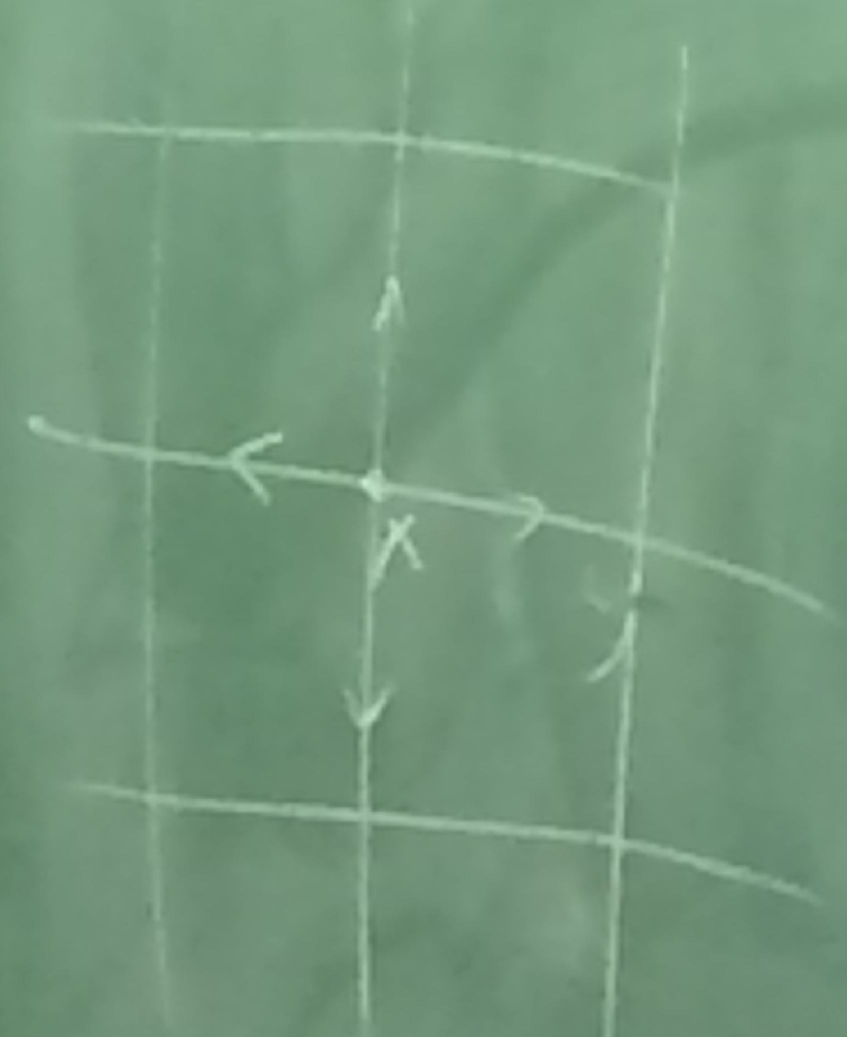
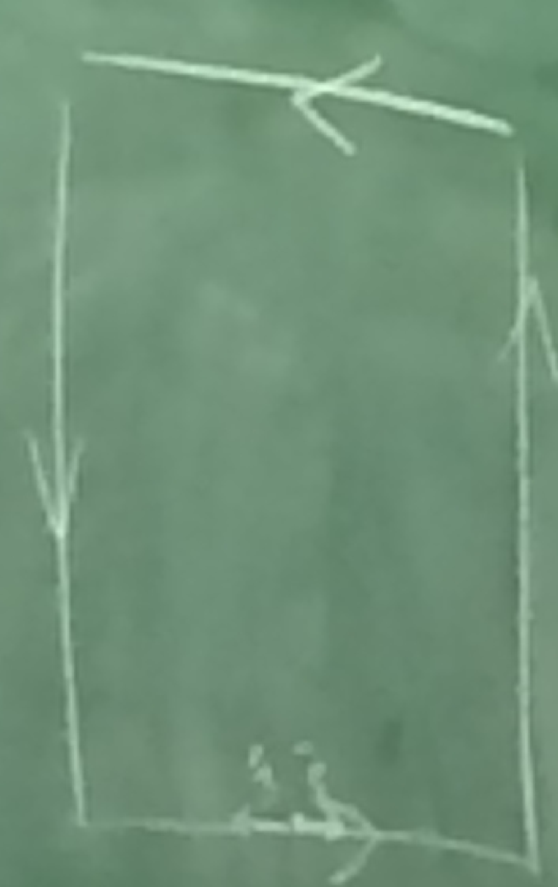
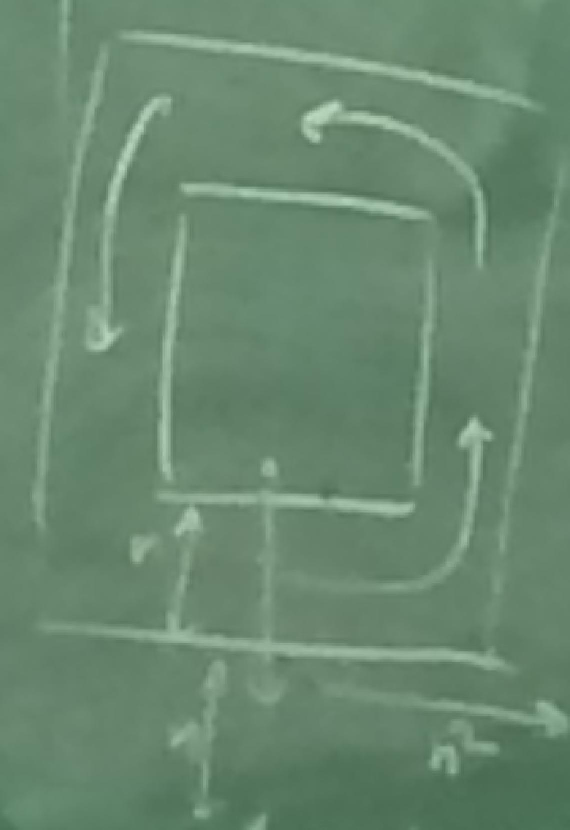


$$P_{xy} + P_{yx} = \frac{1}{d}$$





1 starting

$\boxed{1 \text{ is stationary}}$

Rev

1 stationary

CLT

Thm Under H_1 there is an annealed CLT

Stationary Ergodic div free random environment

$$\underbrace{\mathbb{E}}\left(\left| \sum_{|x| < R_k} \underbrace{v_i(x)}_{\text{drift at } x} |x|^{1-d} \right|^2 \right) < \underbrace{C}_{\text{indep. of } R}$$

$\sum x^2 \to$

$i \in \{1, \dots, d\}$

Kipnis - Varadhan

Ω - prob. space

P - Markov chain gen. Stationary

$v \in L^2(\Omega)$

Thm Under H_{-1}

$$\frac{1}{\sqrt{N}} \sum_{n=1}^{N} v(R_n) \longrightarrow \mathcal{N}$$

q — prediction operator

$f \in L^1(\Omega)$ $\qquad (qf)(\omega) = \mathbb{E}^{\omega} f(R_1)$

$$(I - q)\, u = v$$

$$\sum v(R_n) = \sum u(R_n) - \underbrace{(qu)(R_n)}_{u(R_{n+1}) + D}$$

telescopic

Martingale

$(I(1+\varepsilon) - q) u_\varepsilon = v$

$-1 \quad 1 \quad 1+\varepsilon$

$\| u_\varepsilon \|^2 = o\left(\frac{1}{\varepsilon}\right)$

H_{-1} condition ver 2

Ver 3. in the spectral rep. of q

$q^{-1/2} v \in L^2$

$\int \frac{v^2}{1-x} d\mu < \infty$??? $\frac{\partial^2}{\partial x^2}$

$\frac{q+q^*}{2}$

$H_1 = \{ f' \in L^2 \} \qquad H_{-1} = \{ \int f \in L^2 \}$

Horrath-Tóth-Vető

$A = \frac{q - q^*}{2} \qquad S = I - \frac{q+q^*}{2}$

$B = S^{-1/2} A S^{-1/2} \qquad B^* = -B$

Thm

~~Kipnis - Varadhan~~ RSC

Ω - prob. space

P - Markov chain gen

$v \in L^2(\Omega)$ Stationary ~~reversible~~

Under H_{-1}

Thm $\checkmark$ $\frac{1}{\sqrt{N}} \sum_{n=1}^{N} v(R_n) \longrightarrow \mathcal{N}$

spec $B \subset i\mathbb{R}$

$\exists\, \mathcal{C} \subset \mathcal{H}$ $B: \mathcal{C} \to \mathcal{H}$ essen^t skew-self-adjoint

$(\varepsilon I + S)^{-1/2} A (\varepsilon I + S)^{-1/2} \longrightarrow B$ pointwise on $\mathcal{C}$ (strongly)

$\mathrm{Ker}(B^* \pm I) = \{0\}$

$S^{-1/2} \nabla$

$B^* f = f$

$- \quad A S^{-1/2} f = S^{1/2} f = S(S^{-1/2} f)$

$(A + S)(S^{-1/2} f) = 0$

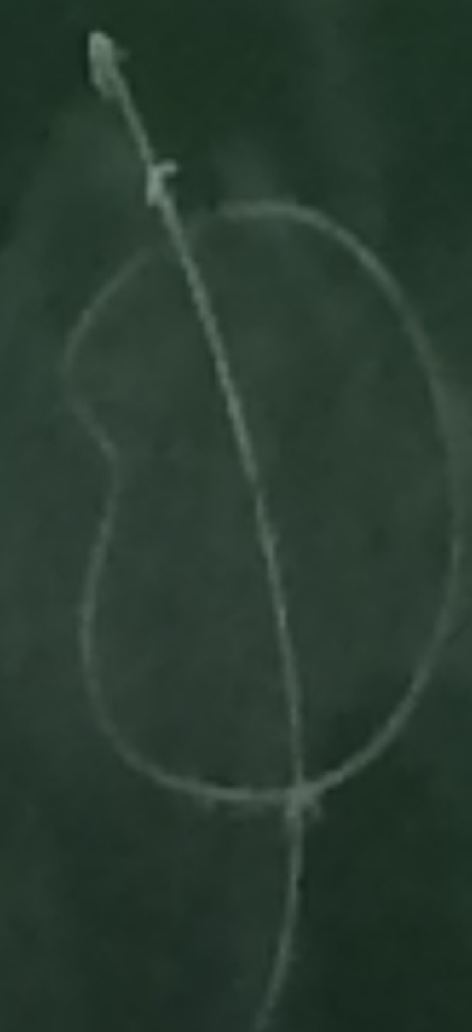
$\nabla(S^{-1/2} f)$

Enough to show:

H harmonic $(\mathbb{Z}^d \to \mathbb{R})$ with stationary gradients $\mathbb{E}(\nabla H)=0$

Ergodic. Then $H \equiv 0$

$H(R_n)$ Martingale



Fact: d-dim free RW on $\mathbb{Z}^d$

$$\mathbb{P}(R_n = x) \le c n^{-d/2}$$