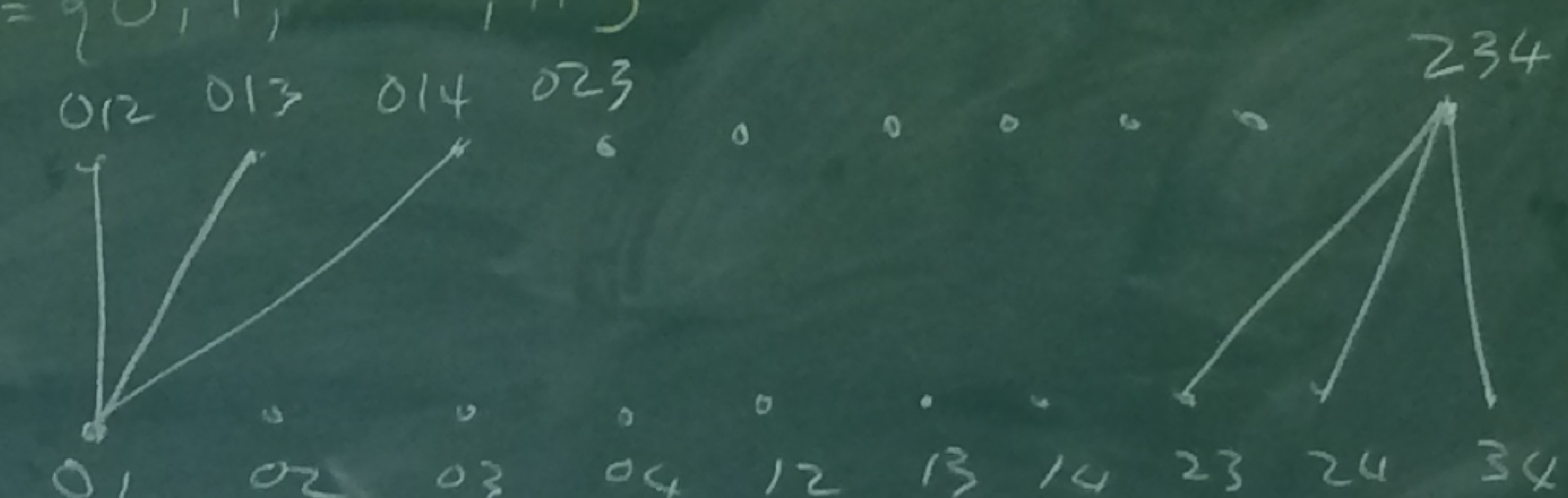
Question

Let G be the bipartite graph on $10+10$ vertices

$E=\{0,1,2,3,4\}$ $\quad E=\{0,1,\text{—},n\}$



$$\binom{n+1}{k+1}$$

$$\binom{n+1}{k}$$

Show that the 20×20 matrix

$$L_{ij}=\begin{cases} 2 & \text{if } i=j \\ -1 & \text{if } i\sim j \\ 0 & \text{if otherwise} \end{cases}$$

has exactly one negative eigenvalue

$L = (\text{usual Laplacian}) - \text{Identity}$

Usual Laplacian

G abstract graph

w pos. edge weight

$$L_{ij} = \begin{cases} d_i & i=j \\ -w_{ij} & i\sim j \\ 0 & \text{otherwise} \end{cases}$$

$$d_i = \sum_{i\sim j} w_{ij}$$

Tropical Laplacian

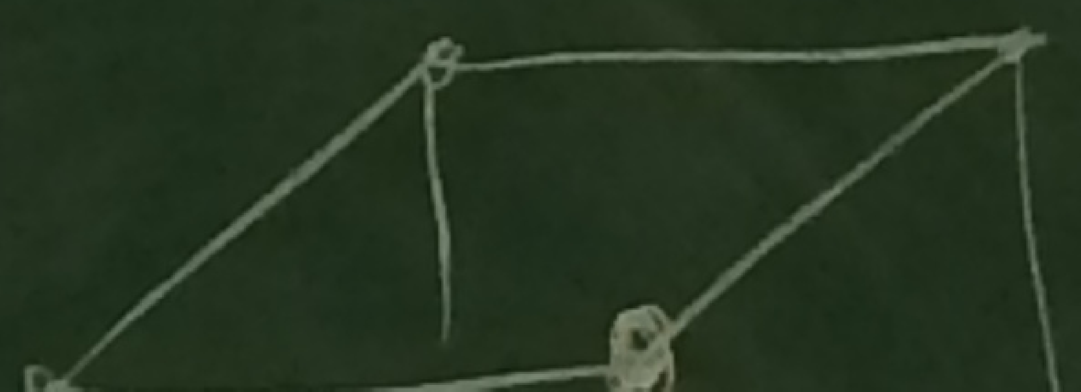
$\underline{G}$ geometric graph in $\mathbb{R}^N$

w = edge weights, <u>balanced</u>

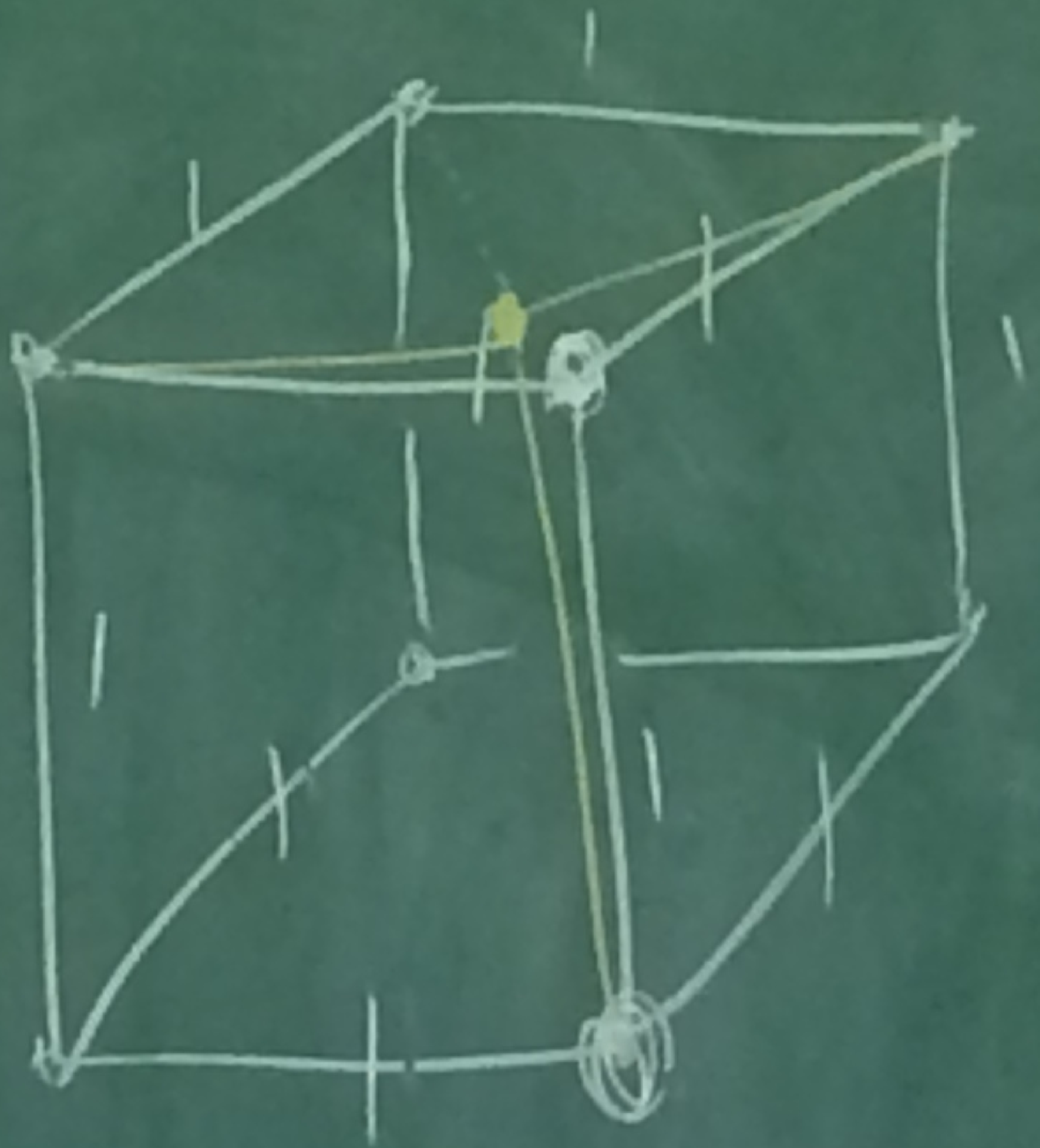
"For every vertex $\underline{v}_i$, there is d_i"

$$d_i \underline{v}_i = \sum_{i\sim j} w_{ij} \underline{v}_j$$

<u>Ex</u>



$(\pm 1, \pm 1, \pm 1)$



$(\pm 1, \pm 1, \pm 1)$

$$\square(1,1,1) = \square(-1,1,1) + \square(1,-1,1) + \square(1,1,-1)$$