

Lecture Titles

Surname	First Name	Affiliation	Title
Ainsworth	Mark	Strathclyde	Rigorous numerical enclosures on the limit load in the analysis of multibody structures
Barrett	John	Imperial	Parametric approximation of elastic flow for curves and curve networks
Bertozi	Andrea	California	Diffuse interface methods in image processing and data analysis
Blowey	James	Durham	Analysis of a nonlinear parabolic cross diffusion population model
Deckelnick	Klaus	Magdeburg	Identification of matrix parameters in elliptic PDEs
Dziuk	Gerhard	Frieburg	Time discretizations for ESFEM
French	Donald	Cincinnati	Numerical approximation of solutions to nonlinear inverse problems arising in olfaction experimentation
Garcke	Harald	Regensburg	On a thermodynamically consistent Cahn-Hilliard/Navier-Stokes model for incompressible two-phase flows with different densities
Giga	Yoshikazu	Tokyo	Hamilton-Jacobi equations with discontinuous source terms
Hinze	Michael	Hamburg	A residual-based a posteriori approach to simulation and control of the Cahn-Hilliard Navier Stokes system
Kay	David	Oxford	Numerical methods for fractional diffusion
Kornhuber	Ralf	Berlin	Time discretizations of anisotropic Allen-Cahn equations
Larsson	Stig	Chalmers	Finite element approximation of the Cahn-Hilliard-Cook equation
Luckhaus	S	Leipzig	Blow up in chemotaxis and beyond
Niethammer	Barbara	Oxford	Self-similar solutions for a kinetic model for grain growth
Novick-Cohen	Amy	Technion	Upperbounds for coarsening for various Cahn-Hilliard type equations
Ockendon	John	Oxford	The perennial problem of alloy solidification
Richardson	Giles	Southampton	The reversing of interfaces in slow diffusion processes with strong absorption
Rodrigues	J	Lisbon	Remarks on the variational inequality approach to constrained Cahn-Hilliard type systems
Sprekels	Juergen	Berlin	Technological and mathematical problems in the industrial growth of semiconductor bulk single crystals
Stinner	Bjorn	Warwick	On a surface finite element method for biomembranes with phase separation
Turner	Matthew	Warwick	Diffusion and patterning on living membranes

Abstracts

(Ainsworth)

Rigorous numerical enclosures on the limit load in the analysis of multibody structures

A rigorous finite element numerical procedure is proposed for the computation of guaranteed lower and upper bounds for the limit load of failure in a system of linear-elastic blocks in mutual non-penetrative contact with given friction. Two numerical approaches are presented. The first approach uses the static limit load problem involving stresses in conjunction with a non-standard conforming finite element method to obtain a linear program from which one can derive a lower bound for the limit load and an expression for the corresponding stress field. The second approach uses the kinematic limit load problem to obtain a linear optimization problem from which one can determine an upper (unsafe) bound for the limit load and an expression for the failure mode. Together, these procedures give rise to rigorous numerical enclosures on the limit load.

(Joint work with Angela Mihai.)

(Barrett)

Parametric approximation of elastic flow for curves and curve networks

We introduce a parametric finite element approximation of the Willmore/elastic flow of a closed curve in \mathbb{R}^d , which has good mesh properties.

We show that in the continuous-in-time semidiscrete case it satisfies a stability bound and an equidistribution mesh property.

We extend this approach to an anisotropic elastic energy, a single open curve that satisfies various boundary conditions and curve networks.

(Joint work with Harald Garcke (Universitat Regensburg) and Robert Nurnberg (Imperial College London))

(Bertozi)

Diffuse interface methods in image processing and data analysis

TBC

(Blowey)

Analysis of a nonlinear parabolic cross diffusion population model

A mathematical and numerical analysis will be carried out for a cross diffusion system. The system appears in modelling the movement of two interacting cell populations whose kinetics are of competition type. A fully practical piecewise linear finite element approximation is proposed and studied. Finally, a practical algorithm for computing the numerical solutions of each system is described and some numerical experiments are performed to illustrate and verify the theoretical results.

(Deckelnick)

Identification of matrix parameters in elliptic PDEs

We consider the inverse problem of identifying the matrix-valued diffusion coefficient in an elliptic PDE from multiple interior measurements.

Employing a Tikhonov regularisation we are led to a PDE-constrained minimisation problem for which we derive an existence result using the concept of H-convergence. Discretisation of the elliptic PDE by linear finite elements gives rise to a discrete minimisation problem. We show that a subsequence of the discrete minimisers converges strongly to a solution of the continuous problem.

(Joint work with Michael Hinze, Hamburg.)

(Dziuk)

Time discretizations for ESFEM

We discuss time discretizations of parabolic PDEs on moving surfaces. The motion of the surface is smooth and given. Conservation and diffusion of a scalar quantity u on a moving surface $\Gamma(t)$ is modelled by

$$u_t + u \nabla_{\Gamma} \cdot \mathbf{v} - \nabla_{\Gamma} \cdot (A \nabla_{\Gamma} u) = 0.$$

Here \mathbf{v} is the velocity of the surface and A is a given diffusion tensor. A discretization with evolving surface finite elements (ESFEM) leads to an ODE for the coefficient vector of the discrete solution. We prove error estimates of optimal order for the backward Euler scheme in combination with piecewise linear finite elements. For this we need an optimal L^2 error estimate for the spatial discretization, which we discuss shortly. (Joint work with Charlie Elliott, Warwick.)

(French)

Numerical approximation of solutions to nonlinear inverse problems arising in olfaction experimentation

The identification of detailed features of neuronal systems is an important challenge in the biosciences today. An interdisciplinary research team has been working to determine the distributions of ion channels in frog olfactory cilia. The cilia are long thin processes that extend from the olfactory receptor neurons. The first step in the transduction of an odor into an electrical signal occurs in the membranes of the cilia and is controlled primarily by ion channels.

Mathematical models and simple approximation methods are derived to obtain estimates of the spatial distributions of the ion channels along the length of a cilium.

In this talk we review our results and describe a specialized problem involving a constrained integral equation. (Joint work with S.J. Kleene, University of Cincinnati College of Medicine.)

(Garcke)

On a thermodynamically consistent Cahn-Hilliard/Navier-Stokes model for incompressible two-phase flows with different densities

A new diffuse interface model for the two-phase flow of two incompressible fluids with different densities is introduced using methods from rational continuum mechanics. The model fulfills local and global dissipation inequalities and is frame indifferent.

Using the method of matched asymptotic expansions we derive various sharp interface models in the limit when the interfacial thickness tends to zero. Depending on the scaling of the mobility in the diffusion equation we either derive classical sharp interface models or models where bulk or surface diffusion is possible in the limit. In the latter case a new term resulting from surface diffusion appears in the momentum balance at the interface.

Finally, existence of weak solutions is shown using an implicit time discretization. Here several new technical difficulties arise which result from a coupling of the momentum balance and the Cahn-Hilliard equation via the diffusive flux.

(Joint work with Helmut Abels and partly with Guenther Gruen and Daniel Depner)

(Giga)

Hamilton-Jacobi equations with discontinuous source terms

We are interested in the initial-value problem for a class of Hamilton-Jacobi equations whose Hamiltonian is discontinuous with respect to the spatial variable. Our problems stem from a model describing the two-dimensional nucleation, which arises in crystal growth phenomena. A typical example is an equation with semicontinuous source terms. For a general framework including such an example we introduce a new notion of solutions by extending the theory of viscosity solutions. We establish a unique global existence result for general Lipschitz initial data. Our solution agrees with what we expect for zero initial data for point source problems.

(Joint work with Nao Hamamuki (U. Tokyo))

(Hinze)

A residual-based a posteriori approach to simulation and control of the Cahn-Hilliard Navier Stokes system

We consider simulation and control of the Cahn-Hilliard Navier-Stokes system (CHNSS) in the phase field approximation with the double obstacle potential. For time discretization a semi-implicit scheme is used. We relax the variational inequalities appearing in every time step by a penalization approach and develop reliable and effective residual based a posteriori error estimators for the resulting PDE system along the lines of [1]. Furthermore we propose a model-predictive concept for control of the CHNSS. Numerical experiments are presented which illustrate the performance of our approach.

[1] M. Hintermüller, M. Hinze, H. Tber: An adaptive finite element Moreau-Yosida-based solver for a non-smooth Cahn-Hilliard problem, to appear in OMS.

(Joint work with Christian Kahle)

(Kay)

Numerical methods for fractional diffusion

Fractional differential equations are becoming increasingly used as a modelling tool for coping with anomalous diffusing processes or spatial heterogeneity. However, the presence of a fractional differential operator causes memory (time fractional) or nonlocality (space fractional) issues and this imposes a number of computational constraints.

We present scalable techniques for solving fractional-in-space reaction diffusion equations using the finite element method and robust techniques for computing the fractional power of a matrix times a vector. The method will be applied to fractional models of the Fisher, Allen-Cahn and monodomain equations in two spatial dimensions.

(Kornhuber)

Time discretizations of anisotropic Allen-Cahn equations

We consider anisotropic Allen-Cahn equations with interfacial energy induced by an anisotropic surface energy density γ .

Assuming that γ^2 is convex and that γ is positive, positively homogeneous of degree one, and sufficiently smooth, we show stability of various time discretizations. In particular, we consider a fully implicit and a linearized time discretization of the interfacial energy combined with implicit and semi-implicit time discretizations of the double-well potential. In the semi-implicit variant, concave terms are taken explicitly. The arising discrete spatial problems are solved by globally convergent truncated nonsmooth Newton methods. Numerical experiments show the accuracy of the different discretizations as well as efficiency and robustness of the spatial solver. We also illustrate that pinch-off under anisotropic mean curvature flow is no longer frame invariant, but depends on the orientation of the initial configuration.

(Larsson)

Finite element approximation of the Cahn-Hilliard-Cook equation

We study the Cahn-Hilliard-Cook equation, i.e., the Cahn-Hilliard equation perturbed by additive colored noise. We show almost sure existence and regularity of solutions. We introduce spatial approximation by a standard finite element method and prove error estimates of optimal order on sets of probability arbitrarily close to 1. We also prove strong convergence without known rate.

(Joint work with M. Kovacs and A. Mesforush)

(Luckhaus)

Blow up in chemotaxis and beyond

We are looking at the degenerate Keller Segel model for chemotaxis. Extending the notion of weak solution to measures, we are able to characterize blow up points and show nonuniqueness after blow up. The open questions that remain have to do mainly with the evolution of correlation measures.

(Joint work with S.Sugiyama and J.J.L.Velazquez.)

(Niethammer)

Self-similar solutions for a kinetic model for grain growth

We prove existence of self-similar solutions to a model for two-dimensional grain growth, which consists of an infinite number of nonlocally coupled transport equations for the number densities of grains with given area and number of neighbours (topological class).

For the proof we introduce a finite maximal topological class and study an appropriate upwind-discretization of the time dependent problem in self-similar variables. We first show that the resulting nonlinear ODE system has nontrivial steady states. In a second step we let the discretization parameter tend to zero and prove that the steady states converge to a self-similar solution for the model with finite maximal topological class. Finally we let the maximal topology class tend to infinity and obtain self-similar solutions to the original system. We also use the upwind discretization to compute self-similar solutions numerically.

(joint work with M. Herrmann and P. Laurencot)

(Novick-Cohen)

Upper bounds for coarsening for various Cahn-Hilliard type equations

Upper bounds for coarsening for various Cahn-Hilliard type equations,

The Cahn-Hilliard equation

$$(1) \quad u_t = \nabla \cdot (1 - u^2) \nabla \left[\frac{\theta}{2} \{ \ln(1 + u) - \ln(1 - u) \} - \alpha u - \epsilon^2 \Delta u \right],$$

is a well established model for phase separation, as well as for population dynamics, galaxy formation, biofilm structure formation, swarm formation, and more [2, 4]. A standard scenario for phase separation which is well captured by the model is the initiation of phase separation via short length scale instabilities (spinodal decomposition), then followed by the growth of domains in which $u(x, t) \approx u_{\pm}$, where u_{\pm} denote the "equilibrium phases"; this latter process is known as coarsening. The degree of coarsening can be quantified in terms of a characteristic length scale, $l(t)$, for the domains in which $u(x, t) \approx u_{\pm}$. Upper bounds on coarsening rates were obtained by Kohn & Otto [3] for (1) at late times, in the deep quench limit in which $\Theta \approx 0$ and in the shallow quench limit in which $\Theta \approx \Theta_{cr}$, with $\bar{u} = 0$ where $\bar{u} := \frac{1}{|\Omega|} \int_{\Omega} u \, dx$, based on three lemmas involving two functionals, $E(t)$, a Liapunov free energy, and $L(t)$, a $W^{1,\infty}$ predual norm of $u(x, t)$, and differential inequalities. Recently [5, 6], we have extended the results in [3], enabling consideration of all mean concentrations, $\bar{u} \in (u_-, u_+)$, all temperatures $\Theta \in (0, \Theta_{cr})$, and for early as well as late times. See also [1].

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- [3] R.V. Kohn & F. Otto, Upper bounds on coarsening rates, *Commun. Math. Phys.* **229** 375-395 (2002).
- [4] A. Novick-Cohen, *The Cahn-Hilliard Equation*, pages 201-228 in *Handbook of Differential Equations. IV Evolutionary Partial Differential Equations*, Editors: C. Dafermos and M. Pokorný, Elsevier 2008.
- [5] A. Novick-Cohen & A. Shishkov, Upper bounds for coarsening for the degenerate Cahn-Hilliard equation, *DCDS-A* **25** 251-272 (2009).
- [6] A. Novick-Cohen, Upper bounds for coarsening for the deep quench obstacle problem, *J. Stat. Phys.* **141**, 142-157 (2010).

(Ockendon)

The perennial problem of alloy solidification

Even for one-dimensional solidification, the Stefan Problem for an alloy still poses unanswered mathematical questions. These will be reviewed briefly and mention will be made of a new challenge coming from gas turbine blade heat treatment. Mention may also be made of Tyndall Stars.

(Richardson)

The reversing of interfaces in slow diffusion processes with strong absorption

This work is a contribution towards the theory of a family of one-dimensional nonlinear diffusion equations with absorption. In particular, the solutions that have interfaces that change their direction of propagation are examined. Although this phenomenon of reversing interfaces has been seen numerically, and some special exact solutions have been obtained there is no analytical insight into how this occurs in the general case. The approach taken is to look for self-similar solutions local to the interface and local to the reversing time. The analysis is split into two parts, one for the solution prior to the reversing time and the other for the solution after the reversing time. In each case the governing PDE is reduced to an ODE by introducing a self-similar coordinate system. These ODEs do not readily admit any exact solutions and so the asymptotic behaviour of solutions are studied. By doing this the adjustable parameters, or degrees of freedom, which may be used in a numerical shooting scheme are determined. A numerical algorithm is then proposed to furnish solutions to the governing PDE in the limit of interest. As an example of a physical problem in which an equation of this type may be used as a model we study the spreading of a viscous film under gravity and subject to evaporation. The numerical algorithm is demonstrated using this example. One notable result is that if a solution to a particular equation has a reversing interface at some point in its evolution, then, this reversing behaviour is generic. (Joint work with J.M. Foster, C.P. Please, A.D. Fitt)

(Rodrigues)

Remarks on the variational inequality approach to constrained Cahn-Hilliard type systems.

TBC

(Sprekels)

Technological and mathematical problems in the industrial growth of semiconductor bulk single crystals

Many free boundary value problems and phase field systems that have been studied in the mathematical literature were supposed to model solidification processes in nature and industry. In this contribution, we will discuss realistic models for the industrial growth of bulk single crystals for semiconductor materials like GaAs or Si. In particular, we will discuss important physical effects such as turbulent melt flows and traveling electromagnetic fields. A special emphasis has to be given to the fact that the temperatures in the growth apparatus are so high that the main mechanism of heat transfer within gas cavities is radiation. This leads to both nonlocal and nonlinear boundary conditions that necessitate a special analytical and numerical treatment. Also, many of the involved material parameters depend strongly on temperature, the geometry is multi-scale and nonsmooth, and we have jumping coefficients and boundary conditions. We report in this contribution how these difficulties have been dealt with in industrial research projects that have been conducted in Berlin in the past years together with industrial companies and under the leadership of the Leibniz Institute for Crystal Growth (IKZ).

(Stinner)

On a surface finite element method for biomembranes with phase separation

We consider vesicles formed by lipid bilayers with intra-membrane phase interfaces. A gradient flow dynamics to compute equilibrium shapes has been defined coupling a geometric evolution law for the membrane hypersurface to a surface Allen-Cahn equation for the interfaces. The discretization is based on $\mathcal{S}H^1$ conforming quadratic isoparametric surface finite elements and a semi-implicit time discretization. Issues are the sharp interface limit, grid regularity, adaptive mesh refinement, and the influence of various physical parameters.

(Turner)

Diffusion and patterning on living membranes

The function of biological membranes depends on the spatial distribution of their different components. It remains an open question how cell membranes maintain domains, 10-100nm in size, in which some components are enriched (and others depleted). Thermodynamic arguments suggest that immiscible components should macrophase separate, like oil in water, leading to domains that are much larger than is observed. It has been suggested that the cell regulates its membrane composition to be near a critical point, where critical fluctuations in composition could (perhaps) be of the right size. We discuss non-equilibrium mechanisms that may help to explain the experiments. These don't require the membrane to be particularly close to a critical point. Finally we discuss a recent study of diffusion in a different membrane geometry and argue that this provides a unique test of Saffman & Delbruck's seminal predictions on membrane diffusion first made in 1975.