Warwick Symposium Grant GR/M93789/01 for
Stochastic Partial Differential Equations and Related Topics

Final report
October 2003

1 Overview

1.1 Organization

The Warwick Symposium for the year 2001-2002 was titled *Stochastic Partial Differential Equations and Related Topics*. The principal organisers were David Elworthy, Andrew Stuart and Roger Tribe (from Warwick), assisted by a Scientific Advisory Board: Arnaud Debussche (Rennes), Mark Friedlin (Maryland), Istvan Gyongy (Edinburgh) and Nic Krylov (Minnesota).

This grant funded two main workshops in the symposium year. A one-week workshop *Discrete and Continuous Stochastic Evolutions* in March (47 participants) concentrated on the links between discrete and continuous models, in particular the analysis of models using particle methods. A two week general workshop *Stochastic Partial Differential Equations* in August 2001 (52 participants) was followed by a ten-day period of emphasis on stochastic fluid models titled *Flows, Fluids, Filtrations and Filaments*. In addition we ran a series of 10 mini-sessions, over the period October 2000 to July 2001, on specific topics. One aim of these was to include some introductory talks in each mini-session to allow non-specialists a route into the area. These were mostly two-day events and averaged 28 registered participants. The largest of these mini-sessions *Infinite dimensional models in finance* was jointly funded by an LMS grant.

Overall 220 individuals visited one or more of the above workshops, of whom 96 were from the UK. An appeal to 20 of the key participants for papers which arose principally as a result of the symposium activity yielded a list of 35 papers (listed, along with all participants, in the full report at www.maths.warwick.ac.uk/research/reports/index.html).

There were a large number of 'long stay' participants, apart from the permanent Warwick staff with interests in the area. Jerzy Zabczyk (Warsaw) was awarded a Leverhulme Visiting Professorship to spend the whole year at Warwick. Other researchers who spent a month or more at Warwick during the symposium included Sandra Cerrai (Florence), Benadetta Ferrario (Bonn), Salah Mohammed (Cordoba), Carl Mueller (Rochester), Andrei Piatnitski (Moscow), Szymon Peszat (Krakow), Marco Romito (Pisa), Francesco Russo (Paris) and Yushiaki Otobe (Nagano). Outreach funding from this grant allowed these, and other visitors (Bogomolov, Glicichk, Hairer, Kwiecinska, Maskowski, Simao), to make research and seminar visits to other UK universities (including Edinburgh, Hull, Loughborough, Manchester, Sussex, Swansea).

Activity at Warwick in SPDE continued throughout the three years of the grant. Four postdoctoral students spent a year or more at Warwick. Dirk Blomker, Martin Hairer, Greg Pavliotis, Petter Wiberg. Stella Brassesco is spending 10 months sabbatical at Warwick during 2003-4. At the end of the three-year period, the symposium grant funded a two-week workshop *Stochastic Partial Differential Equations and Related Topics* in August 2003 (66 participants) intended to review the work done since the main symposium year, and allow us to invite certain workers who were unable to come in 2000-2001. We consider this continuing activity in SPDE at Warwick one of the major outcomes of this grant.

As usual the Symposium was very efficiently organized by the staff of the Warwick Mathematics Research Centre in their customary relaxed and friendly manner, and benefitted considerably from the unique atmosphere of the Warwick Mathematics Institute.

1.2 Related activity

The symposium workshops combined with various other activity in stochastic analysis at Warwick during 2000-2003.
• Computational aspects were a recurrent theme throughout the year and we applied successfully for a separate EPSRC grant to fund a one-week workshop *Computational Stochastic Differential Equations* in March 2001.

• The UK-Japan Winter School ran in January 2001 with key speakers Terry Lyons (Oxford) and John Keating (Bristol).

• A one-week workshop on Levy Processes (see http://science.muni.ac.uk/imsor/conf/Levy/) ran in March 2001.

• Two more mini-sessions on Fractional Brownian motion and Two scale analysis of stochastic systems ran in 2002 and 2003 respectively.

• Eugene Dynkin was awarded an honourary degree in 2003 and we held *Dynkin day* - a day of talks related to his research interests.

• The Warwick stochastic analysis seminar ran throughout the year.

SPDE visitors were involved in running, as well as participants in, all this activity.

### 1.3 Other funding

Other sources of funding for the symposium were:

• **London Mathematical Society** Support for UK participants via the Warwick British Visitors Fund;

• **INTRA** funding for visitors from the former Soviet Union;

• **Marie Curie Training Site** allowing five European PhD students to spend between 3 and 12 months training in Stochastic Analysis at Warwick during this period.

In addition many participants received full or partial support from their own institutions and/or national research funding agencies.

## 2 The Programme

### 2.1 Main Workshops

#### 2.1.1 Discrete and Continuous Stochastic Evolutions: 19 - 23 March 2001

There were two main series of talks (4 each)

| Tom Kurtz (Madison) | Particle representations for stochastic PDEs |
| Andreas Greven (Erlangen) | Longtime behaviour of interacting spatial multitype systems |

The other talks given in this workshop were:

- **Robert Adler** Technion
- **Slava Belkivin** Nottingham
- **Dan Crisan** Imperial
- **Pierre del Moral** Toulouse
- **Alison Etheridge** Oxford
- **Klaus Fleischmann** Berlin
- **Jurgan Gartner** Berlin
- **Geoffrey Grimmett** Cambridge
- **Peter Kotelkoetz** Cleveland
- **Sylvie Meleard** Paris
- **James Norris** Cambridge
- **Stephan Olla** Cergy
- **Terry Lyons** Oxford
- **Michael Rockner** Bielefeld
- **Richard Sowers** Illinois
- **Yuri Suhov** Cambridge
- **Jonathan Warren** Warwick
- **Shinzo Watanabe** Kyoto
- **Oleg Zaboronsky** Warwick

- **Robert Adler** The geometry of Gaussian fields on manifolds
- **Slava Belkivin** Continuous stochastics as boundary value problems in Fock space
- **Dan Crisan** Exact rates of convergence for particle representations for the Zakai equation
- **Pierre del Moral** Genealogies and increasing propagations of chaos for Feynman-Kac and genetic models
- **Alison Etheridge** Survival and extinction in a locally regulated population
- **Klaus Fleischmann** Clumping of a super-Brownian reactant with a stable catalyst
- **Jurgan Gartner** Aspects of intermittency in the parabolic Anderson model
- **Geoffrey Grimmett** Stochastic evolution of ferromagnets
- **Peter Kotelkoetz** Derivation of correlated Brownian motions from Hamiltonian systems of particles
- **Sylvie Meleard** A probabilistic approach to the Boltzmann equation without for non Maxwell molecules
- **James Norris** Existence and uniqueness for spatial coagulation equations
- **Stephan Olla** Diffusive fluctuations in interacting particle systems
- **Terry Lyons** Characterization and identification of path processes
- **Michael Rockner** Weak Poincare inequalities and $L^2$ convergence rates of Markov semigroups
- **Richard Sowers** On Hamiltonian systems with small noise
- **Yuri Suhov** Convergence to equilibria and hydrodynamic limits for hyperbolic equations
- **Jonathan Warren** Some simple examples of sensitivity
- **Shinzo Watanabe** Stochastic flows in duality and noises
- **Oleg Zaboronsky** Statistical field theory of stochastic coalescence
2.1.2 Stochastic PDEs and Related Topics: 16 - 29 July 2001

In this general workshop the talks given were:

- Dirk Bloenker, Augsburg: Ginzburg-Landau Formalism for Stochastic PDEs
- Stella Brasasco, Caracas: Interface fluctuations for stochastic phase field equations in \( d = 1 \)
- Zdzislaw Brzezniak, Hull: Attractors for stochastic Navier Stokes in unbounded domains
- Jan van Casteren, Aalborg: Problems in semigroup theory and the HJB equation
- Sandra Cerrai, Florence: The Fleming Viot operator in \( L^1 \) spaces
- Jinghai Du, Chicago: SPDEs for geophysical fluid dynamics
- Erik Vanden Eijnden, Courant Institute: Generalized flows, intrinsic stochasticity and turbulent transport
- Istvan Gyongy, Edinburgh: On regularization by noise
- Martin Hairer, Geneva: Exponential mixing for a stochastic PDE driven by degenerate noise
- Erica Hausenblas, Salzburg: Numerical approximation of SPDE
- Paul Horridge, Warwick: Stationary distributions for a noisy reaction diffusion equation
- Kostya Khanin, Newton Institute: SPDEs and Burgers turbulence
- Nic Krylo, Minnesota: \( L^p(L^q) \) theory for stochastic PDEs
- Anna Kwiecinska, Warsaw: Stabilization of PDEs by noise
- Olivier Leveque, Lausanne: Hyperbolic equations driven by boundary noises
- Richard Liu, Princeton: Gaussian dynamics and invariant measures for dissipative SPDEs
- Sergey Lototski, USC: Stochastic parabolic equations in domains; weighted spaces and regularity
- Bohdan Maslowski, Prague: SPDE's driven by fractional Brownian motion
- Jon Mattingley, Stanford: Ergodic theory for dissipative SPDEs
- Carl Mueller, Rochester: A superprocess with singular mass creation
- Leonid Mytnik, Technion: PDE driven by stable noise
- David Nualart, Barcelona: SDEs driven by fractional Brownian motions
- Hans Oettinger, Zurich: Probabilistic models for vortex filaments based on fractional Brownian motion
- Martin Ondrejat, Nancy: Descriptions of fluctuations in nonequilibrium thermodynamics
- Andrey Piatnitski, Moscow: Yamada-Watanabe theory in Banach spaces
- James Robinson, Warwick: Homogenization of random reaction-diffusion equations
- Michael Rockner, Bielefeld: Stability of random attractors under perturbations and approximations
- Boris Rosovski, USC: Infinite systems of Brownian motions with singular interactions
- Bjorn Schmalfuss, Marseberg: On Krylov's \( L^p \) theory
- Wilhelm Stannat, Bielefeld: Invariant manifolds for SPDE
- Anna Talarczyk, Warsaw: On the regularity of transition semigroups for Fleming Viot processes
- Alexander Veretennikov, Leeds: Self intersection local time for Gaussian processes in \( S'(\mathbb{R}^d) \)
- Aaron Yip, Purdue: On large deviations for SDE approximations
- Jerry Zabczyk, Warsaw: Noise and uniqueness of motion by mean curvature
- Meike Zakai, Technion: Wong Zakai approximations for a class of stochastic evolution equations
- Lorenzo Zambotti, Pisa: Tangent processes
- Tusheng Zhang, Manchester: Integration by parts on the 3-d Bessel bridge and SPDE's with reflection
- Huazhong Zhao, Loughborough: On Backward Stochastic Partial Differential Equations
- Random travelling waves for stochastic reaction diffusion equations

2.1.3 Diffusions, Flows, Fluids, Filtrations, and Filaments: 30 July - 10 August 2003

After the main August workshop many participants stayed on at Warwick to attend this more specialized period, especially those working on fluid models. There were two main series of 5 talks each:

- **Boris Rosovski** (USC) Stochastic Fluid Mechanics: Stochastic diffeomorphisms and fluid dynamic; Stochastic Stokes equation; Stochastic Navier-Stokes Equations; Propagation of Gaussian Chaos by equations of fluid mechanics and moment theory.

- **Boris Tsirelson** (Tel Aviv) Brownian motions in groups and semigroups; Filtrations in the light of general classification theory; From non-Brownian filtrations to harmonic measures; Off white noises and product systems; Stability and sensitivity on cubes and trees.

The other talks given in this period were:
2.1.4 Stochastic PDEs and Related Topics: 4 - 15 August 2003

This workshop allowed us to see what work had been done since the main symposium year and to invite certain specialists who were unable to attend in 2000-2001. A new organizing committee (Dirk Bloemker, David Elworthy, Martin Hairer, Andrew Stuart, Roger Tribe) reflected the long-time visitors we had during this year. The talks given in this workshop were:

Sigurd Assing  Edinburgh  On the scaling of asymmetric exclusion processes
Peter Bazendale  USC  Lyapunov exponents and stability for the stochastic Duffing - Van der Pol equation
Dirk Bloemker  Warwick  Multiscale expansion of invariant measures near a bifurcation
Zdzislaw Brzezniak  Hull  Approximation for stochastic NSE in unbounded domains
Sandra Cerrai  Florence  Large deviations for stochastic reaction diffusion systems
Steven Evans  Berkeley  Geometry of the space of real-trees and tree-valued processes
Markov mortality models: some implications of quasi-stationarity
Benedetta Ferrario  Pavia  Some uniqueness results for the 2D Navier-Stokes equation with additive noise
Yuri Gliklikh  Voronezh  On conditions for global existence of solutions of various differential equations
Giuseppe Guatteri  Milan  On the backward stochastic Riccati equation in infinite dimensions
Christoph Gugg  Stuttgart  Ergodicity of SDEs driven by fractional Brownian motion
Martin Hairer  Warwick  Numerical approximation of the stochastic Navier Stokes
Erika Hausenblas  Salzburg  Stochastic Partial Differential Equation Driven by Fractional Noise
Yoosheng Hu  Kansas  Exponential stable stationary solutions for stochastic evolution equations
Peter Kloeden  Frankfurt  Between coalescence and diffusion
Yves Le Jan  Paris  Random attractors for SPDEs
Hannelore Lisei  Berlin  On Markov chain approximations to stochastic PDEs driven by Poisson measure noise
Hongwei Long  Edmonton  Wiener Chaos solution of stochastic evolution equations
Sergey Lototsky  USC  Kink dynamics: from an SPDE to diffusion-limited reaction
Grant Lythe  Leeds  Stochastic Navier Stokes Equation: Ergodicity and Malliavin Calculus
Jonathan Mattingly  Duke  On Cauchy-Dirichlet problem for parabolic SPDEs in weighted Hoelder spaces
Renugaik Mikuilevicius  Vilnius  The Stable Manifold Theorem for Semi-Linear Stochastic PDEs
Salah Mohammed  Carbondela  SPDEs driven by stable noise
Carl Mueller  Rochester  Stochastic PDEs with a type of infinitely deep square potential well
Leopold Makridakis  Thessaloniki  White noise limits for inertial particles in a random field
Toshiyuki Otsu  Nagano  A probabilistic representation for the vorticity of a 3D viscous fluid
Grigoris Paoli  Warwick  Stochastic PDEs and spirals
Marco Romito  Florence  Randomly forced CGL equation: stationary measures and the inviscid limit
Tony Shardlow  Durham  Homogenization for fully nonlinear PDE in stationary ergodic media
Armen Shirikyan  Paris  Quadratic Wiener functionals and solitons
Takeo Sugano  Osaka  Approximations for SDEs
Setsuo Taniguchi  Fukuoka  Stochastic comparison theorems for SPDEs
Roger Tribe  Warwick  Representation theorems for historical interacting Fisher-Wright diffusions
Alexander Veretennikov  Leeds  Stochastic Burgers equation with Levy space-time white noise
Anita Winter  Erlangen  Finite element method for stochastic parabolic partial differential equations
Jiang-Lun Wu  Swansea  Lévy noise on harmonic functions
Yubin Yan  Manchester  Fluctuations of some interacting particle systems
Jurek Zabczyk  Warsaw  Approximations of strong solutions of SDEs and SPDEs
Lorenzo Zambotti  Paris  Stochastic elementary formula and asymptotics with caustics in one-dimension

2.2 Mini-sessions

Part of the aim of these mini-sessions was to explain problems and techniques from stochastic PDEs to non-specialists. Thus in each mini-session a speaker was invited to give one or two introductory talks. These mini-sessions usually started
at friday lunchtime and ended saturday evening to make it easier for UK workers to attend during term. They proved so popular that we extended the number we had originally planned, partly reflecting the interests of our long-term visitors.

2.2.1 Stochastic Functional Differential Equations: 10 - 11 November 2000

Saleh Mohammed Carbondale Stochastic Functional DEs as dynamical systems, I and II
Bernt Oksendal Oslo A maximum principal for controlled stochastic delay systems
Rachel Kuske Minnesota Applications of stochastic delay equations to finance
Tony Shardlow Durham Stochastic modulation equations for SDEs
Xuerong Mao Strathclyde Weak approximations of SDEs
Hannelore Lisei Berlin Attraction for solutions of SFDEs

2.2.2 Schrodinger Equations with Random Forcing: 17-18 November 2000

Considering the effects of random forcing, as for example in the Belavkin equation, and not the literature on Schrodinger equations with random potentials.

Thomas Zastavniak Hull Introduction to Feynman path integrals
Arnaud Debussche Rennes Stochastic Mehler kernels via path integration
Vassili Kolokoltsov Nottingham Existence and Blow-up for non-linear stochastic Schrodinger equations
Zdzisław Brzezniak Hull Quasi-classical asymptotics for the Belavkin equation
O. Smolianov Moscow Scattering for stochastic Schrodinger and Newton equations

2.2.3 Ergodicity for spatial dynamics: 1 - 2 December 2000

In particular one aim was comparing different methods to establish ergodicity of spatial models. This was a recurrent theme during the year which contributed later to the simultaneous breakthrough by Kuksin and Shirikyan, Mattingly and Hairer on ergodicity of various dissipative systems under suitable noises.

Yuri Kondratiev Bonn Gibbs measures for lattice and continuous systems: characterisation and existence
Bogoslov Zegarlicki Imperial Applications of coercive inequalities to ergodicity and other problems
Andrew Stuart Warwick Geometric ergodicity via coupling
Jerzy Zabczyk Warsaw Strong Feller processes and invariant measures
Roger Tribe Warwick Coupling and comparisons - some examples

2.2.4 Stochastic Fluid Equations: 19-20 January 2001

Franco Flandoli Pisa The problems of singularities for 3-D fluids
Sergei Kuksin Herriot Watt Probabilistic approach and results under noise perturbations
Armen Shirikyan Herriot Watt Stochastic models of 3-D fluid vortex structures
Zdzisław Brzezniak Hull Deterministic and stochastic Navier Stokes equations I
Ambrey Truman Swansea Deterministic and stochastic Navier Stokes equations II

2.2.5 Homogenization: 2 - 3 February 2001

Alexander Veretennikov Leeds Averaging for Stochastic Differential Equations I and II
Andrei Itsinitski Moscow Homogenization of random parabolic operators with lower order terms I and II
Tomasz Komorowski Pontoise Diffusions in non-mixing Ornstein-Uhlenbeck flows
Stephano Olla Pontoise Bulk diffusion
2.2.6 Approximation methods for SPDEs: 16 - 17 February 2001

Andrew Stuart, Warwick: An Introduction to strong approximation for SDEs
Tony Shardlow, Durham: An introduction to weak approximation
Andrew Stuart, Warwick: An Introduction to strong approximation for SPDEs
Istvan Gyongy, Edinburgh: Approximation of SPDEs via Green's functions
Istvan Gyongy, Edinburgh: Approximation of SPDEs via $L^2$ theory
Jean-Sebastien Giet, Warwick: Speed of convergence for an Euler scheme for a rough functional of an SDE
Michael Tretynakov, Swansea: Weak and mean square approximations of SDEs in a bounded domain
Ben Haubly, Oxford: A Wong-Zakai theorem for reversible Markov processes

2.2.7 Random Dynamical Systems: 9-10 May 2001

Hans Crauel, Exeter: Random Dynamical Systems
Bjorn Schmalfuss, Marburg: Qualitative questions of SPDEs and climate theory
Kostiya Khanin, Newton Institute: Random dynamical systems and Burgers turbulence
James Robinson, Warwick: A stochastic pitchfork bifurcation in a reaction diffusion equation
Jose Lange, Sevilla: Attractors for stochastic partial differential equations
Hans Crauel, Exeter: Random set attractors versus random point attractors
Peter Imkeller, Berlin: Stochastic DEs as RDSs via random coordinate changes
Igor Chueshov, Berlin: Equilibria and attractors for cooperative systems of semilinear parabolic PDEs
Szymon Peszat, Krakow: Lagrangian dynamics for a passive tracer in a class of Gaussian Markovian flows

2.2.8 Infinite dimensional models in finance: 23 - 29 May 2001

There was one preparatory series of talks by Ronen Carmona (Princeton):
1. The Mechanics of the Fixed Income Markets; 2. Analysis of the data, and first mathematical models;

The other talks given were:

Chris Rogers, Bath: Monte Carlo valuation of American Options
Bernt Christensen, Aarhus: Infinite dimensional interest rate dynamics, stochastic volatility and yield curve calibration
Rene Cont, Paris: Term structure dynamics and parabolic SPDEs
Valdo Durlemann, Princeton: Implied correlation and spread options
Dimitri Filippovic, ETH: Consistency problems for Heath-Jarrow-Morton interest rate models
Sam Jacka, Warwick: No arbitrage for infinite dimensional term structure models
Marek Musiela, Paris: Pricing and risk management of derivatives written on non-traded assets
Bernt Oksendal, Oslo: Optimal control of SPDEs and applications to portfolio problems with partial observation
Josef Teichmann, Vienna: Interest rate models and infinite dimensional geometry - the classification result
Jerry Zabczyk, Warsaw: Variational inequalities and applications to optimal stopping

2.2.9 Stochastic Reaction Diffusion Equations: 15 - 16 June 2001

Using the outreach part of the funding this mini-session was held in Loughborough.

Sigurd Assing, Edinburgh: On the comparison method for SPDEs
Sandra Cerrai, Florence: Optimal control problems for reaction diffusion systems
Martin Hairer, Warwick: Exponential mixing for a stochastic P.D.E. driven by degenerate noise
Roger Tribe, Warwick: The KPP equation with multiplicative 'branching' noise
Huazhong Zhao, Loughborough: Ergodic and pathwise properties of random travelling waves for the stochastic KPP equation

2.2.10 Stochastic Stability Day: 29 July 2001

Xuexong Mao, Strathclyde: Asymptotic stability in distribution of SDEs with Markovian switching
Yuhong Li, Hull: Asymptotic compactness of 2D Stochastic Navier-Stokes equations on unbounded domains
Tomas Caraballo, Seville: Asymptotic behaviour of infinite-dimensional dynamical systems perturbed by random terms