Large deviations in the system size for some globally coupled maps

by Gerhard Keller

Dept. Mathematik, Universität Erlangen

In a recent joint paper with Jean-Baptiste Bardet and Roland Zweimüller [1] we introduced a very simple model of globally coupled interval maps: the individual maps are uniformly expanding parametric perturbations of the $2x$ mod 1–map, and they are coupled by a common parameter which is a function of the „mean field“ of the configuration. In the thermodynamic limit the dynamics are described by a (nonlinear) self-consistent Perron-Frobenius operator acting on the space of all probability densities on $[0, 1]$. The operator undergoes a bifurcation: for small coupling strengths it has a unique stable fixed point attracting every initial density, while for stronger couplings this fixed point becomes hyperbolic and two new stable fixed points bifurcate from it. This happens at coupling strengths for which the finite systems are always exponentially mixing w.r.t. a unique invariant measure equivalent to Lebesgue.

This model may serve as a testing ground for ideas on large deviations relating properties of the finite systems to the dynamics of the nonlinear operator. So I will discuss perspectives of large deviations principles for fixed times when the system size goes to infinity. There are many open questions, but for some simple cases I will show how the rate function is related to the asymptotic dynamics of the self-consistent Perron-Frobenius operator. Large deviations for the invariant measures is another open question.

Bibliography

[1] J.-B. Bardet, G. Keller, and R. Zweimüller. Stochastically stable globally coupled maps with bistable thermodynamic limit. *Commun. Math. Phys.*, 292:237–270, 2009.