general HD MMP, on which there have been major recent breakthroughs.

We also plan regular seminars at Warwick on these topics during the term, and a further introductory workshop over the weekend Thu 1st–Mon 5th Nov to mount a more concerted attack on this material, in preparation for the major research conference Wed 12th–Thu 20th Dec 2007.

Research workshop: Toric degenerations, tropical geometry and mirror symmetry

Monday 17th – Wednesday 19th September 2007

The themes will include

1. Understanding mirror symmetry via affine structures on manifolds
2. Tropical geometry and its relations to enumerative geometry
3. Mirror symmetry for Fano manifolds
4. Homological mirror symmetry
5. Other topics: Open Gromov-Witten invariants, cluster varieties...

The workshop aims to explain unifying themes underyling recent ideas that are becoming prevalent in the study of mirror symmetry, based around the idea that mirror pairs are controlled by an underlying real affine manifold. This has shown up in various guises: in the work of Gross/Siebert and Kontsevich/Soibelman exploring the B-side of mirror symmetry; in the work of Mikhalkin, Nishinou/Siebert, Gathmann/Markwig and others on enumerative aspects of tropical geometry, in work of Abouzaid on homological mirror symmetry for toric varieties, and so on.

(Note that Mark Gross will give colloquial lecture the previous Fri 14th Sep.)

Program for week 1: Introduction to explicit methods in algebraic geometry (XPL)

Monday 3rd September

11:00 – 12:00 **Craw** 1

2:00 – 3:00 **Maclagan** 1

3:30 – 4:30 Problem Session 1

5:00 – 6:00 Problem Session 2

Tuesday 4th September

9:30 – 10:30 **Maclagan** 2

11:00 – 12:00 **Craw** 2

1:30 – 3:00 Problem Session 1

3:30 – 5:00 Problem Session 2

5:00 – 6:00 Invited lecture:

Adrian Langer (Warsaw), *On D-affinity of quadrics*

6:30 Buffet supper in common room

Wednesday 5th September

9:00 – 10:00 **Craw** 3

10:30 – 11:30 **Maclagan** 3

12:00 – 1:00 Problem sessions

Free afternoon

Thursday 6th September

9:30 – 10:30 **Maclagan** 4

11:00 – 12:00 **Craw** 4

1:30 – 3:00 Problem Session 1

3:30 – 5:00 Problem Session 2

6:30 Party in common room

Friday 7th September

9:30 – 10:30 **Craw** 5

11:00 – 12:00 **Maclagan** 5

1:30 – 3:00 Problem Session

4:00 – 5:00 Colloquium lecture:

Jaroslaw Wisniewski (Warsaw), *Phylogenetic trees and algebraic geometry*

5:15 Snacks and drinks in common room

Program for week 2: Topics in HD MMP - an instructional workshop

Monday 10th September
11 – 12 **Reid**, General introduction to Mori theory and flips
2 – 3 TBA
4 – 5 **Corti**, Introduction to HD MMP and Shokurov algebras

Tuesday 11th September
9:30 – 10:30 **Brown**, Hypersurface flips
11 – 12 **Lazic**, Shokurov algebras (1)
2 – 3 **Caibar**, 3-fold canonical and terminal singularities (1)
4 – 5 **Reid**, Toric and diptych varieties (1)
6:30 Buffet supper in common room

Wednesday 12th September
9 – 10 **Brown**, Toric and diptych varieties (2)
10:30 – 11:30 TBA
12 – 1 **Corti**, Geometry of Fano 3-folds (1)
Free afternoon

Thursday 13th September
9:30 – 10:30 **Lazic**, Shokurov algebras (2)
11 – 12 **Reid**, Toric and diptych varieties (3)
2 – 3 **Corti**, Geometry of Fano 3-folds (1)
4 – 5 **Birkar**, HD MMP and termination (1)
6:30 Party in common room

Friday 14th September
9:30 – 10:30 **Kaloghiros**, Geometry of Fano 3-folds
11 – 12 **Birkar**, HD MMP and termination (2)
2 – 3 **Caibar**, 3-fold canonical and terminal singularities (2)
4 – 5 Colloquium: **Mark Gross** (San Diego), *The tropical vertex*
5:15 Snacks and drinks in common room

Saturday 15th September
10 – 11 TBA
12 – 1 **Birkar**, HD MMP and termination (3)
1:30 Pub lunch

Program for week 3: Research workshop - Toric degenerations, tropical geometry and mirror symmetry

Monday 17th September
11:00 **Gross**, Toric degenerations and mirror symmetry 1
2:00 **Ilia Itenberg**
3:30 TBA
5:00 **Alessio Corti**
6:15 Reception and supper in common room

Tuesday 18th September
9:30 **Melissa Liu**, Nekrasov's conjecture for toric surfaces
11:00 **Gross**, Toric degenerations and mirror symmetry 2
2:00 **Katzarkov**, Mirror symmetry and irrationality criteria 1
3:30 TBA
5:00 **Paul Seidel**

Wednesday 19th September
9:30 **Tom Coates**
11:00 **Siebert**
2:00 **Katzarkov**, Mirror symmetry and irrationality criteria 1
4:00 **Gross**, Toric degenerations and mirror symmetry 3
6:30 Party in common room