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Course Regulations for Year 4

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/)

Note: The modules below are for the current academic year only, it is not guaranteed that they will run next year, or in future years, due to their highly specialised nature.

MASTER OF MATHEMATICS MMATH G103 4th Years

Normal Load = 120 CATS. Maximum Load = 150 CATS.

Students are required to take at least 90 CATS from the Core plus Lists A, C and D and, in their third and fourth years combined, at least 105 CATS from the Core plus Lists C and D.

[For example, a typical MMath student might satisfy this last requirement by including two **List C** modules in their offering for Year 3, and then including MA4K8/9 Project and three other **List C** modules in their offering for Year 4.]

4th Year MMath students will not be allowed to take second year modules, except as unusual options and even then only with a valid reason for doing so.

Direct link to MA4K8/9 Projects.

Many **List A** Year 3 Mathematics modules have a support class timetabled in weeks 2 to 10. This is your opportunity to bring the examples you have been working on, to compare progress with fellow students, and where several people are stuck or confused by the same thing, to get guidance from the graduate student in charge. **List C** and **D** modules tend to have fewer students and support classes are less common; in these cases you are more than usually encouraged to discuss problems or concerns directly with the lecturer, either during or after lectures, or in office hours.
For a full list of available modules see the relevant course regulation page.

Maths Modules

Optional Modules - List A
As the Third year option List A for G103 Mathematics (not including MA395 Third Year Essay nor MA397 Consolidation) with the exception of second year modules (coded MA2xx for example).

Optional Modules - List B
As the Third Year option List B for G103 Mathematics with the exception of second year modules (coded MA2xx for example).

Optional Modules - List C and D:

Note: Modules with an asterix (*) after them are deemed particularly suitable for 3rd year MMath students to consider taking, but this should be done taking the advice given on the Year 3 Regulations page on board.

Term	Code	Module	CATS	List

Term 1	MA424	Dynamical Systems	15	List C
	MA433	Fourier Analysis	15	List C
	MA453	Lie Algebras *	15	List C
	MA4A2	Advanced PDEs	15	List C
	MA4A5	Algebraic Geometry	15	List C
	MA4A7	Quantum Mechanics: Basic Principles and Probabilistic Methods	15	List C
	MA4C0	Differential Geometry	15	List C
	MA4E0	Lie Groups	15	List C
	MA4H0	Applied Dynamical Systems	15	List C
	MA4H4	Geometric Group Theory	15	List C
	MA4J1	Continuum Mechanics *	15	List C
	MA4J3	Graph Theory	15	List C
	MA4J5	Structures of Complex Systems	15	List C
	MA4J8	Commutative Algebra II	15	List C
	MA4L0	Advanced Topics in Fluids	15	List C
	MA4L6	Analytic Number Theory	15	List C
	MA4L9	Variational Analysis and Evolution Equations	15	List C
	MA4M5	Geometric Measure Theory *	15	List C
	MA4M7	Complex Dynamics	15	List C
	MA4M8	Theory of Random Graphs *	15	List C
	PX408	Relativistic Quantum Mechanics	7.5	List C
	PX425	High Performance Computing in Physics	7.5	List C
	PX430	Gauge Theories for Particle Physics	7.5	List C
	PX436	General Relativity	15	List C
Terms 1 & 2	MA4K8 MA4K9	Projects (Research/Maths in Action)	30	Core
	MA472	Reading Module	15	List C

Term 2	MA426	Elliptic Curves	15	List C
	MA427	Ergodic Theory	15	List C
	MA442	Group Theory *	15	List C
	MA448	Hyperbolic Geometry	15	List C
	MA473	Reflection Groups	15	List C
	MA482	Stochastic Analysis	15	List C
	MA4E7	Population Dynamics: Ecology and Epidemiology	15	List C
	MA4F7	Brownian Motion	15	List C
	MA4H8	Ring Theory	15	List C
	MA4H9	Modular Forms *	15	List C
	MA4J0	Advanced Real Analysis	15	List C
	MA4J7	Cohomology and Poincare Duality	15	List C
	MA4L2	Statistical Mechanics	15	List C
	MA4L3	Large Deviation Theory	15	List C
	MA4L7	Algebraic Curves	15	List C
	MA4M1	Epidemiology by Example	15	List C
	MA4M2	Mathematics of Inverse Problems *	15	List C
	MA4M4	Topics in Complexity Science	15	List C
	MA4M6	Category Theory *	15	List C
	MA4M9	Mathematics of Neuronal Networks	15	List C

Common Unusual Options

Term	Code	Module	CATS	List
Terms 1/2	STxxx	ST4 modules offered by the Statistics Department (note ST401, ST402 and ST404 are **only** available to Statistics Students and ST407 is List B).	15 or 18	Unusual Option

Interdisciplinary Modules (IATL and GSD)

Second, third and fourth-year undergraduates from across the University faculties are now able to work together on one of IATL's 12-15 CAT interdisciplinary modules. These modules are designed to help students grasp abstract and complex ideas from a range of subjects, to synthesise these into a rounded intellectual and creative response, to understand the symbiotic potential of traditionally distinct disciplines, and to stimulate collaboration through group work and embodied learning.

Maths students can enrol on these modules as an Unusual Option, you can register for a maximum of TWO IATL modules but also be aware that on many numbers are limited and you need to register an interest before the end of the previous academic year. Contrary to this is GD305 Challenges of Climate Change , form filling is not required for this option, register in the regular way on MRM (this module is run by Global Sustainable Development from 2018 on).

Please see the IATL page for the full list of modules that you can choose from, for more information and how to be accepted onto them, but some suggestions are in the table below:

Term	Code	Module	CATS	List
Term 1	IL105	Applied Imagination	12/15	Unusual
	GD305	Challenges of Climate Change	15	Unusual

Term 2	IL108	Reinventing Education	12/15	Unusual
	IL131	Serious Tabletop Game Design and Development	12/15	Unusual
	IL116	The Science of Music	7.5/15	Unusual
	IL123	Genetics: Science and Society	12/15	Unusual

Languages

The Language Centre offers academic modules in Arabic, Chinese, French, German, Japanese, Russian and Spanish at a wide range of levels. These modules are available for exam credit as unusual options to mathematicians in all years. Pick up a leaflet listing the modules from the Language Centre, on the ground floor of the Humanities Building by the Central Library. Full descriptions are available on request. Note that you may only take one language module (as an Unusual Option) for credit in each year. Language modules are available as whole year modules, or smaller term long modules; both options are available to maths students. These modules may carry 24 (12) or 30 (15) CATS and that is the credit you get. We used to restrict maths students to 24 (12) if there was a choice, but we no longer do this.

Note: 3rd and 4th year students cannot take beginners level (level 1) Language modules.

There is also an extensive and very popular programme of lifelong learning language classes provided by the centre to the local community, with discounted fees for Warwick students. Enrolment is from 9am on Wednesday of week 1. These classes do not count as credit towards your degree.

The Transnational Resources Centre provides resources in the FAB building for all students registered with the Language Centre, more information can be found here.

A full module listing with descriptions is available on the Language Centre web pages.

Important note for students who pre-register for Language Centre modules

It is essential that you confirm your module pre-registration by coming to the Language Centre as soon as you can during week one of the new academic year. If you do not confirm your registration, your place on the module cannot be guaranteed. If you decide, during the summer, NOT to study a language module and to change your registration details, please have the courtesy to inform the Language Centre of the amendment.

Information on modules can be found at the Language Centre page

Objectives

After completing the fourth year of the MMath degree the students will have

- covered advanced mathematics in greater depth and/or breadth, and be in a position to decide whether they wish to undertake research in mathematics, and to ascertain whether they have the ability to do so
- achieved a level of mathematical maturity which has progressed from the skills expected in school mathematics to the understanding of abstract ideas and their applications
- developed
    - investigative and analytical skills,
    - the ability to formulate and solve concrete and abstract problems in a precise way, and
    - the ability to present precise logical arguments
- been given the opportunity to develop other interests by taking options outside the Mathematics Department in all the years of their degree course.

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4-template/)
Lecturer:

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Three hour examination

Prerequisites:

Leads to:

Content:

This is a second course on ordinary representations of finite groups, which only assumes the basics covered in Groups and Representations. Representation Theory studies ways in which a group can act on vector spaces by linear transformations. This has important applications in algebra, in number theory, in geometry, in topology, in physics, and in many other areas of pure and applied mathematics. We will begin by reviewing the basics of representation and character theory, covered in MA3E1. Then, we will introduce new powerful representation theoretic techniques, including:

* Symmetric and alternating powers, Frobenius-Schur indicators, and definability over R. For example, we will be able to study the following questions:

Given an element g in a finite group G, count the number of elements x in G whose square is g.Given a complex representation of G, is there a change of basis after which all matrices are defined over the reals?

* Representations of the symmetric groups following Vershik-Okounkov approach.

* Schur-Weyl duality and representations of the general linear groups.

* If time permits: induction theorems, Brauer induction and Artin induction.

Aims:

To introduce some techniques in the theory of ordinary representations of finite groups that go beyond the basics and that are important in other areas of mathematics.

Objectives:

By the end of the module the student should be able to:

- quickly compute the full character table of some important groups
- investigate real, complex and quaternionic fields representations
- understand characters of symmetric and general linear groups

Books:
Isaacs, *Character Theory of Finite Groups*
Curtis and Reiner, *Methods of Representation Theory, with Applications to Finite Groups and Orders*, Vols. 1 and 2
Fulton, Harris, *Representation Theory: a first course*
Ceccherini-Silberstein, Scaraborti, Tolli, *Representation Theory of the Symmetric Groups:* the Okounkov–Vershik Approach, Character Formulas, and Partition Algebras

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

MA474 Representation Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma474/)

Not Running 2019/20

Lecturer:

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Three hour examination

Prerequisites: MA3E1 Groups and Representations

Leads to: Postgraduate work in Algebra, Combinatorics, Geometry and Number Theory

Content:

This is a second course on ordinary representations of finite groups, which only assumes the basics covered in Groups and Representations. Representation Theory studies ways in which a group can act on vector spaces by linear transformations. This has important applications in algebra, in number theory, in geometry, in topology, in physics, and in many other areas of pure and applied mathematics. We will begin by reviewing the basics of representation and character theory, covered in MA3E1. Then, we will introduce new powerful representation theoretic techniques, including:

* Symmetric and alternating powers, Frobenius-Schur indicators, and definability over R. For example, we will be able to study the following questions:

Given an element g in a finite group G, count the number of elements x in G whose square is g.Given a complex representation of G, is there a change of basis after which all matrices are defined over the reals?

* Representations of the symmetric groups following Vershik-Okounkov approach.

* Schur-Weyl duality and representations of the general linear groups.

* If time permits: induction theorems, Brauer induction and Artin induction.

Aims:

To introduce some techniques in the theory of ordinary representations of finite groups that go beyond the basics and that are important in other areas of mathematics.

Objectives:

By the end of the module the student should be able to:

- quickly compute the full character table of some important groups
- investigate real, complex and quaternionic fields representations
- understand characters of symmetric and general linear groups

Books:
Isaacs, *Character Theory of Finite Groups*
Curtis and Reiner, *Methods of Representation Theory, with Applications to Finite Groups and Orders*, Vols. 1 and 2
Fulton, Harris, *Representation Theory: a first course*
Ceccherini-Silberstein, Scaraborti, Tolli, *Representation Theory of the Symmetric Groups:* the Okounkov–Vershik Approach, Character Formulas, and Partition Algebras

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

MA4E3 Asymptotic Methods

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4e3/)

Not running in 2019/20

Lecturer:

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: 3 hour examination

Prerequisites: All the core Analysis modules of Years 1 and 2; MA3B8 Complex Analysis is desirable but may be taken in parallel.

Content:
The classical analysis mainly deals with convergent series in spite of the fact that an attempt to solve a problem using series often leads to divergence. If treated in a consistent way, a divergent solution may provide even more information about the original problem than a convergent one. Asymptotic series has been a very successful tool to understand the structure of solutions of ordinary and partial differential equations.

Divergent series: summation of divergent series, divergent power series, analytic continuation of a convergent series outside the disk of convergence, asymptotic series, an application to ODEs.
Laplace transform: basic properties, Borel transform, Gevrey-type series, Borel sums, Watson theorem.
Stokes phenomenon: examples, asymptotics in sectors of a complex plane, an application - asymptotic of Airy function.
Multivalued analytic functions: analytic continuation, multivalued functions, introduction to Riemann surfaces.
Formal convergence: space of formal series, formal convergence, an application to ODEs.
Rapidly oscillating integrals: asymptotics of rapidly oscillating integrals, method of stationary phase, examples.

Aims:
To introduce a systematic approach to analysis of divergent series, their interpretation as asymptotic series, and application of these methods to study of ordinary differential equations and integrals.

Objectives:
At the end of the module the student should be familiar with the methods involving analysis of asymptotic series and to acquire basic techniques in studying asymptotic problems. The student should be able to perform analysis of divergent series and to be able to correctly interpret them as asymptotic series.

Books:
We will not follow any particular book, but most of the material can be found in:
C.F. Carrier, M. Krook and C.E. Pearson, *Functions of a Complex Variable: theory and technique*, Hodbooks.
N.G. De Bruijn, *Asymptotic Methods in Analysis*, North-Holland Publishing co. (3d ed.) (1970).
P.P.G. Dyke, *An Introduction to Laplace Transforms and Fourier Series*, Springer Undergraduate Mathematics Series (2000).
G. Hardy, *Divergent Series*, Clarendon Press, 1963/American Mathematical Society, 2000.
R.B. Dingle, *Asymptotic Expansions: Their Derivation and Interpretation*, Academic Press (1973).

Additional Resources

MA4G5 Analytical Fluid Dynamics

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4g5/)

Not Running 2019/20

Lecturer:

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: 100% Exam (3 hours)

Prerequisites: MA3G7 Functional Analysis I is required.
A few selected results from MA359 Measure Theory, MA3G1 Theory of PDEs and MA3G8 Functional Analysis II may be reviewed briefly, as required. MA433 Fourier Analysis, MA4A2 Advanced PDEs, and MA4J0 Advanced Real Analysis may make good companion courses.

Content:
Topics include:

- The equations (brief derivation and key properties) 1
- The vorticity formulation and Biot–Savart law
- Local-in-time existence and uniqueness results in Rn, n = 2, 3, via energy estimates
- An alternative approach to local well-posedness for Euler, using particle trajectory methods
- Global-in-time existence results in 2D and comparisons to 3D
- Criteria for blowup of solutions e.g. the celebrated Beale-Kato-Majda theorem
- An introduction to weak solutions of the Navier–Stokes equations
- A global existence result for weak solutions of the Navier–Stokes equations (time permitting)
- Other selected topics, according to student interest (time permitting)

Aims:
This course aims to give an introduction to the rigorous analytical theory of the PDEs of fluid mechanics. In particular we will focus on the incom-pressible Euler and Navier-Stokes equations in R2 and R3, which are widely used models for inviscid and viscous flow, respectively. The questions of global existence and uniqueness of solutions to these systems form the basis for a great deal of current research. In this course we will study a few of the fundamental results in this field, which will give students a chance to apply knowledge from Functional Analysis and PDE modules to these highly-relevant non-linear systems.

Objectives:
By the end of the module, students will:

- Be familiar with the Euler and Navier–Stokes and the physical meaning of the terms therein, for classical and vorticity-stream formulations.
- Have explored, in these particular cases, some of the typical issues arising in the study of PDEs (local vs global existence, uniqueness, blowup criteria, 2D vs 3D behaviour etc.)
- Have learnt two approaches to proving local existence and uniqueness restults: via an energy methods (featuring Sobolev estimates), and a particle-trajectory method (using H¨older spaces).
- Have seen the definition of a weak solution of the Navier–Stokes equa-tions and a discussion of further well-known existence results (at least in summary).

Books:

- Primary text: A.J. Majda and A.L. Bertozzi. Vorticity and incom¬pressible flow. CUP, Cambridge, 2002.
- A.J. Chorin and J.E. Marsden. A mathematical introduction to fluid mechanics, volume 4 of Texts in Applied Mathematics. Springer-Verlag, New York, third edition, 1993.
- J.C. Robinson, J.L. Rodrigo, and W. Sadowski. The three–dimensional Navier–Stokes equations. Classical Theory. Cambridge University Press, Cambridge, 2016.
- P. Constantin and C. Foias. Navier-Stokes Equations. The University of Chicago Press, Chicago, 1988.

Additional Resources

MA4G6 Calculus of Variations

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4g6/)

Not Running 2020/21

Lecturer:

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Written Examination (85%), Assignments (15%)

Prerequisites:
MA209 Variational Principles (useful, but not required)
MA3G7 Functional Analysis 1 (parts of)
MA3G8 Functional Analysis 2 (parts of, can be heard concurrently, not absolutely required)

Leads To:
MA4A2 Advanced PDEs can be heard concurrently or before/after.
MASDOC A1 MA912 Analysis for Linear PDEs.
MASDOC A2 MA914 Topics in PDEs.
PhD-level courses.

Content:

- Sobolev spaces.
- The Direct Method of the Calculus of Variations and lower semicontinuity.
- Convexity and aspects of Convex Analysis (duality).
- Existence of solutions for scalar problems.
- Polyconvexity and existence of solutions semicontinuity for vector-valued problems.
- Regularity theory for minimisation problems.
- Optimal control theory and Young measures.
- Quasiconvexity, laminates and microstructure.
- Variational convergence of functionals (Γ convergence).

If time permits:

- Other variational principles (Ekeland etc.).
- Functions of bounded variations and applications.

Aims:
The Calculus of Variations is both old and new. Starting from Euler's work up to very recent discoveries, this sub-field of Mathematical Analysis has proven to be very successful in the analysis of physical, technological and economical systems. This is due to the fact that many such systems incorporate some kind

of variational (minimum, maximum, extremum) principle and understanding this structure is paramount to proving meaningful results about them. Applications range from material sciences over geometry to optimal control theory. The aim of this course is to give a thoroughly modern introduction and to lead from the basics to sophisticated recent results.

Objectives:
By the end of the module the student should be able to:

- Understand why variational problems are important
- See several examples of variational problems in physics and other sciences.
- Appreciate that (and why) some problems have "classical" solutions and some do not.
- Be able to prove the existence of solutions to convex variational problems.
- Know which kinds of problems are not convex and why convexity is often an unrealistic assumption for vector-valued problems.
- Have an insight into generalised convexity conditions, such a quasiconvexity and polyconvexity and their applications.
- Be able to prove existence of solutions to quasiconvex/polyconvex variational problems.
- Have seen simple optimal control problems and can understand them as a special case of general variational problems.
- Know what microstructure is, why it forms, and what its physical significance is.
- Have seen how regularised functionals converge to a limit functional as the regularisation parameter tends to zero.

Books:
B. Dacorogna: *Introduction to the Calculus of Variations*. Imperial College Press 2004.
B. Dacorogna: *Direct Methods in the Calculus of Variations*. 2nd edition. Springer, 2008.
L. C. Evans: *Partial Differential Equations*. 2nd edition. AMS, 2010 (some chapters).
I. Fonseca and G. Leoni: *Modern Methods in the Calculus of Variations: Lp -spaces*. Springer, 2007.
E. Giusti: *Direct Methods in the Calculus of Variations*. World Scientific, 2002.

Additional Resources

Archived Pages: 2014 2016 2018

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4J7 Cohomology and Poincaré Duality

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j7/)
Lecturer: Goncalo Tabuada

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: 85% 3 hour examination in the summer, 15% by assignments

Formal registration prerequisites: None

Assumed knowledge:

- MA3F1 Introduction to Topology
- MA3H6 Algebraic Topology

Useful knowledge: A certain level of mathematical maturity (comfort with proofs and routine computations). Some category theory (categories, functors, natural transformations) will make learning the material much easier.

Synergies: This is a capstone of the undergraduate mathematics programme. It is a valuable course for anybody with an interest in "modern topology" (bringing us up to the 1960's). It is essential background for postgraduate study in geometry or topology, as well as many areas of algebra, number theory, and applied mathematics.

Leads to: The following modules have this module listed as **assumed knowledge** or **useful background:**

Content:

- Cochain complexes and cohomology
- The duality between homology and cohomology
- Chain approximations to the diagonal and products in cohomology
- The cohomology ring
- The cohomology ring of a product of spaces and applications
- The Poincaré duality theorem
- The cohomology ring of projective spaces and applications
- The Hopf invariant and the Hopf maps
- Spaces with polynomial cohomology
- Further applications of cohomology

Aims:

- To introduce cohomology and products as an important tool in topology
- Give a proof of the Poincaré duality theorem and go on to use this theorem to compute products
- There will be many applications of products including using products to distinguish between spaces with isomorphic homology groups
- To use products to study the classical Hopf maps

Objectives: By the end of the module the student should be able to:

- Define cup and cap products
- Use the Poincaré duality theorem
- Compute the cohomology ring of many spaces including product spaces and projective spaces
- Apply the cohomology ring to get topological results
- Define, calculate and apply the Hopf invariant

Books:
Algebraic Topology, Allen Hatcher, CUP 2002
Algebraic Topology A First Course, Greenberg and Harper, Addison-Wesley 1981

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4J8 Commutative Algebra II

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j8/)
Lecturer: Miles Reid

Term(s): Term 1

Status for Mathematics students: List A

Commitment: 30 lectures

Assessment: 85% by 3-hour examination, 15% coursework

Formal registration prerequisites: None

Assumed knowledge: Divisibility and ideals from MA249 Algebra II: Groups and Rings, the first half of the MA3G6 Commutative Algebra module or my book "Undergraduate Commutative Algebra".

Useful background: Material from, or an interest in:

- MA3A6 Algebraic Number Theory
- MA3G6 Commutative Algebra
- MA377 Rings and Modules
- MA3D5 Galois Theory
- MA3H6 Algebraic Topology with basic ideas on Homology

Synergies: The module runs in parallel with MA4A5 Algebraic Geometry and the overlap between the two may serve as useful repetition and reinforcement. The material of commutative algebra has many basic and more advanced links with Algebraic Geometry, Algebraic Number Theory and next term's MA4L7 Algebraic Curves.

Content:

- Review of MA3G6 Commutative Algebra
- Completion and dimension of Noetherian local rings
- Regular local rings, free resolutions and projective dimension
- Regular sequences, Cohen-Macaulay and Gorenstein rings

Aims:

The module treats selected topics in commutative algebra, as a continuation of MA3G6 Commutative Algebra. Commutative algebra takes as its model 19th century work in arithmetic and algebraic geometry, as unified in the work of Dedekind and Weber, and later David Hilbert. In the modern era, commutative algebra is viewed as foundational for algebraic number theory, algebraic geometry (especially its scheme theoretic aspects) and computer algebra. Introductory texts on Algebraic Geometry commonly assume results from Commutative Algebra going beyond the first MA3G6 Commutative Algebra module, and my course will cover many of these results, together with topics in the algebra of commutative rings that are of independent interest.

Objectives:
By the end of the module the student should:

- Have developed a sophisticated command of many facets of a major branch of algebra with important applications across the whole of mathematics
- Be in a position to read standard texts and research papers on commutative algebra
- Be able to apply commutative algebra methods to problems in algebra, arithmetic and geometry, both on paper and in computer algebra
- Understand the dimension of rings and the relations between regular local rings and nonsingular points of algebraic varieties
- Know how to work with Cohen-Macaulay, Gorenstein and complete Intersection rings

Website:
Example sheets, assessed worksheets and additional material will be available from my webpage:
https://homepages.warwick.ac.uk/~masda/MA4J8/
I may produce notes on some topics, including colloquial “general interest” or “propaganda” items, and technical appendices.

Books:
M.F. Atiyah and I.G. MacDonald: *Introduction to Commutative Algebra*, Warwick Library QA 251.3.A8, available as e-book
David Eisenbud: *Commutative Algebra with a View Towards Algebraic Geometry*, Warwick Library QA 251.3.E4, available as e-book
Hideyuki Matsumura: *Commutative Ring Theory*, Warwick Library QA 251.3.M2, available as e-book
Miles Reid: Undergraduate Commutative Algebra, Warwick Library QA 251.3.R3, available as e-book
Marco Schlichting, Commutative algebra II (notes of the 2013 module by Florian Bouyer), get from:
https://homepages.warwick.ac.uk/~masda/MA4J8/

Additional Resources

MA4K0 Introduction to Uncertainty Quantification

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4k0/)

Not running 2021/22

Lecturer: Dr Tim Sullivan

Term(s): Term 2

Status for Mathematics students: List A

Commitment: 30 hours of lectures

Assessment: Three hour exam in summer (50%), term-time assessments (50%)

Assessment in term-time will be a mix of written assessed work and a min-project.

Prerequisites:
Essential: ST112 Probability B, MA3G7 Functional Analysis I and either MA359 Measure Theory or ST318 Probability Theory.

Useful or related:
MA4A2 Advanced PDEs, ST407 Monte Carlo Methods.
Some programming background in e.g. C, Mathematica, Matlab, Python, or R.

Leads to:
Graduate study in a range of problems at the interface of differential equations and probability, including UQ theory, data assimilation, inverse problems and filtering. These subjects may be studied within mathematics departments, or in applications departments throughout the sciences and engineering.

Content: This is a list of *possible* topics, not all of which will necessarily be covered in the module. In particular, sections marked *** are likely to be omitted.

1. **Introduction and Course Outline**
   1. Typical UQ problems and motivating examples: uncertainty propagation, inverse problems, certification, prediction
   2. Epistemic and aleatoric uncertainty. Bayesian and frequentist interpretations of probability
2. **Preliminaries**
   1. Recap of Hilbert space theory [e.g. from MA3G7 Functional Analysis I]: direct sums; orthogonal decompositions and orthogonal projection; compact operators
   2. Recap of measure/probability theory [e.g. from MA359 Measure Theory or ST318 Probability Theory]: basic axioms for measure/probability spaces, Lebesgue integration of real-valued functions
   3. More Hilbert space theory: tensor products; trace-class and Hilbert–Schmidt operators; scales of Hilbert spaces
   4. More probability theory (on function spaces): Bochner integration of vector-valued functions; probability measures (especially Gaussian measures) on function spaces; various representations of random functions, e.g. Karhunen–Loève expansions, random series, gPC expansions, Gaussian processes and Gaussian mixtures
3. **Inverse Problems and Bayesian Perspectives**
   1. Examples of inverse problems (linear and nonlinear, static and dynamic) and their ill-posedness
   2. Deterministic solution of linear inverse problems in Hilbert spaces; the Moore–Penrose pseudo-inverse
   3. Regularisation of inverse problems: regularisation of operators; convergence of regularisation schemes; variational regularisation; perspectives on nonlinear inverse problems

    4. Bayesian inverse problems: formulation and well-posedness; special treatment of linear Gaussian problems and the Kálmán filter
4. **Computational Methods**
    1. Deterministic numerical evaluation of integrals: univariate and multivariate quadrature rules; sparse quadratures
    2. Random quadratures: Monte Carlo; Markov chain Monte Carlo; dimension-independent proposals.
    3. Particle and sequential Monte Carlo methods for sampling: ensemble Kálmán filtering and inversion
    4. Pseudo-random methods***: quasi-Monte Carlo, low-discrepancy sequences, Koksma–Hlawka inequality
    5. Intrusive and non-intrusive calculation of gPC expansions: stochastic Galerkin, non-intrusive spectral projection, stochastic collocation
    6. Visualisation of uncertainty***
    7. Computing with Gaussian processes***
5. **Sensitivity Analysis***
    1. Estimation of derivatives
    2. "L^∞" sensitivity indices, e.g. McDiarmid subdiameters; associated concentration-of-measure inequalities
    3. ANOVA and "L^2" sensitivity indices, e.g. Sobol' indices
    4. Active subspaces and model reduction
6. **Second-Order ("Knightian") Uncertainty***
    1. Mixed epistemic/aleatoric uncertainty; the robust Bayesian paradigm
    2. Finite-dimensional parametric studies; convex programs
    3. Optimal UQ / distributionally-robust optimization: formulation, reduction, computation

Aims:
Uncertainty Quantification (UQ) is a research area of theoretical and practical importance at the intersection of applied mathematics, probability, statistics, computational science and engineering (CSE) and many application areas. UQ can be seen as the theory and numerical application of probability/statistics to problems and models with a strong "real-world" (especially physics- or engineering-based) setting.

This course will provide an introduction to the basic problems and methods of UQ from a mostly mathematical point of view, with numerical exercises so that the methods can be seen to work in (small) practical settings. More generally, the aim is to provide an introduction to some relatively diverse methods of applied mathematics and applied probability as they are used in practice, through the particular unifying theme of UQ.

Objectives:
By the end of the module students should be able to understand both the basic theory of, and in example settings perform:

- deterministic and Bayesian solution of inverse problems
- forward propagation of uncertainty
- orthogonal systems of polynomials and their diverse applications
- data assimilation and filtering
- finite- and infinite-dimensional optimization methods
- sensitivity and variance analysis

Literature:
The following books may be of interest:

1. T. J. Sullivan. *Introduction to Uncertainty Quantification*, Texts in Applied Mathematics, Springer, 2015. doi:10.1007/978-3-319-23395-6
2. R. C. Smith. *Uncertainly Quantification: Theory, Implementation, and Applications*, SIAM, 2013. ISBN:978-1-611973-21-1
3. K. J. H. Law, A. M. Stuart, and J. Voss. *Data Assimilation: A Mathematical Introduction*, Texts in Applied Mathematics, Springer, 2015. doi:10.1007/978-3-319-20325-6
4. O. P. Le Maître and O. M. Knio. *Spectral Methods for Uncertainty Quantification. With Applications to Computational Fluid Dynamics*. Scientific Computation, Springer, 2010. doi:10.1007/978-90-481-3520-2
5. D. Xiu. *Numerical Methods for Stochastic Computations. A Spectral Method Approach*. Princeton University Press, 2010. ISBN:978-0-691-14212-8

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

MA4K2 Optimisation and Fixed Point Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4k2/)

Not Running 2019/20

Lecturer:

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Three hour written examination (100%)

Prerequisites: MA3G7 Functional Analysis I and MA3G1 Theory of PDEs

Leads To: Graduate studies in Applied Mathematics (eg MASDOC)

Content:
We will cover some of the following topics:-

- Optimisation in Banach spaces.
- Optimisation in Hilbert spaces with and without constraints.
- Optimality conditions and Lagrange multipliers.
- Lower semi-continuity.
- Convex functionals.
- Variational inequalities
- Gradient descent and iterative methods.
- Banach, Brouwer Schauder fixed point theorems.
- Monotone mappings.
- Applications in differential equations, inverse problems, optimal control, obstacle problems, imaging.

Aims:
The module will form a fourth year option on the MMath Degree.It builds upon modules in the second and third year like Metric Spaces, Functional Analysis I and Theory of PDEs to present some fundamental ideas in nonlinear functional analysis with a view to important applications, primarily in optimisation and differential equations. The aims are: introduce the concept of unconstrianed and constrained optimisation in Banach and Hilbert spaces; existence theorems for nonlinear equations; importance in applications to calculus of variations, PDEs, optimal control and inverse problems.

Objectives:
By the end of the module the student should be able to:-

- Recognise situations where existence questions can be formulated in terms of fixed point problems or optimisation problems.
- Recognise where the Banach fixed point approach can be used.
- Apply Brouwers and Schauders fixed point theorems.
- Apply the direct method in the calculus of variations.
- Apply elementary iterative methods for fixed point equations and optimisation.

Books:
The instructor has own printed lecture notes which will provide the primary source. The printed lecture notes will also have a bibliography.

List A (These books contain material directly relevant to the module):-

- G. Allaire, Numerical analysis and optimisation, Oxford Science Publications 2009
- P.G. Ciarlet, Linear and nonlinear functional analysis with applications. SIAM 2013
- P. G. Ciarlet, Introduction to numerical linear algebra and optimisation, Cambridge 1989

- L.C. Evans, Partial Differential Equations , Graduate Studies in Mathematics 19, AMS, 1998.
- F. Troltzsch, Optimal control of partial differential equations AMS Grad Stud Math Vol 112 (2010)

List B (The following texts contain relevant and more advanced material):-

- G. Aubert and P. Kornprobst. Mathematical problems in Image Processing, Applied Mathematical Sciences (147). Springer Verlag 2006.
- M. Chipot. Elements of nonlinear analysis . Birkhauser, Basel-Boston-Berlin, 2000.
- D. Kinderleher and G. Stampacchia, An introduction to variational inequalities and their applications Academic Press 1980
- E. Zeidler, Nonlinear functional analysis and its applications I, Fixed Point theorems , Springer New York, 1986

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4K3 Complex Function Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4k3/)

Not Running 2019/20

Lecturer:

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Assignments 15%, 3 hour written exam 85%

Prerequisites:
MA3B8 Complex Analysis is essential.
MA359 Measure Theory and MA3G7 Functional Analysis I are desirable but not essential.

Leads To: PhD level research in function spaces.

Content:
1. Problems on the Hardy space.
1.1. Overview. Understanding functions through problems in Complex Function Theory.
1.2. Review of Complex Analysis I, Functional Analysis I and Measure Theory.
1.3. The Hardy space. Basic properties. Other important spaces.

2. Problems on functions.
2.1. Evaluation at one point. Reproducing kernel.
2.2. Multipliers. Bounded functions.
2.3. Existence of boundary values.
2.4. Zero sets. Blaschke products.
2.5. Inner-outer factorization.

3. Problems on operators and functionals.
3.1. Examples of operators. Boundedness. Spectrum. Spectral theorem.
3.2. The shift operator. Subspaces. Polynomials. Cyclicity. Invariant subspaces.
3.3. The restriction operator. Interpolation and sampling. Embeddings.
3.4. Optimization of functionals. Distances. Extremal problems. Cyclicity revisited.

4. What else?
4.1. More spaces and operators. Domains. Several variables. Meromorphic and entire functions. Dirichlet series. Banach spaces. Random functions. More operators.
4.2. More problems. Approximation. Corona. Growth. Other operator properties. Univalence. Completeness. Conformal representations.

Aims:
To provide to the students a variety of roads they can follow on their private further research.
To introduce them to the results in analytic function spaces through a fundamental example.
To show to the students how natural problems motivate this study.

Objectives:
By the end of the module the student should be able to:

Understand the fundamental properties of the Hardy space.
Understand the fundamental properties of the Hardy space, that this is the case for complex function theory.
Produce proofs of simple facts and solve particular cases of the classical problems.

Books:
P. L. Duren, Theory of Hp spaces.
J. E. Garnett, Bounded Analytic Functions.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4K4 Topics in Interacting Particle Systems

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4k4/)

Not Running 2019/20

Lecturer:

Term(s): Term 2

Lectures:

- Wed 11-12 in B1.01
- Thurs 12-1 in B3.01
- Fri 3-4 in B2.03 (sci-conc.)

Support Classes:

- Wednesday 12-1 in A1.01

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: 3 Hour written Exam 85%, Assignments 15%

Prerequisites: Undergraduate Probability Theory, Linear Algebra and Markov Processes (eg. MA3H2 Markov Processes and Percolation Theory or ST333 Applied Stochastic Processes)

Leads To:

Content:

1] Interacting Particle Systems

- Construction and definitions (graphical construction, semigroups and generators).
- Revision of basic concepts like stationary distributions and reversibility.
- Classical examples (to be used throughout the course).

2] Relaxation and Mixing Times

- Introduction and definitions of mixing times.
- Basic bounds and techniques.
- Spectral methods and relaxation times.
- Basics of Potential Theoretic approach, resistor networks for reversible Markov Processes.

3] Large Deviations

- Introduction with examples.

4] Metastability

- Application of Large Deviations.
- Application of Potential Theory.

Aims:

The principle aim is firstly to introduce basic stochastic models of collective phenomena arising from the interactions of a large number of identical components, called interacting particle systems. The module will then introduce several key topics which are currently at the forefront of mathematical research in interacting particle systems. In particular we fill focus on the study of large-scale dynamics.

Objectives:

By the end of the module the student should be able to:

- Have a good working knowledge of key prototypical models of interacting particle systems such as the Ising model, the exclusion process and the zero-range process.
- Understand the main concepts used in current research into the large scale dynamics of interacting particle systems.
- Work in an independent and practical manner on topics related to interacting particle systems. Students should gain an advanced-level understanding of continuous time Markov processes on finite state spaces.
- Build and run stochastic simulations using their preferred method (simple examples of C-code will be given, requiring straightforward adaptation, for those who do not have a strong background in this area). This module should also help students building team working skills.

Books:

- Levin, Peres, Wilmer: *Markov Chains and Mixing Times*, AMS (2009) [Available Online]
- T.M. Liggett: *Interacting Particle Systems - An Introduction*, ICTP Lecture Notes 17 (2004) [Available Online]
- F. den Hollander: *Large Deviations*, AMS (2000)
- A. Bovier: *Metastability, Lecture notes* - Prague (2006) [Available Online]
- Montenegro, Tetali: *Mathematical aspects of mixing times in Markov chains* (2006) [Available Online]

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4K5 Introduction to Mathematical Relativity

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4k5/)

Not Running in 2019/20

Lecturer:

Term(s):

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Written Examination 100%

Prerequisites:
MA3H5 Manifolds; MA3G1 Theory of PDEs (strongly recommended)
MA4C0 Differential Geometry (recommended)
PX148 Classical Mechanics & Relativity

Leads To:

Content:
* The wave equation and Special Relativity (Propagation of signals: the light-cone; finite speed of propagation; Transformations preserving the wave equation; the Lorentz group; Minkowski spacetime)

* Brief review of (pseudo-)Riemannian geometry (Vectors, one-forms and tensors; the metric tensor; the Levi-Civita connection and curvature; Stoke's theorem)

* Lorentzian geometry (Lorentzian metrics; causal classification of vectors and curves; global hyperbolicity; The d'Alembertian operator; Energy-momentum tensor for a scalar field; finite speed of propagation for a scalar field)

* General Relativity (Einstein's equations; discussion of local well posedness; Example: The Schwarzschild black hole; The Cauchy problem; discussion of open problems)

Aims:
One of the crowning achievements of modern physics is Einstein's theory of general relativity, which describes the gravitational field to a very high degree of accuracy. As well as being an astonishingly accurate physical theory, the study of general relativity is also a fascinating area of mathematical research, bringing together aspects of differential geometry and PDE theory. In this course, I will introduce the basic objects and concepts of general relativity without assuming a knowledge of special relativity. The ultimate goal of the course will be a discussion of the Cauchy problem for the vacuum Einstein equations, including a statement of the relevant well-posedness theorems and a discussion of their relevance. We will take a 'field theory' approach to the subject, emphasising the deep connection between Lorentzian geometry and hyperbolic PDE. In contrast to the course PX436 General Relativity offered by the department of physics, we concentrate on the mathematical structure of the theory rather than its physical implications.

Objectives:
By the end of the module the student should be able to:

- Understand how the Minkowski geometry and Lorentz group arise from considerations of signal propagation for the scalar wave equation.
- Understand the basics of Lorentzian geometry: the metric; causal classification of vectors; connection and curvature; hypersurface geometry; conformal compacti cations; the d'Alembertian operator.
- Be able to state the well-posedness theorems for the Cauchy problem for the Einstein equations and sketch the proof of local well posedness.

Books:
General Relativity and the Einstein Equations, Yvonne Choquet-Bruhat, Oxford University Press, 2009. (Available as an electronic resource.)
The large scale structure of spacetime, S.W. Hawking and G.F.R. Ellis, Cambridge University Press, 1973.
Gravitation, Charles W. Misner, Kip S. Thorne and John Archibald Wheeler.
General Relativity, Robert M. Wald, University of Chicago Press, c1984.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

MA4K6 Data Assimilation

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4k6/)

Not Running in 2019/20

Lecturer:

Term(s):

Status for Mathematics students: List C

Commitment: 30 hours of lectures

Assessment: Written Examination 70%, MATLAB based Coursework 30%

Prerequisites: ST112 Probability B and MA254 Theory of ODEs

Leads To: Graduate studies in Applied Mathematics (eg MASDOC)

Content:
1. Problem Formulation
(i) Dynamical Systems: iterated maps, Markov kernels, time-averaging and ergodicity, explicit examples.
(ii) Bayesian Probability: joint, marginal and conditional probabilities; Bayes' formula.
(iii) Smoothing, Filtering: formulation of these off-line and on-line probability distributions using Bayes' theorem and the links between them.
(iv) Well-Posedness: introduction of metrics on probability measure and demonstration that smoothing and filtering distributions are Lipschitz with respect to data, using these metrics.

2. Smooting Algorithms
(i) Monte Carlo Markov Chain: Random walk Metropolis, Metropolis-Hastings, proposals tuned to the data assimilation scenario.
(ii) Variational Methods: relationship between maximizing probability and minimizing a cost function; demonstration of multi-modal behaviour.

3. Filtering Algorithms
(i) Kalman filter: derivation using precision matrices and use of Sherman-Woodbury identity to formulate with covariances.
(ii) 3DVAR: derivation as a minimization principle compromising between fit to model and to data.
(iii) Extended and Ensemble Kalman Filter. Generalize 3DVAR to allow for adaptive estimation of (covariance) weights in the minimization principle.
(iv) Particle Filter. Sequential importance sampling, proof of convergence.

Aims:
The module will form a fourth year option on the MMath Degree. Data Assimilation is concerned with the principled integration of data and dynamial models to produce enhanced predictive capability. As such it finds wide-ranging applications in areas such as weather forecasting, oil reservoir management, macro and micro economic modelling and traffic flow. This module aim is to describe the mathematical and computational tools required to study data assimilation.

Objectives:
By the end of the module the student should be able to understand a range of important subjects in modern applied mathematics, namely:

- Stochastic dynamical systems
- Long-time behaviour of dynamical systems
- Bayesian probability
- Metrics on probability measures
- Monte Carlo Markov Chain
- Optimization
- Control
- Matlab programming

Books:
Instructor has has own printed lecture notes (draft of a book) which will provide the primary source. These notes have an extensive bibliography and include matlab codes which will be made available to the students.

Additional Resources

MA4L0 Advanced Topics in Fluids

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l0/)
Lecturer: Thomasina Ball

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 100% Examination

Formal registration prerequisites: None

Assumed knowledge:

- MA3D1 Fluid Dynamics:

Familiarity with the mathematical description of fluid dynamics is necessary.

- MA250 Introduction to PDEs:

Experience with partial differential equations and methods of their solutions.

Useful background:

- MA3J4 Mathematical Modelling with PDE:

Some experience of modelling with PDEs.

Synergies: The following modules would go well with Advanced Topics in Fluids:

- MA3G1 Theory of Partial Differential Equations
- MA3H0 Numerical Analysis and PDEs
- MA4H0 Applied Dynamical Systems
- MA4J1 Continuum Mechanics

Content:

Fluid dynamics forms a core subject with applications in a number of disciplines including, engineering, nanotechnology, biology, medicine and geosciences. Principles of fluid dynamics serves as an anchor to describe natural phenomena by providing a common language and set of tools for describing, analyzing and understanding observations and experiments in such a diverse array of disciplines. Continuing on from MA3D1: Fluid dynamics, in this module we will study selected advanced topics in fluid dynamics that provides a core understanding of fluid dynamical phenomena.

The module will cover the following topics:

- **Stokes flow:** Properties of Stokes Flows, spherical harmonic solutions, flow around a translating rigid sphere
- **Lubrication theory:** Thin-film approximation, squeeze flows, surface tension, free-surface flows, gravity-driven flows
- **Complex fluids and non-Newtonian rheology:** Constitutive laws for shear thinning, yield stress and viscoelastic fluids, squeeze flows and free surface flows for non-Newtonian rheologies
- **Hydrodynamic instability:** The Rayleigh instability, Kelvin-Helmholtz instability

Aims:

- Students will be able to apply the governing principles of fluid dynamics to specific phenomena, possibly involving some systematic simplification methods.

- They will be introduced to some advanced techniques for analyzing fluid flow.
- They will be able to relate observations in nature to the aforementioned analysis techniques.

Learning outcomes:

- Be able to state and prove properties of Stokes flow and use these properties to formulate solutions in spherical geometries.
- Be able to describe the approximations of lubrication theory and derive the governing equations.
- Be able to apply lubrication theory to squeeze flow and free surface flows and solve the associated partial differential equations for their flow characteristics.
- Be able to identify different non-Newtonian rheologies and describe their physical characteristics.
- Be able to solve for the flow field for generalised non-Newtonian fluids in Poiseuille and Taylor-Couette flow, and for viscoelastic fluids in simplified shear and extensional flows.
- Be able to describe the method of linear stability analysis.
- Be able to use linear stability analysis to classify the stability of layered inviscid and viscous flows.

Further reading:

D.J. Acheson, *Elementary Fluid Dynamics*, Oxford University Press.

G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press

H. Ockendon, J. R. Ockendon, Viscous Flow, Cambridge University Press

A. Morosov, S. E. Spagnolie, Complex Fluids in Biological Systems, Chapter 1: Introduction to Complex Fluids, Springer

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4L1 Mathematical Modelling in Biology and Medicine

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l1/)

Not Running 2019/20

Lecturer:

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: Written examination (50%), Project work (50%)

Prerequisites: no formal requirements

Leads To:

Content:
Part A Mathematical Modeling in the Life Sciences

Week 1: Mathematical Foundations (Repetition as warming up)
Lecture 1 Introduction to graph theory, relevance for the Life Sciences, degree distributions and their characteristics, examples.
Lecture 2 Random variables and probability distributions, stochastic processes, examples.
Lecture 3 Statistics and data analysis

Week 2: Biochemical Reaction Systems and Rule Based Systems
Lecture 1 Introduction to reaction schemes.
Lecture 2 Hypergraphs and chemical complexes.
Lecture 3 Extended reaction schemes.

Part B Applications.

Week 3: Morphogenesis, Cellular Transport Processes
Lecture 1 Dynamical systems, semi-Flows and functional analysis.
Lecture 2 Reaction-diffusion equations and models of pattern formation/morphogenesis.
Lecture 3 Qualitative behaviour, more pattern formation, modeling transport and reaction.

Week 4: Cell Biology and Cell Cultures
Lecture 1 Modeling in Genetics.
Lecture 2 The Cell Nucleus.
Lecture 3 The Chemostat.

Week 5: Cell Cultures and Physiology
Lecture 1 Physiologically Structured Populations.
Lecture 2 The Cell Cycle.
Lecture 3. Structured Populations in the Chemostat.

Week 6: Future Medicine
Lecture 1 Learning Algorithms I.
Lecture 2 Learning Algorithms II.
Lecture 3. Data mining in medicine.

Week 7: Future Medicine
Lecture 1 Numerical simulation in medicine.
Lecture 2 Numerical simulation in medicine.
Lecture 3 Numerical simulation in medicine.

Week 8: Global Ecology
Lecture 1 Population Dynamics and Global Disturbances.
Lecture 2 Models of Biodiversity.
Lecture 3 The Growth of Cities and Landscape Patterns.

Week 9: Evolutionary theory
Lecture 1 Models of evolution.
Lecture 2 Examples of complex evolving systems, biology and language.
Lecture 3 Examples of complex evolving systems, game theory.

Week 10: Climate Change and Feedback to Living Systems
Lecture 1 The global climate and its modeling.
Lecture 2 The global climate and oceans.
Lecture 3 The global climate and vegetation.

Aims:
• Introduce the student to advanced mathematical modelling in the Life Sciences in a systematic way.
• Making the student aware how to choose and use different modelling techniques in different areas of the Life Sciences.
• A clarification about the mathematical content and structure of mathematical models in the Life Sciences.
• A general introduction to modern systems analysis tailored to the Life Sciences.

Objectives:
By the end of the module the student should be able to:
Orient in the latest research on Mathematical Biology
Apply methods learned in the module to new problems inside the scope of Mathematical Biology.
Quickly solve standard problems occurring in Mathematical Biology

Books:
Newman, M. 2010 Networks: an introduction. Oxford University Press.
Metz, J. A. J. and Diekmann, O. 1986. The dynamics of physiologically structured populations. Lecture Notes in Biomathematics. 68.

Keener, J. and Sneyd, J. 1998 Mathematical Physiology. Springer-Verlag.
Murray J.D. 2002. Mathematical Biology. New York: Springer.
Iannelli, M., Martcheva, M., and Milner, F. A. 2005 Gender-Structured Population Modeling: Mathematical Methods.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4L2 Statistical Mechanics

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l2/)
Lecturer: Yuchen Liao

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: 100% by 3 hour examination

Formal registration prerequisites: None

Assumed knowledge: Basic probability theory and some combinatorics:

ST111 Probability (Part A):

- Notions of events and their probability
- Conditional probabilities
- Law of large numbers

ST112 Probability (Part B):

- Random variables
- Joint distributions
- Independence of random variables
- Moment generating functions
- Law of large numbers

MA241 Combinatorics:

- Basic counting
- Binomial and multinomial theorems
- Generating functions
- Basics of graph theory

MA3H2 Markov Processes and Percolation Theory:

- Notion of Markov process

Useful background: In addition, some notions of measure theory can be useful indeed:

MA359 Measure Theory:

- Fatou's lemma
- Monotone and dominated convergence theorems
- Fubini's theorem
- Riesz representation theory

Synergies: The models of statistical mechanics provide excellent illustrations for the following module:

- ST318 Probability Theory

Content: Statistical mechanics describes physical systems with a huge number of particles.

In physics, the goal is to describe macroscopic phenomena in terms of microscopic models and to give meaning to notions such as temperature or entropy. Mathematically, it can be viewed as the study of random variables with spatial dependence. Models of statistical mechanics form the background for recent advances in probability theory and stochastic analysis, such as Schramm-Loewner evolutions and the theory of regularity structures. So, they form an important background for understanding these topics of modern mathematics.

This module is aimed at giving a thorough mathematical introduction to equilibrium statistical mechanics, mainly focusing on concrete models of lattice spin systems. Topics will include:

- The Curie-Weiss model (thermodynamic limit and critical exponents).
- The Ising model (van Hove theorem, infinite-volume Gibbs states, correlation inequalities, phase diagram, Peierls' argument, Lee-Yang theorem, Kramers-Wannier duality).
- The Gaussian free field (infinite-volume Gibbs states, random walk representation).
- Models with continuous symmetry (Mermin-Wagner theorem).

Time permitting, we will be discussing some additional topics based on the interests of the students. Here are some possible topics:

- Exactly solvable models (dimer models and random tiling, Onsager's solution of the 2d Ising model, six vertex model and Yang-Baxter equation, etc).
- More on quantum spin systems (XY model, Heisenberg model, quantum Ising model, etc).
- Disordered systems (spin glasses, random field Ising model, etc).

Aims: To familiarise students with statistical mechanics models, phase transitions, and critical behaviour.

Objectives: By the end of the module students should be able to:

- Apply basic ideas of phase transitions and critical behaviour to lattice systems of statistical mechanics.
- Understand how large complex systems at equilibrium can be described from microscopic rules.
- Have understood basic ideas of phase transitions and critical behaviour in several concrete examples.

Books: We will mainly follow Chapters 2, 3, 8, 9 of the new introductory textbook:

Sacha Friedli and Yvan Velenik, *Equilibrium Statistical Mechanics of Classical Lattice Systems: a Concrete Introduction*. Available at: http://www.unige.ch/math/folks/velenik/smbook/index.html

Additional topics will be accompanied by lecture notes.

Interested students can also look into:

David Ruelle, *Statistical Mechanics: Rigorous Results*, World Scientific, 1999.

James Sethna: *Statistical Mechanics: Entropy, Order Parameters and Complexity*, Oxford Master Series in Physics, 2006.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

MA4L3 Large Deviation Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l3/)
Lecturer: Stefan Adams

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: 85% Exam and 15% Homework

Formal registration prerequisites: None

Assumed knowledge:

- MA359 Measure Theory
- ST342 Mathematics of Random Events
- MA250 Introduction to Partial Differential Equations

Useful background:

- MA3H2 Markov Processes and Percolation Theory
- MA209 Variational Principles
- MA3G7 Functional Analysis I
- MA3G1 Theory of PDEs

Synergies:

- MA4F7 Brownian Motion
- MA427 Ergodic Theory
- MA424 Dynamical Systems

Content:

- Basic understanding of large deviation techniques (definition, basic properties, Cramer's theorem, Varadhan's lemma, Sanov's theorem, the Gärtner-Ellis Theorem).
- Large deviation approach to Gibbs measure theory (free energy; entropy; variational analysis; empirical process; mathematics of phase transition).
- Large deviation theory for stochastic processes and its connections with PDEs (Fleming semi group; viscosity solutions; control theory).
- Applications of large deviation theory (at least one of the following list of topics: interface models; pinning/wetting models; dynamical systems; decay of connectivity in percolation; Gaussian Free Field; Free energy calculations; Wasserstein gradient flow; renormalisation theory (multi-scale analysis)).

Aims:

- Basic understanding of large deviation theory (rate function; free energy; entropy; Legendre-transform).
- Understanding that large deviation principles provide a bridge between probability and analysis (PDEs, convex and variational analysis).
- Large deviation theory as the mathematical foundation of mathematical statistical mechanics (Gibbs measures; free energy calculations; entropy-energy competition).
- Understanding large deviation in terms of the nonlinear Fleming semi group and its links to control theory.
- Discussion of the role of large deviation methods and results in joining different scales, e.g. as the micro-macro passage in interacting systems.
- Connection of large deviation theory with stochastic limit theorems (law of large numbers; ergodic theorems (time and space translations); scaling limits).

Objectives: By the end of the module students should be able to:

- Derive basic large deviation principles
- Be familiar with the variational principle and the large deviation approach to Gibbs measure
- Distinguish all three level of large deviation
- To calculate Legendre-Fenchel transform for most relevant distributions
- Understand basic variational problems
- Be familiar with some application of large deviation theory

- Link basic large deviation principle for stochastic processes to PDEs
- Compute of rare probabilities via large deviation rate functions given as variational problems in analysis and PDE theory. Be able to use Legendre-transform techniques, basic convex analysis and Laplace integral methods.
- Understand the role of free energy calculations and representations in analysis (PDEs and control problems and variational problems). Be able to provide a variational description of Gibbs measures.
- Be able to analyse the minimiser of large deviation rate functions of basic examples and to provide interpretation of the possible occurrence of multiple minimiser.
- Explain the role of the free energy in interacting systems and its link to stochastic modelling. Be able to provide different representations of the free energy for some basic examples.
- Be able to estimate probabilities for interacting systems using Laplace integral techniques and basic understanding of Gibbs distributions.
- Apply large deviation theory to one topic from the following list: interface models; pinning/wetting models (random walk models); dynamical systems; decay of connectivity in percolation; Gaussian Free Field; Free energy calculations; Wasserstein gradient flow; renormalisation theory (multi-scale analysis).

Books: We won't follow a particular book and will provide lecture notes. The course is based on the following three books:

[1] Frank den Hollander, *Large Deviations* (Fields Institute Monographs), (paperback), American Mathematical Society (2008).

[2] Amir Dembo & Ofer Zeitouni, *Large Deviations Techniques and Applications* (Stochastic Modelling and Applied Probability), (paperback), Springer (2009).

[3] Jin Feng and Thomas G. Kurtz, *Large Deviations for Stochastic Processes*, American Mathematical Society (2006).

Other relevant books and lecture notes:

[a] Hans-Otto Georgii, *Gibbs Measures and Phase Transitions*, De Gruyter (1988).

[b] Stefan Adams, Lectures on mathematical statistical mechanics, Communications of the Dublin Institute for Advanced Studies Series A (Theoretical Physics), No. 30 , available online http://www2.warwick.ac.uk/fac/sci/maths/people/staff/stefan adams/lecturenotestvi/cdias-adams-30.pdf

[c] Stefan Adams, *Large Deviations for Stochastic Processes*, EURANDOM reports 2012-25, (2012); available online http://www.eurandom.tue.nl/reports/2012/025-report.pdf

Additional Resources

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MA4L4 Mathematical Acoustics

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l4/)
Lecturer: Ed Brambley

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: 100% Exam (3 hours)

Prerequisites: MA244 Analysis III (for contour integration); MA250 Introduction to PDEs (for Green's functions). MA3D1 Fluid Dynamics is useful but not necessary.

Content:

- Some general acoustic theory
- Sound generation by turbulence and moving bodies (including the Lighthill and Ffowcs Williams-Hawkings acoustic analogies)
- Wave scattering (including the scalar Wiener-Hopf technique applied to the Sommerfeld problem of scattering by a sharp edge)
- Long-distance sound propagation, including nonlinear and viscous effects
- Wave-guides.

Aims:
The application of wave theory to problems involving the generation, propagation and scattering of acoustic and other waves is of considerable relevance in many practical situations. These include, for example, underwater sound propagation, aircraft noise, remote sensing, the effect of noise in built-up areas, and a variety of medical diagnostic applications. This course aims to provide the basic theory of wave generation, propagation and scattering, and an overview of the mathematical methods and approximations used to tackle these problems, with emphasis on applications to aeroacoustics.

Objectives:
By the end of the module the student should be able to:

- Reproduce standard models and arguments for sound generation and propagation
- Apply mathematical techniques to model sound generation and propagation in simple systems
- Understand and apply Wiener-Hopf factorisation in the scalar case

Books:

- A.D. Pierce, "Acoustics", McGraw-Hill 1981
- D.G. Crighton, A.P. Dowling, J.E. Ffowcs Williams, et al, "Modern Methods in Analyticial Acoustics", Springer 1992
- L.D. Landau & E.M. Lifshitz, "Fluid Mechanics", Elsevier 1987

Additional Resources

Archived Pages: 2017 2018

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MA4L6 Analytic Number Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l6/)
Lecturer: Adam Harper

Term(s): Term 1

Status for Mathematics Students: List C

Commitment: 30 one hour lectures

Assessment: 100% 3 hour examination

Formal registration prerequisites: None

Assumed knowledge: Some basic real and complex analysis, including: uniform convergence, the Identity Theorem from complex analysis and especially Cauchy's Residue Theorem. This is covered in the modules MA244 Analysis III and (ideally) MA3B8 Complex Analysis. Although the module will not assume much specific content or results, it will have a serious "analytic" flavour of estimating objects and handling error terms. The most important thing is to be comfortable with this style of mathematics, which might be familiar from previous courses in analysis, measure theory or probability.

Useful background: The module will assume very little from number theory, but in a few places it will be useful to know things like: the Chinese Remainder Theorem, the structure of the multiplicative group mod q. These are covered in e.g. MA249 Algebra II and MA257 Introduction to Number Theory.

Synergies: This is fundamentally an analysis module, and would go well (in terms of the general style of the mathematics) with modules like: MA433 Fourier Analysis, MA4J0 Advanced Real Analysis and possibly MA427 Ergodic Theory. Those interested in number theory will probably also enjoy MA426 Elliptic Curves.

Content:
The course will cover some of the following topics, depending on time and audience preferences:

- Warm-up:
  The counting functions $\pi(x), \Psi(x)$ of primes up to x. Chebychev's upper and lower bounds for $\Psi(x)$.
- Basic theory of the Riemann zeta function:
  Definition of the zeta function $\zeta(s)$ when $\Re(s) > 1$, and then when $\Re(s) > 0$ and for all s. The connection with primes via the Euler product. Proof that $\zeta(s) \neq 0$ when $\Re(s) \geq 1$, and deduction of the Prime Number Theorem (asymptotic for $\Psi(x)$).
- More on zeros of zeta:
  Non-existence of zeta zeros follows from estimates for $\sum_{N<n<2N} n^{it}$. The connection with exponential sums, and outline of the methods of Van der Corput and Vinogradov. Wider zero-free regions for $\zeta(s)$, and application to improving the Prime Number Theorem. Statement of the Riemann Hypothesis.
- Primes in arithmetic progressions:
  Dirichlet characters χ and Dirichlet L-functions $L(s, \chi)$. Non-vanishing of $L(1, \chi)$. Outline of the extension of the Prime Number Theorem to arithmetic progressions.

Aims:
Multiplicative number theory studies the distribution of objects, like prime numbers or numbers with "few" prime factors or "small" prime factors, that are multiplicatively defined. A powerful tool for this is the analysis of generating functions like the Riemann zeta function $\zeta(s)$, a method introduced in the 19th century that allowed the resolution of problems dating back to the ancient Greeks. This course will introduce some of these questions and methods.

Objectives:
By the end of the module the student should be able to:

- Consolidate existing knowledge from real and complex analysis and be able to place in the context of Analytic Number Theory
- Have a good understanding of the Riemann zeta function and the theory surrounding it up to the Prime Number Theorem
- Understand and appreciate the connection of the zeros of the zeta function with exponential sums and the statement of the Riemann Hypothesis
- Demonstrate the necessary grasp and understanding of the material to potentially pursue further postgraduate study in the area

Books:

- H. Davenport. Multiplicative Number Theory. Third edition, published by Springer Graduate Texts in Mathematics. 2000
- A. Ivi'c. The Riemann Zeta-Function. Theory and Applications. Dover edition, published by Dover Publications, Inc.. 2003
- H. Montgomery and R. Vaughan. Multiplicative Number Theory I. Classical Theory. Published by Cambridge studies in advanced mathematics. 2007
- E. C. Titchmarsh. The Theory of the Riemann Zeta-function. Second edition, revised by D. R. Heath-Brown, published by Oxford University Press. 1986

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
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MA4L7 Algebraic Curves

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l7/)
Lecturer: Diane Maclagan

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: 85% exam, 15% assessed worksheets

Formal registration prerequisites: None

Assumed knowledge: Some familiarity with basic ideas of commutative algebra is a prerequisite. As a rough guide, the lectures need the first half of MA3G6 Commutative Algebra. More specifically, the main technical items are localisation (partial rings of fraction of an integral domain), local rings. The definitions and ideas from the first half of MA4A5 Algebraic Geometry are also prerequisites.

Useful background: Material such as field extensions and ideals in the polynomial ring from MA3D5 Galois Theory will serve as useful motivation. The idea of integral elements of a number field MA3A6 Algebraic Number Theory is a good warm-up for integral closure that will be used in the course. The idea of meromorphic function from MA3B8 Complex Analysis will be mentioned in explaining the purely algebraic discussion of zeros and poles of a rational function. In a similar way, the Cauchy integral theorem is good motivation for the full statement of the Riemann-Roch theorem, although it is not needed for the proof.

Synergies: The course is a basic introduction to the study of algebraic varieties (and schemes) and their cohomology. The Riemann-Roch theorem for curves is a first major step towards the classification of algebraic curves, surfaces and higher dimensional varieties, that makes up a large component of modern algebraic geometry, and has applications across the mathematical sciences and theoretical physics.

Leads to: The following modules have this module listed as **assumed knowledge** or **useful background:**

- MA426 Elliptic Curves

Content: The module covers basic questions on algebraic curves. The first sections establishes the class of non-singular projective algebraic curves in algebraic geometry as an object of study, and for comparison and motivation, the parallel world of compact Riemann surfaces. After these preliminaries, most of the rest of the course focuses on the Riemann-Roch space $\mathcal{L}(C, D)$, the vector space of meromorphic functions on a compact Riemann surface or a non-singular projective algebraic curve with poles bounded by a divisor D - roughly speaking, allowing more poles gives more meromorphic functions.

The statement of the Riemann-Roch theorem

$$\dim \mathcal{L}(C, D) \geq 1 - g + \deg D.$$

It comes with sufficient conditions for equality. The main thrust of the result is to provide rational functions that allows us to embed C into projective space $\mathbb{P}^n$. The formula involves an invariant called the genus $g(C)$ of the curve. In intuitive topological terms, we think of it as the ''number of holes''. However, it has many quite different characterisations in analysis and in algebraic geometry, and is calculated in many different ways. The logical relations between these treatments is a little complicated. A middle section of the course emphasizes the meaning and purpose of the theorem (independent of its proof), and give important examples of its applications.

The proof of RR is based on commutative algebra. Algebraic varieties have many different types of rings associated with them, including affine coordinate rings, homogeneous coordinate rings, their integral closures, and their localisations such as the DVRs that correspond to points of a non-singular curve. Footnote to the course notes include (as non-examinable material) references to high-brow ideas such as coherent sheaves and their cohomology and Serre--Grothendieck duality.

Learning Outcomes:

By the end of the module the student should be able to:

- Demonstrate understanding of the basic concepts, theorems and calculations related to projective curves defined by homogeneous polynomials of low degree
- Demonstrate understanding of the basic concepts, theorems and calculations that relate the zeroes and poles of rational functions with the general theory of discrete valuation rings and divisors on projective curves
- Demonstrate knowledge and understanding of the statement of the Riemann-Roch theorem and an understanding of some of its applications
- Demonstrate understanding of the proof of the Riemann-Roch theorem

Books:

Frances Kirwan, *Complex Algebraic Curves*, LMS student notes

William Fulton, *Algebraic Curves: An Introduction to Algebraic Geometry* online at www.math.lsa.umich.edu/~wfulton/CurveBook.pdf

I.R. Shafarevich, *Basic Algebraic Geometry* (especially Part 1, Chapter 3, Section 3.7)

Robin Hartshorne, *Algebraic Geometry*, (Chapter 4 only)

The lecturer's notes will be made available during the course.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

MA4L8 Numerical Analysis and Nonlinear PDEs

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l8/)

Not Running in 2020/21

Lecturers:

Term(s):

Status for Mathematics students:

Commitment: 10 x 3 hour lectures + 9 x 1 hour support classes10 x 3 hour lectures + 9 x 1 hour support classes

Assessment: 3hr Written Examination 100%

Prerequisites: Ideally one would take at least two from:

-MA3G7 Functional Analysis I

-MA359 Measure Theory

-MA3G1 Theory of PDEs

-MA3H0 Numerical analysis and PDEs.

Leads To:

Content:

1. Review analysis for PDEs
2. Review numerical discretisation
3. Finite element theory for linear problems
4. Concepts for discretises problems
5. Semilinear monotone equations
6. Obstacle problems

7. Time dependent problems

Aims:

The goal of this course is to introduce some fundamental concepts, methods and theory associated with the numerical analysis of partial differential equations and in particular nonlinear equations.

Finite element theory will provide the core machinery for devising methods and their analysis. Abstract notions of stability, consistency and convergence will be introduced as applied to convergence of minimisers, approximation of equilibrium points and the solution of nonlinear discrete problems

Objectives:

By the end of the module the student should be able to:

- Recognise the nature of the problem to be approximated

- Recognise where the Galerkin method is appropriate for use

- Formulate discrete approximations of nonlinear PDEs

- Obtain stability estimates in a variety of settings

- Apply elementary iterative methods for fixed point equations and optimisation.

Books:

- Soren Bartels *Numerical methods for nonlinear PDEs* Springer Series in Computtaional Mathematics Vol 47 (2015)
- S Larrsson and V. Theme

***PDEs and numerical methods* Springer Texts in Applied Maths Vol 45 (2005)**

Additional Resources

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MA4L9 Variational Analysis and Evolution Equations

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4l9/)
Lecturer: Charles Elliott

Term: Term 1

Status for Mathematics students: List C

Commitment: 10 x 3 hour lectures + 9 x 1 hour support classes

Assessment: 100% Examination

Formal registration prerequisites: None

Assumed knowledge:

- MA3G7 Functional Analysis I Banach and Hilbert spaces, Dual spaces, Linear operators, Riesz Representation Theorem
- MA359 Measure Theory Lebesgue integration, properties and convergence theorems
- MA259 Multivariable Calculus Differentiable functions of several variables, Divergence theorem

Useful background:

- MA3G1 Theory of Partial Differential Equations
- MA3G7 Functional Analysis II

Synergies:

The module fits well with other modules in Analysis and Applications involving partial differential equations, particularly MA4A2 Advanced PDEs. Essential or useful for research in much of analysis, dynamical systems, probability and applied mathematics etc.

Content:

Because of the ubiquitous nature of PDE based mathematical models in biology, advanced materials, data analysis, finance, physics and engineering much of mathematical analysis is devoted to their study. Often the models are time dependent; the state evolves in time. Although the complexity of the models means that finding formulae for solutions is impossible in most practical situation one can develop a functional analysis based framework for establishing well posedness in a variety of situations.

This course covers some of the main material behind the most common evolutionary PDEs. In particular, the focus will be on functional analytical approaches to find well posed formulations and properties of their solutions.

This course is particularly suitable for students who have liked analysis and differential equation courses in earlier years and to students interested in applications of mathematics. Many students intending graduate studies will find it useful. There are not too many prerequisites, although you will need some functional analysis, some knowledge of measure theory and an acquaintance with partial differential equations. Topics include:

- Abstract formulation of linear equations, Bochner spaces
- Hille-Yosida Theorem, Lions-Lax-Milgram Theorem
- Gradient flows
- PDE examples
- Applications

Books: There will be typed lecture notes with a bibliography. For example, there will be will be material related to chapters in the following:-

H. Brezis *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer Universitext (2011)

A.Ern and J.-L. Guermond, *Finite Elements III, Texts in Applied Mathematics*, Springer (2021)

L. C. Evans *Partial Differential Equations*, AMS Grad Studies in Maths Vol 19

S. Bartels *Numerical Methods for Nonlinear PDEs*, Springer (2015)

Additional Resources

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MA4M1 Epidemiology by Example

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m1/)
Lecturer: Dr Kat Rock and Dr Simon Spencer

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 2 formal lectures per week plus 1 structured, lecturer-lead lab session plus 1 support lab

Assessment: 100% assessed through coursework

Formal registration prerequisites: None

Assumed knowledge:

- Knowledge of general behaviour/steady states of ODEs
- Basic programming skills (ODE solvers, functions, for/if/while loops, plotting, vectors and matrices) - the course is taught in Matlab but the first 2 weeks is aimed at getting everyone to a basic level either by refreshing their knowledge or by learning Matlab given prior programming experience in another language
- Basic knowledge of key probability distributions and their properties (including gamma, Erlang, Poisson, binomial, beta, uniform, exponential, normal)

Useful background: There are no strict prerequisites, but other modules that could provide a useful background include those on modelling such as:

- MA254 Theory of ODEs
- MA256 Introduction to Systems Biology
- ST202 Stochastic Processes
- MA390 Topics in Mathematical Biology
- MA3J4 Mathematical Modelling with PDE

For programming:

- MA124 Maths by Computer

- MA117 Programming for Scientists
- MA261 Differential Equations: Modelling and Numerics

Synergies: The following module goes well together with Epidemiology by Example:

- MA4E7 Population Dynamics - this is a complementary course which is recommended to be taken before or simultaneously with Epidemiology by Example. Population Dynamics provides a lot more detail on development of different ecological and epidemiological models and their behaviour (especially qualitative) and is assessed 100% through exam. Epidemiology by Example focuses on programming to tackle a series of real-world-inspired epidemiological problems, bringing in components like model fitting and health economics, which would not be able to be assessed through traditional written examination.

Content: Epidemiology by Example was a new module in 2020/21 which focuses on the application of numerical methods to address real-world problems in infectious diseases. Starting with programming for basic infectious disease models, the module will progress on to implementation of stochastic models, fitting models to real-world data, adaptive management of diseases and health economic analyses for decision making. The module is designed to give an overview of key methods currently used in epidemiology research and will be 100% assessed through coursework.

Programming language: Matlab

Aims: Students taking this module will acquire hands-on experience of manipulating mathematical models, implementing appropriate numerical methods and fitting models to data, all of which are essential components of
modern-day modelling for research or industry. By the end of the module, students will have encountered a range of model types which can describe a broad range of important infection systems such as influenza, malaria, measles and soil transmitted helminths. Students will understand how to perform predictive analyses which could inform policy decision making - such as assessing future control interventions including adaptive strategies and health economic analyses.

Objectives:

By the end of the module the student will be able to:

- Adapt or create infection models within Matlab and perform simulations
- Perform fitting to data using both frequentist and Bayesian approaches
- Implement and explain deterministic and stochastic modelling approaches and their situational appropriateness
- Demonstrate how modelling predictions can be performed and contrast future interventions including adaptive strategies
- Utilise basic health economic concepts (disability-adjusted life years, willingness to pay, etc.) and methodology
  communicate modelling outcomes in a clear and informative manner
- Appraise the suitability of different models and their predictions for real-world decision making
  evaluate the role of assumptions in influencing model outcomes

Outline of the module:

This 10-week programme will be partitioned into five, 2-week topics:

- Simple infectious disease model dynamics simulation and prediction
- Deterministic vs stochastic modelling approaches (endemic vs outbreak or elimination)
- Modelling fitting to data (frequentist and Bayesian methods)
- Health economics for dynamic models and decision making
- Adaptive management for improved intervention efficacy

There will be 2 formal lectures per week plus 1 structured, lecturer-lead lab session plus 1 support lab.

Coursework: Assessment will take the format of five worksheets to be submitted in weeks 3, 5, 7, 9 and 11. Weighting is 20% for each sheet. Marks will be returned in weeks 4, 6, 8, 10 and during the Easter break so feedback is received before submitting the next worksheet. Submitted documents will be a mixture of LaTeXed solutions and Matlab code.

Suggested reading:

General: M.J. Keeling and P. Rohani, *Modelling Infectious Diseases in Humans and Animals*, Princeton University Press, 2007 (ISBN 0691116172)

Briggs, Claxton and Sculpher, *Decision Modelling for Health Economic Evaluation* (2006) (available in print through Warwick library)

Topic-specific research articles will be suggested as reading during the course.

Additional Resources

Year 1 regs and modules
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Year 2 regs and modules

MA4M2 Mathematics of Inverse Problems

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m2/)
Lecturer: Florian Theil

Term(s): Term 2

Status for Mathematics students:

Commitment: 30 lectures

Assessment: 100% written examination, oral exam for postgraduate students can be arranged

Formal registration prerequisites: None

Assumed knowledge:

MA3G7 Functional Analysis I:

- Linear operators
- Compact operators
- Dual spaces

Useful background:

MA3G8 Functional Analysis II:

- Compactness and weak convergence

Synergies:

- MA4A2 Advanced PDEs
- MA4J1 Continuum Mechanics

Content: Inverse problems play an increasingly important role for modern data oriented applications. Classical examples are medical imaging and tomography where one attempts to reconstruct the internal structure from transmission data.
Using the theory of partial differential equations it is possible to map the unknown internal structure to the observed data. The task of inverting this map is called 'Inverse Problem'.
We will study the mathematical theory that underpins the construction of the forward operator and devise regularisation techniques that will result in well posed inverse problems.

- Review of Functional Analysis and some basic ideas from PDE theory
- Modelling of simple physical systems, Radon transform
- Loss functions and the direct method
- Regularisation: Tikhonov and Total Variation
- Convergence of solutions for vanishing noise

Aims: Students will be able to identify inverse problems in applications like acoustics. They will be become aware of the connections between the theory of partial differential equations and parameter estimation problems, as well as being able to devise regularisations for simple inverse problems so that the regularised problem admits solutions.

By the end of the module, students should be able to:

- Understand the difference between forward problems and inverse problem
- Derive the Tikhonov functional for specific applications
- Apply the direct method to establish the existence of solutions of regularised inverse problems

- Become competent in using Hilbert space methods
- Apply knowledge to model simple physical systems

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MA4M3 Local Fields

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m3/)
Lecturer: Not running in 2022-23

Term(s): Term 2

Status for Mathematics students:

Commitment: 30 lectures

Assessment: 85% by 3-hour examination and 15% coursework

Formal registration prerequisites: None

Assumed knowledge:

- MA260 Norms, Metrics and Topologies (or MA222 Metric Spaces): I will freely use material from this entire course, with a vital role played by norms, their associated metrics, and topological spaces, including notions such as compactness and completeness.
- MA3A6 Algebraic Number Theory: material from the first half of the course will be essential, particularly including: algebraic number fields, rings of algebraic integers, norms and traces, prime ideals and their factorisation in extensions of number fields.
- MA3D5 Galois Theory: (finite) field extensions, Galois extensions and Galois groups. The theory of finite fields (and their Galois extensions) will be particularly crucial.

Useful background: MA3G6 Commutative Algebra: this course gives a thorough (and general) background to concepts such as valuations, local rings and localisation. These will all be defined/recalled during the Local Fields course, but some familiarity with these concepts will be useful.

Synergies: Local Fields is a topic in algebraic number theory. Whilst there will not be direct overlap in the course material, on a deeper level MA426 Elliptic Curves and MA4H9 Modular Forms are strongly related courses, and all three are likely to be useful for students looking to pursue future study (e.g. a PhD) in algebraic number theory (or, for example, an MA4K9 Research Project in this area). This module will also go well with MA4J8 Commutative Algebra II, which will treat topics useful for studying Local Fields in a general abstract context.

Content: The real numbers R are defined as the completion of the rational numbers Q in the usual metric. However, this metric is not that well-suited to arithmetic study; for example, the integers are discrete in R.

In number theory, one is often more interested in p-adic numbers Qp, the completion of Q in the p-adic metric. In the p-adic metric, a number is very close to zero if it is highly divisible by a prime p (for example, whilst 1,000,000,000 is 'large' in the usual metric, it is highly divisible by 2 and 5, so it is very small in the 2-adic and 5-adic metrics). The integers are not discrete in the p-adic metric (as e.g. one can arbitrarily approximate 0 by integers p-adically), so p-adic numbers are much better suited to arithmetic, and have accordingly become fundamental in number theory and arithmetic geometry.

The real and p-adic numbers are examples of local fields. This module will give an introduction to local fields, with an emphasis on the p-adic numbers/non-archimedean local fields, and describe some of their beautiful properties, including: the classification of local fields, Hensel's lemma and applications to solubility of polynomials, and extensions and Galois theory of local fields.

The course will also treat some notable applications in number theory and arithmetic geometry, in particular the Kronecker—Weber theorem on abelian extensions of Q and the Hasse—Minkowski theorem on solubility of quadratic forms.

Aims: To give students a grounding in the theory of local fields (e.g. the p-adic or real numbers) and their relationship with global fields (e.g. the rationals), and to gain insight into the use of local methods to solve global problems.

Objectives: By the end of the module, students should be able to:

- Explain the definition, basic properties and classification of valuations and local fields
- Understand inverse limits and the topology of the p-adic integers
- Use Hensel's lemma to determine solubility of polynomial equations over local fields
- Use the Hasse—Minkowski theorem to determine solubility of rational quadratic forms
- Describe the Galois theory of local fields, including solubility of the Galois group and classification of abelian extensions

Books: Primary resources will include the books *Local Fields* by Serre and *Local Fields* by Cassels.

Additional Resources

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MA4M4 Topics in Complexity Science

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m4/)
Lecturer: Marya Bazzi

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 20 lectures

Assessment: Essay Draft 5%, Essay 80% and Presentation 15%

Formal registration prerequisites: None

Assumed knowledge: The module MA398 Matrix Analysis and Algorithms provides some methodological foundations in linear algebra and matrix algorithms as well as hands-on experience in programming that would be relevant for the module. The particular concepts from MA398 that will be important are vector and matrix norms, singular value decomposition, eigenvalue decomposition, and algorithmic computational cost.

Useful background: Some notions from ST202 Stochastic Processes such as Markov processes and Markov chains, basic concepts in graph theory as covered in MA241 Combinatorics, and algorithms in MA252 Combinatorial Optimisation for tackling NP-hard problems would be helpful but are not required.

Synergies: The module links with MA4J5 Structures of Complex Systems and its application focus links well with modules such as MA4E7 Population Dynamics: Ecology and Epidemiology and MA4M1 Epidemiology by Example, as well as many other application areas (e.g., Sociology, Economics, Neuroscience).

Content: This course aims to provide an introduction to network science, which can be used to study complex systems of interacting entities. Networks are interesting both mathematically and computationally, and they are pervasive in sociology, biology, economics, physics, information science, and many more fields. Networks have grown in importance over the last few decades and most of the topics to be considered are active modern research areas. Possible topics in complexity include:

- Network science
- Selfish routing
- Interacting particle systems
- Reduction of dynamical systems

- Dynamics of networks of oscillators
- Large deviation theory
- Representation and inference of many-variable probabilities
- Analogues for many-body quantum systems
- Aggregation methods
- Data assimilation
- Biophysical modelling
- Fluid dynamic models

Aims: By the end of the module, students should be able to:

- Have a sound knowledge of and appreciation for some of the tools, concepts, models, and computations used in the study of networks
- Read and understand current research papers in the field
- Gain some experience with communicating scientific research
- Gain some experience working with real-world data

The module overlaps with several disciplines other than Mathematics, such as Computer Science and Statistics. The applications (which students may pursue in more depth in their essays) may also intersect with further disciplines, such as Sociology, Economics, and Biology.

Books:

1. M. E. J. Newman, *Networks: An Introduction*, Oxford University Press, 2010
2. A. Barrat et al, *Dynamical Processes on Complex Networks*, Cambridge University Press, 2008
3. Various papers and review articles to be specified by the instructor.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4M5 Geometric Measure Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m5/)

Not running in 2023-24

Lecturer: Filip Rindler

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Three hour examination

Formal registration prerequisites: None

Assumed knowledge:

- MA3G7 Functional Analysis I: Banach spaces, Lebesgue spaces, dual spaces, linear operators
- MA359 Measure Theory: General measures, Lebesgue measure & integral, properties of Lebesgue integral, convergence theorems

Useful background:

Synergies:

- MA4A2 Advanced PDEs: Applications & motivation
- Graduate courses in Analysis

Content:

Geometric measure theory is the study of geometric objects with the tools of measure theory. It occupies a central place in modern Geometric Analysis, where it has led to the resolution of many conjectures such as Plateau's problem and intriguing questions about soap bubbles. It is also extremely useful as a toolkit of methods that have enabled many new discoveries in other fields of Mathematics, spanning from Mathematical Material Science, over the Theory of PDEs and the Calculus of Variations, all the way to Number Theory. This course will give an introduction to this important area.

Outline:

- Motivation: Plateau's Problem
- Hausdorff Measures
- Area & Coarea Formula
- Rectifiability
- Forms and Stokes' Theorem
- Currents
- Integral Currents
- Deformation Theorem
- Resolution of Plateau's Problem

Books:

- Ambrosio, N. Fusco and D. Pallara, *Functions of Bounded Variation and Free- Discontinuity Problems*, Oxford Mathematical Monographs, Oxford University Press, 2000.
- H. Federer, *Geometric Measure Theory*, Grundlehren der mathematischen Wis- senschaften, vol. 153, Springer, 1969.
- S. G. Krantz and H. R. Parks, *Geometric Integration Theory*, Birkhauser, 2008.
- P. Mattila, *Geometry of Sets and Measures in Euclidean Spaces*, Cambridge Studies in Advanced Mathematics, vol. 44, Cambridge University Press, 1995.
- F. Morgan, *Geometric Measure Theory*, 5th ed., Elsevier, 2016.
- L. Simon, *Lectures on Geometric Measure Theory*, Proceedings of the Centre for Mathematical Analysis, vol. 3, Australian National University, Canberra, 1983.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4M6 Category Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m6/)
Lecturer: Emanuele Dotto

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: 85% examination by 3 hour written exam, 15% coursework

Formal registration prerequisites: None

Assumed knowledge:

- MA249 Algebra II
- MA3F1 Introduction to Topology

Useful background: The module will explore examples coming from various areas of mathematics. Some level of comfort with basic definitions from the following modules will be helpful to better appreciate the module content:

- MA3G6 Commutative Algebra
- MA377 Rings and Modules
- MA3E1 Groups and Represenations
- MA3H6 Algebraic Topology

Synergies: There are set theoretic subtleties at the foundations of category theory, as well as beautiful connections with logic, that we will not dive into. MA3H3 Set Theory connects well with this module. Category theory was introduced by Eilenberg and MacLane to formalise the notion of "naturality", and had its first applications in topology and homological algebra. This module is well complemented by MA3H6 Algebraic topology and MA4J7 Cohomology and Poincaré Duality.

Leads to: This module provides essential background for postgraduate studies in modern algebraic topology and algebraic geometry.

Content: Mathematical structures come equipped with a notion of "map", or "morphism", which are used to compare objects with such structure. For example, we may not want to distinguish between the sets {1,2,3} and {a,b,c} by noticing that we can map the elements of the first set to the second bijectively.
This principle extends to virtually all mathematical objects. To name a few: one studies groups via group homomorphisms, vector spaces via linear maps, spaces via continuous maps, manifolds via smooth maps, probability spaces via measurable functions, paths via homotopies.

A category consists of a collection of objects, a collection of morphisms, and a composition rule. Thus category theory provides a framework to study systematically those properties and constructions which can be formulated purely in terms of maps. Perhaps surprisingly, many common mathematical constructions arise in this manner, for example products, coproducts, direct sums, kernels, quotients, coinvariants, compactifications. The theme of the module will be to investigate how vaguely analogous constructions in various areas of mathematics are in fact instances of the same construction carried out at the level of a category.

The module will cover roughly the first 4 chapters of" Category theory in context", by Emily Riehl. This is a provisional list of contents:

- Categories, functors and natural transformations
- Representable functors and the Yoneda Lemma
- Limits and colimits
- Adjunctions and Freyd's Theorem
- The fundamental groupoid

In the last section we will apply the category theoretical framework to formulate the Seifert Van-Kampen theorem for the fundamental groupoid, and use it to calculate the fundamental group of the circle.

Learning Outcomes: By the end of the module, students should be able to:

- Explain the definitions and properties of the basic notions of category theory
- Understand adjunctions and the importance of Freyd's adjoint functor theorem
- Use the framework of category theory to prove results by universal property
- Recognise common constructions as instances of categorical constructions
- Describe the fundamental groupoid of a gluing of spaces from that of its pieces

Books:

Emily Riehl, *Category Theory in Context* , 2016

Brown, *Topology and Groupoids*, 2006

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

MA4M7 Complex Dynamics

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m7/)
Lecturer: John Smillie

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Three hour examination

Formal registration prerequisites: None

Assumed knowledge:

- MA222 Metric Spaces
- MA260 Norms, Metrics and Topologies
- MA3B8 Complex Analysis

Useful background:

- MA3D4 Fractal Geometry
- MA3F1 Introduction to Topology

Synergies:

- MA424 Dynamical Systems
- MA448 Hyperbolic Geometry
- MA427 Ergodic Theory

Aims: Complex Dynamics is a very active area of the field of Dynamical Systems. This course will be an introduction to the subject focusing on the dynamics of complex quadratic polynomials. This family of examples will be studied using a variety of tools coming from classical and modern techniques in complex analysis, topology, geometry and dynamical systems.

The course will have three main themes. Firstly, to understand the local behaviour of holomorphic transformations in one complex variable. Second, to understand the global behaviour of holomorphic maps focussing on complex quadratic polynomials. These exhibit dynamically important features such as chaotic behaviour. Third, we explore the parameter space of quadratic polynomials and the Mandelbrot set. Here we see examples of structural stability, structural instability and renormalisation behaviour.

Content:

We will cover some of the following topics:

- Local dynamics of holomorphic maps
- Expanding maps, shadowing, closing lemmas
- The theory of external rays
- Global dynamical behaviour of hyperbolic Julia sets, Markov partitions and symbolic dynamics
- Global behaviour of arbitrary Julia sets
- Structural stability, shadowing, closing lemmas
- Global properties of parameter space, the Mandelbrot set and renormalisation

Learning outcomes:

- Use a variety of techniques to analyse complex dynamical systems
- Understand the role of structural stability in dynamical systems
- Understand the role of renormalisation in dynamical systems
- Understand how Markov partitions can be used to understand the behaviour of orbits in dynamical systems

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4M8 Theory of Random Graphs

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m8/)
Lecturer: Richard Montgomery

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one hour lectures

Assessment: Three hour examination (100%)

Formal registration prerequisites: None

Assumed knowledge:

- MA241 Combinatorics

Useful background:

- MA359 Measure Theory
- ST342 Mathematics of Random Events
- ST202 Stochastic Processes

Synergies:

- MA3J2 Combinatorics II
- MA3H2 Markov Processes and Percolation Theory
- MA4J3 Graph Theory

Content:

The study of Random Graphs combines combinatorial and probabilistic techniques, and has applications in areas from Statistical Physics and Computer Science back to Extremal Combinatorics. In this course, we will study the typical properties of a random graph, with topics including the following:

- Different random graph models
- Thresholds for graph properties
- Applications to Extremal Combinatorics problems via the Probabilistic Method
- The evolution of random graphs and the emergence of the giant component

- The chromatic number of a random graph
- Connectedness and Hamiltonicity in random graphs
- The appearance of small subgraphs

Aims:

To give students a good grounding in methods from the theory of random graphs while proving fundamental results in the area.

Objectives:

By the end of the module the student should be able to:

- State and prove key results presented in the module
- Adapt the methods to new settings, including using coupling arguments
- Apply the techniques to determine other likely properties of random graphs

Books:

- Bollobás, B., *Random Graphs*
- Janson, S, Luczak, T and Ruciński, A., *Random Graphs*

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4M9 Mathematics of Neuronal Networks

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4m9/)
Lecturer: Magnus Richardson

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 one hour lectures and 10 tutorials

Assessment: Written exam (100%)

Formal registration prerequisites: None

Assumed knowledge:

- Solutions to standard ordinary and partial differential equations as in MA133 Differential Equations and MA250 Introduction to PDEs
- 2x2-Matrix eigenvalues and eigenvectors as in MA106 Linear Algebra

Useful background:

- More experience with differential equations such as MA3G1 Theory of PDEs
- Some knowledge of probability and statistics such as MA359 Measure Theory or ST342 Mathematics of Random Events or ST202 Stochastic Processes, although basics of stochastic differential equations will be taught in the module itself.

Synergies:

- MA482 Stochastic Analysis

- MA4F7 Brownian Motion
- MA4L3 Large Deviation Theory

Content:

Experimentally verified mathematical models of synapses and neurons will first be developed with a particular emphasis on their noisy dynamics. These components, described in terms of stochastic differential equations, will be coupled to provide a probabilistic Fokker-Planck-based description of emergent phenomena at the population and network levels. Further abstractions of neurons and their synaptic weights will also be introduced to understand how patterns might be stored in attractor networks or learned in feedforward networks. These mathematical insights into natural and artificial neuronal networks will allow for a critical evaluation of whether currently proposed architectures reflect cognitive activity in the brain as we currently understand it.

Aims:

The module will cover:

- Mathematical models of the computational units of natural neuronal networks - stochastic synapses and neurons
- How detailed synaptic and neuronal models can be simplified and combined to describe activity at the level of coupled networks
- How these detailed models can be further simplified to develop abstract models of networks that can store patterns or learn from example

Objectives:

By the end of the module, students should be able to:

- Build testable mathematical models of physiological objects such as synapses and neurons
- Understand basic stochastic differential equations and develop probabilistic descriptions of their dynamics
- Understand how emergent phenomena in networks arise from the characteristics of their coupled lower-level components
- Construct abstract, artificial neuronal networks that can learn to recall or classify patterned inputs

Books:

The course is self-contained; however, the following online books provide background reading:

Theoretical Neuroscience: Computational and Mathematical Modelling of Neural Systems, Dayan and Abbott

Neuronal Dynamics: From Single Neurons to Networks and Models of Cognition, Gerstner, Kistler, Naud and Paninski

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA408 Algebraic Topology

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma408/)
Please note that this module is now being taught as MA3H6

Status for Mathematics students: List C . Suitable for Year 3 MMath

Commitment: 30 one-hour lectures. Suitable for Third Year MMath.

Assessment: Three-hour examination (85%), assessed work (15%)

Prerequisites: MA3F1 Introduction to Topology (keen students can take this module at the same time), MA455 Manifolds (when available) can be taken at the same time as Algebraic Topology

Leads To: MA447 Homotopy Theory and advanced modules in Geometry and Topology

Content: Algebraic topology is concerned with the construction of algebraic invariants (usually groups) associated to topological spaces which serve to distinguish between them. Most of these invariants are "homotopy'' invariants. In essence, this means that they do not change under continuous deformation of the space and homotopy is a precise way of formulating the idea of continuous deformation. This module will concentrate on constructing the most basic family of such invariants, homology groups, and the applications of these homology groups.

The starting point will be simplicial complexes and simplicial homology. An n-simplex is the n-dimensional generalisation of a triangle in the plane. A simplicial complex is a topological space which can be decomposed as a union of simplices. The simplicial homology depends on the way these simplices fit together to form the given space. Roughly speaking, it measures the number of p-dimensional "holes'' in the simplicial complex.

Singular homology is the generalisation of simplicial homology to arbitrary topological spaces. The key idea is to replace a simplex in a simplicial complex by a continuous map from a standard simplex into the topological space. It is not that hard to prove that singular homology is a homotopy invariant but it is quite hard to compute singular homology from the definition. One of the main results in the module will be the proof that simplicial homology and singular homology agree for simplicial complexes. This result means that we can combine the theoretical power of singular homology and the computational power of simplicial homology to get many applications. These applications will include the Brouwer fixed point theorem, the Lefschetz fixed point theorem and applications to the study of vector fields on spheres.

Aims: To introduce homology groups for simplicial complexes; to extend these to the singular homology groups of topological spaces; to prove the topological and homotopy invariance of homology; to give applications to some classical topological problems.

Objectives: To give the definitions of simplicial complexes and their homology groups and a geometric understanding of what these groups measure; to give techniques for computing these groups; to give the extension to singular homology; to understand the theoretical power of singular homology; to develop a geometric understanding of how to use these groups in practice.

Books:

There is no book which covers the module as it will be taught. However, there are several books on algebraic topology which cover some of the ideas in the module, for example:

JW Vick, *Homology Theory*, Academic Press.

MA Armstrong, *Basic Topology*, McGraw-Hill.

Additional references:

CRF Maunder, *Algebraic Topolgy*, CUP.

A Dold, *Lectures on Algebraic Topology*, Springer-Verlag.

C Kosniowski, *A first course in algebraic topology*, CUP.

MJ Greenberg and JR Harper, *Algebraic Topology: A first course*, Addison-Wesley.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA424 Dynamical Systems

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma424/)

Lecturer: Richard Sharp

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures and weekly assignments

Assessment: 3 hour exam 100%

Formal registration prerequisites: None

Assumed knowledge:

MA260 Norms, Metrics and Topologies or MA222 Metric Spaces:

- Metric and topological spaces
- Continuous functions
- Homeomorphisms
- Compactness
- The Cantor set

MA259 Multivariable Calculus:

- Differentiable functions
- Diffeomorphisms

Useful background:

MA3H5 Manifolds:

- Definition of manifold
- Tangent bundle

Synergies:

MA427 Ergodic Theory

MA4M7 Complex Dynamics

Content: Dynamical Systems is one of the most active areas of modern mathematics. This course will be a broad introduction to the subject and will attempt to give some of the flavour of this important area.

The course will have two main themes. Firstly, to understand the behaviour of particular classes of transformations. We begin with the study of one dimensional maps: circle homeomorphisms and expanding maps on an interval. These exhibit some of the features of more general maps studied later in the course (e.g., expanding maps, horseshoe maps, toral automorphisms, etc.). A second theme is to understand general features shared by different systems. This leads naturally to their classification, up to conjugacy. An important invariant is entropy, which also serves to quantify the complexity of the system.

Aims: We will cover some of the following topics:

- circle homeomorphisms and minimal homeomorphisms,
- expanding maps and Julia sets,
- horseshoe maps, toral automorphisms and other examples of hyperbolic maps,
- structural stability, shadowing, closing lemmas, Markov partitions and symbolic dynamics,
- conjugacy and topological entropy,
- strange attractors.

Books:

M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press

B. Hasselblatt and A. Katok, *A First Course in Dynamics: With a Panorama of Recent Developments*, Cambridge University Press

S. Sternberg, *Dynamical Systems*, Dover

P. Walters, *An Introduction to Ergodic Theory*, Springer-Verlag

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

MA426 Elliptic Curves

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma426/)

Lecturer: Han Yu

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 85% by 3-hour examination, 15% coursework

Formal registration prerequisites: None

Assumed knowledge:

MA249 Algebra II: Groups and Rings:

- Basic theory of groups, rings, fields

MA257 Introduction to Number Theory:

- Primes
- Divisibility
- Congruences

Useful background:

MA4L7 Algebraic Curves:

- Affine and projective plane curves
- Bezout's theorem
- Singular points
- Change of coordinates

MA3B8 Complex Analysis:

- Holomorphic and meromorphic functions
- Laurent series
- Poles
- Residues

MA4A5 Algebraic Geometry:

- Algebraic varieties
- Rational maps
- Morphisms

Synergies: The following modules go well together with Elliptic Curves:

- MA3A6 Algebraic Number Theory
- MA4H9 Modular Forms
- MA3D5 Galois Theory

- MA4M3 Local Fields

Content: We hope to cover the following topics in varying levels of detail:

- Non-singular cubics and the group law; Weierstrass equations
- Elliptic curves over the rationals; descent, bounding $E()/2E()$, heights and the Mordell-Weil theorem, torsion groups; the Nagell-Lutz theorem
- Elliptic curves over complex numbers, elliptic functions
- Elliptic curves over finite fields; Hasse estimate, application to public key cryptography
- Application to diophantin equations: elliptic diophantine problems, Fermat's Last Theorem
- Application to integer factorisation: Pollard's $p-1$ method and the elliptic curve method

Leads to: Ph.D. studies in number theory or algebraic geometry.

Books:

Our main text will be Washington; the others may also be helpful:

- Lawrence C. Washington, Elliptic Curves: Number Theory and Cryptography, Discrete Mathematics and its applications, Chapman & Hall / CRC (either 1st edition (2003) or 2nd edition (2008)
- Joseph H. Silverman and John Tate, Rational Points on Elliptic Curves, Undergraduate Texts in Mathematics, Springer-Verlag, 1992.
- Anthony W. Knapp, Elliptic Curves, Mathematical Notes 40, Princeton 1992.
- J. W. S. Cassels, Lectures on Elliptic Curves, LMS Student Texts 24, Cambridge University Press, 1991.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA427 Ergodic Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma427/)
Lecturer: Cagri Sert

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: 100% 3-hour examination

Formal registration prerequisites: None

Assumed knowledge:

MA106 Linear Algebra (Maths):

- Eigenvalues and eigenvectors

MA222 Metric Spaces / MA260 Norms, Metrics and Topologies:

- Metric spaces
- Continuity
- Compactness

MA359 Measure Theory:

- Abstract measures
- Lebesgue measure
- Convergence Theorems
- L^1 and L^2 spaces

Useful background:

ST111 Probability A and ST112 Probability B:

- Probability spaces
- Notion of random variable
- Law of large numbers

MA3G7 Functional Analysis I:

- Hilbert Spaces
- Orthonormal basis
- Dual spaces

MA3G8 Functional Analysis II:

- Banach spaces
- Hanh-Banach theorem
- Convex sets

MA433 Fourier Analysis:

- Fourier series and their properties

Synergies:

- MA424 Dynamical Systems
- MA3D4 Fractal Geometry

Leads To:

Content: Consider the following maps:

1. A fixed rotation of a circle through an angle which is an irrational multiple of 2π.
2. The map of a circle which doubles angles.

If we choose two points of the circle which are close to each other and repeatedly apply the first map the behaviour of each point closely resembles the behaviour of the other point. On the other hand if we apply the second map repeatedly this is no longer the case - the behaviour of each point can be wildly different. The first example can be described as `deterministic' or `rigid' and the second as `random' or `chaotic'. We shall examine many examples of such maps displaying various degrees of randomness, and one of our aims will be to classify different types of behaviour using measure theoretic techniques. A key result (which we will prove) is the ergodic theorem. This is a basic tool in our analysis. We shall also consider applications to number theory and to Markov chains. For most of the module rigorous proofs will be provided. Occasionally we shall give proofs which depend on references which you will be encouraged to read. The written examination will depend only on module lectures.

Aims: To study the long term behaviour of dynamical systems (or iterations of maps) using methods developed in Measure Theory, Linear Analysis and Probability Theory.

Objectives: At the end of the module the student is expected to be familiar with the ergodic theorem and its application to the analysis of the dynamical behaviour of a variety of examples.

Books: **(Recommended reading):**

A. Katok & B. Hasselblatt, *Introduction to the Modern Theory of Dynamical Systems*, CUP, 1995

K. Petersen, *Ergodic Theory*, CUP, 1983

P. Walters, *An Introduction to Ergodic Theory*, Springer, 1982

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

MA433 Fourier Analysis

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma433/)
Lecturer: Professor Ian Melbourne

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 100% 3 hour exam

Formal registration prerequisites: None

Assumed knowledge: Familiarity with measure theory at the level of MA359 Measure Theory especially Fubini's Theorem, Dominated and Monotone Convergence Theorems.

Useful background: Further knowledge of Functional Analysis such as: MA3G7 Functional Analysis I and MA3G8 Functional Analysis II is helpful but not necessary. Topics such as norms of bounded linear operators will be reviewed in the module. Some basics about Hilbert spaces will also be reviewed in the module. The uniform boundedness principle will be stated without proof, but the other major results from functional analysis are not used.

Synergies: The following modules go well together with Fourier Analysis:

- MA4A2 Advanced Partial Differential Equations
- MA4J0 Advanced Real Analysis

Leads to: The following modules have this module listed as **assumed knowledge** or **useful background:**

- MA427 Ergodic Theory
- MA4M2 Mathematics of Inverse Problems
- MA4A7 Quantum Mechanics: Basic Principles and Probabilistic Methods
- MA4J0 Advanced Real Analysis

Content: Fourier analysis lies at the heart of many areas in mathematics. This course is about the *applications* of Fourier analytic methods to various problems in mathematics and sciences. The emphasis will be on developing the ability of using important tools and theorems to solve concrete problems, as well as getting a sense of doing formal calculations to predict/verify results. Topics will include:

- Fourier series of periodic functions, Gibbs phenomenon, Fejer and Dirichlet kernels, convergence properties, etc
- Basic properties of the Fourier transform on R^d, including L^p theory
- Topics on the Fourier inversion formula, including the Gauss-Weierstrass and Abel Poisson kernels, and connections to PDE
- A selection of more advanced topics, including the Hilbert transform and an introduction to Singular Integrals

Aims: The aim of the module is to convey an understanding of the basic techniques and results of Fourier analysis and of their use in different areas of maths.

References (optional): The following books may also contain useful materials:

- Stein, E. & Shakarchi, R. *Fourier Analysis, an Introduction.* Princeton University Press 2003.
- Duoandikoetxea, J. *Fourier Analysis* - American Mathematical Society 2001.
- Körner, T. *Fourier Analysis*, CUP 1988.
- Strichartz, R. *A Guide to Distribution Theory and Fourier Transforms*, CRC Press 1994.
- Folland, G. *Real Analysis: Modern Techniques and their Applications*, Wiley 1999.

- Grafakos, L. *Classical Fourier Analysis*, Springer 2008.
- Grafakos, L. *Modern Fourier Analysis*, Springer 2008.
- Stein, E.M. *Singular Integrals and Differentiability Properties of Functions and Differentiability Properties of Functions*, Princeton University Press.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA442 Group Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma442/)
Lecturer: Derek Holt

Term: Term 2

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: Three-hour written examination (100%)

Prerequisites: MA3K4 Introduction to Group Theory

Content: This module is a continuation of MA3K4 Introduction to Group Theory. The material in MA3K4 will be reviewed as we go through, but it will be assumed that students are already familiar with it. One significant difference in notation is that we shall be using right rather than left actions.

The main emphasis will be on finite groups, and particularly on the classification of simple groups of small order. However, results will be stated for infinite groups too whenever possible.

After reviewing the basic material on permutation groups and group actions, we shall study the Schreier-Sims Algorithm, which is an efficient programmable algorithm for computing the order of a subgroup of the symmetric group Sym(n) generated by a given set of permutations. This is based on and uses the Orbit-Stabilizer Theorem, together with a general theorem of Schreier on generating sets of subgroups of groups.

Soluble groups were studied in MA3K4, and we shall introduce and develop the basic properties of the more restricted class of nilpotent groups, which includes all finite p-groups.

Transitive and doubly transitive groups were introduced in MA3K4, and we shall study the intermediate classes of primitive and imprimitive permutation groups. Imprimitive groups arise naturally when the set X of permuted elements can be partitioned into sets of more than one element that are permuted by the group. For example, the Rubik's Cube group acting on the 24 corner faces of the cube permutes the eight corners, each of which consists of three faces. This leads to a new general construction, the wreath product of two groups, which is based on both direct and semidirect products.

The final part of the module will be on finite simple groups. We shall review the proof (from MA3K4) of the simplicity of the alternating groups Alt(n) for n at least 5, and prove the simplicity of the groups PSL(n,K) (which arise as quotient groups of groups of matrices over a field K) for all n greater than 1 (with a couple of small exceptions).

We finish with a complete classification of simple groups of order up to 500.

Aims: The main aim of this module is to classify all simple groups of order up to 500. The module will give some of the flavour of the greatest achievement in group theory during the 20th century, namely the classification of all finite simple groups.

Objectives: By the end of the module students should be familiar with the topics listed above under `Contents'. In particular, they should be able to use Sylow's Theorems and other techniques as a tool for analysing the structure of a finite group of a given order.

Books: No specific books are recommended for this module. There are many groups on Group Theory in the library, and some of these might be helpful for parts of the module, but no single book is likely to cover the whole syllabus.

MA448 Hyperbolic Geometry

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma448/)
Lecturer: Adam Epstein

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: 100% 3 hour examination

Formal registration prerequisites: None

Assumed knowledge:

- MA259 Multivariable Calculus
- General notions of metric, topology, and continuity as presented in MA260 Norms, Metrics and Topologies or MA222 Metric Spaces

Useful background:

- MA243 Geometry
- MA3B8 Complex Analysis

Synergies:

- MA3D9 Geometry of Curves and Surfaces
- MA4H4 Geometric Group Theory
- MA4C0 Differential Geometry

Leads to:

Content: An introduction to hyperbolic geometry, mainly in dimension two, with emphasis on concrete geometrical examples and how to calculate them. Topics include: basic models of hyperbolic space; linear fractional transformations and isometries; discrete groups of isometries (Fuchsian groups); tesselations; generators, relations and Poincaré's theorem on fundamental polygons; hyperbolic structures on surfaces.

Aims: To introduce the beautiful interplay between geometry, algebra and analysis which is involved in a detailed study of the Poincaré model of two-dimensional hyperbolic geometry.

Objectives: To understand:

- The non-Euclidean geometry of hyperbolic space
- Tesselations and groups of symmetries of hyperbolic space
- Hyperbolic geometry on surfaces

Books:

J.W. Anderson, *Hyperbolic Geometry*, Springer Undergraduate Math. Series.

S. Katok, *Fuchsian Groups*, Chicago University Press.

S. Stahl, *The Poincaré Half-Plane*, Jones and Bartlett.

A. Beardon, *Geometry of Discrete Groups*, Springer.

J. Lehner, *Discontinuous Groups and Automorphic Functions*. AMS.

L. Ford, *Automorphic Functions*, Chelsea (out of print but in library).

J. Stillwell, *Mathematics and its History*, Springer.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA453 Lie Algebras

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma453/)
Lecturer: Adam Thomas

Term(s): Term 1

Status for Mathematics students: List C. Suitable for Year 3 MMath

Commitment: 30 Lectures

Assessment: 100% 3 hour examination

Formal registration prerequisites: None

Assumed knowledge: Linear algebra and ring theory from Year 2: MA251 Algebra I: Advanced Linear Algebra and MA249 Algebra II: Groups and Rings.

Useful background: Any third year algebra module will be useful to have more familiarity with complicated abstract algebra results and proofs. Some examples include: MA3E1 Groups and Representations, MA3G6 Commutative Algebra, MA377 Rings and Modules and MA3D5 Galois Theory.

Synergies: Any algebraic module will go well with Lie Algebras and the module MA4E0 Lie Groups will have some overlap but from a different, more analytic/topological perspective.

Content: Lie algebras are related to Lie groups, and both concepts have important applications to geometry and physics. The Lie algebras considered in this course will be finite dimensional vector spaces over endowed with a multiplication which is almost never associative (that is, the products $(ab)c$ and $a(bc)$ are different in general). A typical example is the n^2-dimensional vector space of all $n \times n$ complex matrices, with *Lie product* $[A, B]$ defined as the commutator matrix $[A, B] = AB - BA$. The main aim of the course is to classify the building blocks of such algebras, namely the simple Lie algebras of finite dimension over .

Books:

J.E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer, 1979

T.O. Hawkes, *Lie Algebras*, Notes available from Maths Dept.

N. Jacobson, *Lie Algebras*, Dover, 1979

K.Erdmann and M. Wildon, *Introduction to Lie Algebras*, Springer 2006

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

MA467 Presentations of Groups

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma467/)
Lecturer: Derek Holt

Term: Not running 2020/21

Status for Mathematics students: List C . This module is suitable for Third Year MMath students

Commitment: 30 one-hour lectures

Assessment: Three-hour written examination (100%).

Prerequisites: MA251 Algebra I and MA249 Algebra II

Leads To: Postgraduate work in Group Theory

Content: This module is about groups that are defined by means of a presentation in terms of generators and relations. This means that a set of generators *X* is given for the group *G*, and a set of defining relations *R*. Defining relations are equations involving the generators and their inverses, which are required to hold in *G*. Then *G* is defined to be essentially the largest group that is generated by a set *X* for which the defining relations hold. For example, the dihedral group of order 6 could be defined as the group with generating set $X = \{x, y\}$and relations $R = \{x^3 = 1, y^2 = 1, yxy = x^{-1}\}$.

This method of defining a group has the advantage that it is often the most concise description of the group possible. Furthermore, groups arising from algebraic topology often appear naturally in this form. The disadvantage of the method is that it can be very difficult (and even theoretically impossible in some cases) to derive important properties of a group *G* that is given only by a presentation, such as whether it is finite, abelian, etc., However, as a result of the frequency with which group presentations crop up in other branches of mathematics, the development of techniques for finding out information about these groups has become a major branch of mathematical research.

In this module, we shall be developing the basic theory of group presentations, and looking at some particular techniques for analysing them. We start with free groups (groups with no defining relations) and prove a fundamental theorem of Schreier, that a subgroup of a free group is itself free. We then move on to presentations in general, and look at lots of examples. In the later part of the module, we shall be looking at some algorithmic methods for studying group presentations, including the Todd-Coxeter algorithm for calculating the index of a subgroup *H* of finite index in *G*, and the Reidemeister-Schreier method for calculating a presentation of *H*. (These algorithms are highly suitable for computer implementation, although we will not be studying that aspect of them in detail in this course.)

Aims: To illustrate the important general notion of definition of an algebraic structure by generators and defining relations in the context of group theory.

To develop some examples of the use of algorithmic methods in pure mathematics.

Objectives: To give a mathematically precise but comprehensible treatment of the definition of a group by generators and relations, and to teach students how to start extracting elementary information about the group from its presentation.

To teach students how to carry out the Todd-Coxeter coset enumeration algorithm by hand in simple examples, and how to compute presentations of subgroups of groups.

Books:

D.L. Johnson, *Presentations of Groups* (Second Edition), LMS Student Texts 15 C.U.P. 1997, Chapters 1,2,4,5,8,9.

Additional Resources

MA472 Reading Course

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma472/)

Lecturer:

Term(s): Terms 1-2

Status for Mathematics students: List C

Commitment:

Assessment: 3 hour exam

This scheme is designed to allow any student to offer for exam any reasonable piece of mathematics not covered by the lectured modules, for example a 4th year or M.Sc. module given at Warwick in a previous year. Any topic approved for one student will automatically be brought to the attention of the other students in the year. Note that a student offering this option will be expected to work largely on his or her own.

The aims of this option are (a) to extend the range of mathematical subjects available for examination beyond those covered by the conventional lecture modules, and (b) to encourage the habit of independent study. In the following outline regulations, the term ``book'' includes such items as published lecture notes, one or more articles from mathematical journals, etc.

1. A student wishing to offer a book for a reading module must first find a member of staff willing to act as moderator. The moderator will be responsible for obtaining approval of the module from the Director of Undergraduate Studies of the Mathematics Department, and for circulating a detailed syllabus to all MMath students before the end of the Term 1 registration period (week 3).

2. The moderator will be responsible for setting a three-hour exam paper, this exam is almost always in the exam session immediately after Easter vacation, regardless of the term(s) in which the particular reading module is carried out.

3. The mathematical level and content of a reading module must be at least that of a standard 15 CATS List C module. A reading module must not overlap significantly with any other module in the university available to MMath students.

4. Students may not take more than one reading module in any one year (MA372, MA472 or a reading module with its own code).

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA473 Reflection Groups

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma473/)

Lecturer: Robert Kropholler

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 100% by 3 hour examination

Formal registration prerequisites: None

Assumed knowledge: The groups theory and some geometric ideas from the second year Maths core:

MA251 Algebra I: Advanced Linear Algebra:

- Euclidean spaces
- Abelian groups

MA249 Algebra II: Groups and Rings:

- Groups, generators and relations.

Useful background: Interest in Group Theory:

MA3K4 Introduction to Group Theory:

- Semidirect products

Synergies: The following modules go well together with Reflection Groups:

- MA3K4 Introduction to Group Theory
- MA3E1 Groups and Representations
- MA453 Lie Algebras

Content: A reflection is a linear transformation that fixes a hyperplane and multiplies a complementary vector by -1. The dihedral group can be generated by a pair of reflections. The main goal of the module is to classify finite groups (of linear transformations) generated by reflections. The question appeared in 1920s in the works of Cartan and Weyl as the Weyl group is a finite crystallographic reflection group. In fact, if you have done MA453 Lie Algebras then you are already familiar with classification of semisimple Lie algebras, which is essentially the classification of crystallographic reflection groups.

Besides classifications, we will concentrate on examples and polynomial invariants.

Reference: R. Goodman, *The Mathematics of Mirrors and Kaleidoscopes*, American Mathematical Monthly.

www.math.rutgers.edu/~goodman/pub/monthly.pdf

Book:

J. E. Humphreys, *Reflection groups and Coxeter groups*, Cambridge University Press, 1992.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA475 Riemann Surfaces

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma475/)

Lecturer: Not running in 2022-23

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 one-hour lectures and fortnightly example sheets

Assessment: 100% three-hour written examination

Formal registration prerequisites: None

Assumed knowledge:

- MA3B8 Complex Analysis
- MA3F1 Introduction to Topology

Useful background: Some familiarity with differential forms would be helpful

Synergies: Riemann surfaces play a role in many areas of modern mathematics. This course might be useful for those with interests in the direction of topology, geometry or dynamical systems.

Leads to: The following modules have this module listed as **assumed knowledge** or **useful background**:

- MA4A5 Algebraic Geometry
- MA3H5 Manifolds

Content: Riemann Surfaces arose naturally in the study of complex analytic functions. They are abstract objects, patched together from open domains of the complex plane according to a rigid set of patching data. The beauty of complex analysis carries over to this abstract setting: the apparently very general definition turns out to constrain the objects in a rather strong way. This leads to interesting geometric, analytic and topological theorems about Riemann surfaces, showing also their ubiquity in much of modern mathematics.

We will first review some of the important features of complex analysis in the plane, before moving on to defining Riemann surfaces as abstract objects modelled on planar domains, and give several examples such as the Riemann sphere, complex tori, and so on. We will explore how Riemann surfaces can be classified and uniformised, along the way taking in such results as the Monodromy theorem, the Riemann mapping theorem and introducing concepts such as universal covers and the covering group of deck transformations. The rest of the module will explore further topics: the degree of a mapping, triangulations and the Riemann-Hurwitz formula, the construction of holomorphic differentials and meromorphic functions on Riemann surfaces, metrics of constant curvature and the pants decompositions of Riemann surfaces, quasiconformal maps and the deformation of complex structures.

Aims:

- To motivate the idea of a Riemann surface along the lines of Riemann's original reasoning
- To introduce the abstract concepts supported by examples
- To explain the modern way of understanding Riemann surfaces and discuss their geometry and topology

Objectives: Students at the end of the module should be able to:

- Define abstract Riemann surfaces with maps between them and give examples
- Use hyperbolic geometry and other geometries to construct Riemann surfaces
- Analyse topological and numerical properties of analytic mappings between Riemann surfaces
- Understand the classification of complex tori
- Have an overall understanding of all Riemann surfaces as quotients of their universal covers using the statement of the Uniformisation Theorem

Books:

L V Ahlfors, *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable*, McGraw-Hill.

A Beardon, *A Primer on Riemann Surfaces*, CUP.

O Forster, *Lectures on Riemann Surfaces*, Chapter I, Springer.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

MA482 Stochastic Analysis

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma482/)
Lecturer: Roger Tribe

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 100% 3 hour final examination

Assumed knowledge:

- Basic ideas of Probability Theory as in ST111 Probability (Part A) or ST112 Probability (Part B): Random variables, expectations, mean and variance, central limit theorem, law of large numbers.
- Some experience of stochastic processes. In past years about 80% of students had taken MA4F7/ST403 Brownian Motion (although this module will recap Brownian motion and only a few properties are needed). The modules ST202 Stochastic Processes or ST333 Applied Stochastic Processes would be valid alternatives.
- Measure Theory: This module will use the key weapons of rigorous measure theory (measurable functions, integrals, Fubini's Theorem, Dominated Convergence Theorem, Fatou's lemma) as seen in MA359 Measure Theory or ST342 Mathematics of Random Events. The module gives a chance to see these ideas in action, but it will not stress measure theoretic aspects.

Useful background: There will be links with material from several other modules: Solutions to elliptic and parabolic linear PDEs are very closely related, and students who have taken MA250 Introduction to Partial Differential Equations, MA3G1 Theory of Partial Differential Equations or MA4A2 Advanced Partial Differential Equations will be motivated for some of these connections. Although we won't lean on the material developed in ST318 Probability Theory, the examples developed here motivated a lot of the theory of martingales.

Synergies: Financial mathematicians make use of the tools developed here so that it will eventually mesh with the ideas in ST339 Introduction to Mathematical Finance (perhaps in Masters level courses).

Leads to:

Content: We will introduce stochastic integration, and basic tools in stochastic analysis including Ito's formula. We will also introduce lots of examples of stochastic differential equations.

Books:
Laurence Evans: *An Introduction to Stochastic Differential Equations.*
Bernt Oksendall: *Stochastic Differential Equations.*

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

MA4A2 Advanced Partial Differential Equations

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4a2/)
Lecturers: Felix Schulze

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 100% Examination

Formal registration prerequisites: None

Assumed knowledge:

MA359 Measure Theory:

- Lebesgue integration
- Fubini's Theorem
- Dominated Convergence Theorem
- Divergence Theorem
- Riesz Representation Theorem

MA259 Multivariable Calculus:

- Differentiable functions
- Partial derivatives
- Chain rule
- Implicit and Inverse Function Theorem

Useful background:

- MA3G1 Theory of Partial Differential Equations
- MA3G7 Functional Analysis I
- MA3G8 Functional Analysis II

Synergies: This module fits well with MA3G1 Theory of PDEs and leads naturally to MA4J0 Advanced Real Analysis and MA4M2 Mathematics of Inverse Problems. Essential for research in much of geometry, analysis, probability and applied mathematics etc.

Leads to: The following modules have this module listed as **assumed knowledge** or **useful background:**

- MA4M2 Mathematics of Inverse Problems
- MA482 Stochastic Analysis

Content: Partial differential equations have always been fundamental to applied mathematics, and arise throughout the sciences, particularly in physics. More recently they have become fundamental to pure mathematics and have been at the core of many of the biggest breakthroughs in geometry and topology in particular. This course covers some of the main material behind the most common 'elliptic' PDE. In particular, we'll understand how analysis techniques help find solutions to second order PDE of this type, and determine their behaviour. Along the way we will develop a detailed understanding of Sobolev spaces.

This course is most suitable for people who have liked the analysis courses in earlier years. It will be useful for many who intend to do a PhD, and essential for others. There are not too many prerequisites, although you will need some functional analysis, and some facts from Measure Theory will be recalled and used (particularly the theory of Lp spaces, maybe Fubini's theorem and the Dominated Convergence theorem etc.). It would make sense to combine with MA3G1 Theory of PDEs, in particular the parts about Laplace's equation, in order to see the relevant context for this course, although this is not essential.

Aims: To introduce the rigorous, abstract theory of partial differential equations.

MA4A5 Algebraic Geometry

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4a5/)
Lecturer: Rohini Ramadas

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures plus assignments

Assessment: Assignments (15%), 3 hour written exam (85%)

Formal registration prerequisites: None

Assumed knowledge: MA3G6 Commutative Algebra: The Module will make free use of the basic concepts of ring and module theory, ideals, prime and maximal ideals, localisation, integral closure. Moreover, Hilbert's Nullstellensatz and primary decomposition will be essential for the foundations.

Useful background: It may be helpful, though not absolutely essential, to be acquainted with basic notions of projective geometry and in particular the concept of projective space from MA243 Geometry. Furthermore, the notion of the exterior algebra of a vector space introduced for example in MA3H5 Manifolds is useful background, but will be fully recalled.

Synergies: The following modules go well together with Algebraic Geometry:

- MA4L7 Algebraic Curves
- MA426 Elliptic Curves
- MA4E0 Lie Groups
- MA4C0 Differential Geometry

Leads to: The following modules have this module listed as **assumed knowledge** or **useful background:**

- MA426 Elliptic Curves
- MA4L7 Algebraic Curves

Content:
Algebraic geometry studies solution sets of polynomial equations by geometric methods. This type of equations is ubiquitous in mathematics and much more versatile and flexible than one might as first expect (for example, every compact smooth manifold is diffeomorphic to the zero set of a certain number of real polynomials in R^N). On the other hand, polynomials show remarkable rigidity properties in other situations and can be defined over any ring, and this leads to important arithmetic ramifications of algebraic geometry.

Methodically, two contrasting cross-fertilizing aspects have pervaded the subject: one providing formidable abstract machinery and striving for maximum generality, the other experimental and computational, focusing on illuminating examples and forming the concrete geometric backbone of the first aspect, often uncovering fascinating phenomena overlooked from the bird's eye view of the abstract approach.

In the lectures, we will introduce the category of (quasi-projective) varieties, morphisms and rational maps between them, and then proceed to a study of some of the most basic geometric attributes of varieties: dimension, tangent spaces, regular and singular points, degree. Moreover, we will present many concrete examples, e.g., rational normal curves, Grassmannians, flag and Schubert varieties, surfaces in projective three-space and their lines, Veronese and Segre varieties etc.

Books:

- Atiyah M.& Macdonald I. G., *Introduction to Commutative Algebra*, Addison-Wesley, Reading MA (1969)
- Harris, J., *Algebraic Geometry, A First Course*, Graduate Texts in Mathematics 133, Springer-Verlag (1992)
- Mumford, D., *Algebraic Geometry I: Complex Projective Varieties*, Classics in Mathematics, reprint of the 1st ed. (1976); Springer-Verlag (1995)
- Reid, M., *Undergraduate Algebraic Geometry*, London Math. Soc. Student Texts 12, Cambridge University Press (2010)
- Shafarevich, I.R., *Basic Algebraic Geometry 1*, second edition, Springer-Verlag (1994)
- Zariski, O. & Samuel, P., *Commutative Algebra*, Vol. II, Van Nos- trand, New York (1960)
- Smith, K.E, Kahanpää, L, Kekäläinen, P & Traves, W., *An Invitation to Algebraic Geometry* ebook available in the Warwick library

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4A7 Quantum Mechanics: Basic Principles and Probabilistic Methods

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4a7/)
Lecturer: Vedran Sohinger

Term(s): Term 1

Status for Mathematics students:

Commitment: 30 lectures

Assessment: 3 hour examination (100%)

Formal registration prerequisites: None

Assumed knowledge: In this module, we will assume the content of MA3G7 Functional Analysis I, most notably the general framework of Hilbert spaces. We will use some notions from first-year Probability, which are taught in ST111 Probability A and ST112 Probability B (or equivalent modules). We will not assume any prior advanced knowledge of physics.

Useful background: It is also useful to have some familiarity with the Fourier transform. A module in which this is presented in a self-contained way is MA433 Fourier Analysis (in Term 1). Taking the module without having previously taken MA433 or an equivalent module is possible, although it would be a good idea to discuss this with the instructor beforehand. We will not directly use material from MA3G8 Functional Analysis II, although ideas from this module could be useful.

Synergies: The following modules could be helpful to take concurrently:

- MA4J0 Advanced Real Analysis
- MA4L2 Statistical Mechanics (We will not directly use any results from either module)

Content:

Quantum mechanics is one of the most successful and most fundamental scientific theories. It provides mathematical tools capable of describing properties of microscopic structures of our World. It is fundamental to the understanding of a variety of physical phenomena, ranging from atomic spectra and chemical reactions to superfluidity and Bose-Einstein condensation.

In the lectures we will discuss mathematical foundations of quantum theory: This includes the concepts of mixed and pure states, observables and evolution operator, a wave function in Hilbert space, the stationary and time-dependent Schrödinger equations, the uncertainty principle and the connections with classical mechanics (Ehrenfest theorem).

We will give simple, exactly soluble examples of both time-dependent and time-independent Schrodinger equations. We will also touch some more advanced topics of the theory.

Aims:
To introduce the basic concepts and mathematical tools used in quantum mechanics, preparing students for areas which are at the forefront of current research.

Objectives:
The students should obtain a good understanding of the basic principles of quantum mechanics, and to learn the methods used in the analysis of quantum mechanical systems.

Books:

Stephen J. Gustafson, Israel M. Sigal: *Mathematical Concepts of Quantum Mechanics*, Springer Universitext 3rd Edition 2020.

Gerald Teschl: *Mathematical Methods in Quantum Mechanics: with Applications to Schrödinger Operators*, American Mathematical Society, Graduate Studies in Mathematics 157, Second Edition 2014.

Michael Reed, Barry Simon: *Methods of Modern Mathematical Physics (Volume 1)*, Academic Press.

Elliott H. Lieb, Michael Loss: *Analysis*, American Mathematical Society, Graduate Studies in Mathematics 14, Second Edition 2001.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4C0 Differential Geometry

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4c0/)
Lecturer: Max Stolarski

Term(s): Term 1

Status for Mathematics students: List C

Commitment:

Assessment: 85% Examination, 15% Homework

Formal registration prerequisites: None

Assumed knowledge:

MA3H5 Manifolds: Key topics from MA3H5 will be covered rapidly in the first few lectures but you should do this module in Term 2 for a thorough discussion of them

MA251 Algebra I:

- Bilinear forms
- Eigenvalues and Eigenvectors

MA259 Multivariable Calculus:

- Differentiation of functions of several variables, including the Chain Rule
- Inverse and Implicit Function theorems

MA260 Norms, Metrics and Topologies OR MA222 Metric Spaces:

- Basic point set topology

MA254 Theory of ODEs:

- Existence and uniqueness of solutions to ODEs and their smooth dependence on parameters and initial conditions

Useful background:

- MA3D9 Geometry of Curves and Surfaces

Synergies:

- MA4E0 Lie Groups

Outline: The core of this course will be an introduction to Riemannian geometry - the study of Riemannian metrics on abstract manifolds. This is a classical subject, but is required knowledge for research in diverse areas of modern mathematics. We will try to present the material in order to prepare for the study of some of the other geometric structures one can put on manifolds.

Summary:

- Review of basic notions on smooth manifolds; tensor fields
- Riemannian metrics
- Affine connections; Levi-Civita connection; parallel transport
- Geodesics; exponential map; minimising properties of geodesics
- The curvature tensor; sectional, Ricci and scalar curvatures
- Training in making calculations: switching covariant derivatives; Bochner/Weitzenböck formula
- Jacobi fields; geometric interpretation of curvature; second variation of length
- Classical theorems in Riemannian Geometry: Bonnet-Myers, Hopf-Rinow and Cartan-Hadamard

Books:
Lee, J. M.: *Riemannian Manifolds: An Introduction to Curvature.* Graduate Texts in Mathematics, 176. Springer-Verlag, 1997
Gallot, S., Hulin, D., Lafontaine, J.: *Riemannian Geometry*. Springer. 2nd edition, 1993
Jost, J.: *Riemannian Geometry and Geometric Analysis* 5th edition. Springer-Verlag, 2008
Petersen, P.: *Riemannian Geometry Graduate Texts in Mathematics*, 171. Springer-Verlag, 1998
Kobayashi, S., Nomizu, K.: *Foundations of Differential Geometry*
do Carmo, M: *Riemannian Geometry*. Birkhäuser, Boston, MA, 1992

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4E0 Lie Groups

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4e0/)
Lecturer: Weiyi Zhang

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: 100% 3 hour exam

Formal registration prerequisites: None

Assumed knowledge:

- MA260 Norms, Metrics and Topologies or MA222 Metric Spaces
- MA3H5 Manifolds

Useful background: A knowledge of calculus of several variables including the Implicit Function and Inverse Function Theorems, as well as the existence theorem for ODEs. A basic knowledge of manifolds, tangent spaces and vector fields will help. Results needed from the theory of manifolds and vector fields will be stated but not proved in the course.

- MA254 Theory of ODEs
- MA3H6 Algebraic Topology

Synergies:

- MA4C0 Differential Geometry

Leads to:

Content: The concept of continuous symmetry suggested by Sophus Lie had an enormous influence on many branches of mathematics and physics in the twentieth century. Created first as a tool in a small number of areas (e.g. PDEs) it developed into a separate theory which influences many areas of modern mathematics such as geometry, algebra, analysis, mechanics and the theory of elementary particles, to name a few.

In this module we shall introduce the classical examples of Lie groups and basic properties of the associated Lie algebra and exponential map.

Books:

The lectures will not follow any particular book and there are many in the Library to choose from. See section QA387. Some examples:

C. Chevalley, *Theory of Lie Groups, Vol I*, Princeton.

J.J. Duistermaat, J.A.C. Kölk, *Lie Groups*, Springer, 2000.

F.W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, (Graduate Texts in Mathematics), Springer, 1983.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4E7 Population Dynamics: Ecology & Epidemiology

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4e7/)
Lecturer: Dr. Louise Dyson

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 one-hour lectures

Assessment: 100% 3-hour examination

Formal registration prerequisites: None

Assumed knowledge: No formal prerequisites. Some practical knowledge of how to analyse and solve differential equations will be assumed. In particular, finding fixed points and their stability in systems of ODEs, drawing phase plane diagrams, solving difference equations, a very small amount of PDEs etc.

Useful background: MA256 Introduction to Systems Biology and MA390 Topics in Mathematical Biology may provide some useful background material.

Synergies: This course complements MA4M1 Epidemiology by Example, and the two courses are designed to work separately, concurrently, or one after the other.

Leads To:

Content: This course deals with the mathematics behind the dynamics of populations; both populations of free-living organisms (from plants to predators) and those that cause disease. Once the basic models and concepts have been introduced attention will focus on understanding the many complexities that can arise, such as age-structure, spatial structure, temporal forcing and stochasticity. The focus of the course will be how mathematical models can help us both predict the future behaviour of populations and understand their dynamics.

Research into the dynamics of ecological populations allows us to understand the conservation of endangered species, make predictions about the effects of global climate change and understand the population fluctuations observed in the natural world. Work on infectious diseases clearly has important applications to public-health, allowing us to predict the spread of an epidemic (such as Foot-and-Mouth or SARS virus) and determine the effect of control measures.

Throughout, use will be made of examples in the recent literature, with a strong bias towards read-world problems. Special attention will be given to the applied use of the models developed and the necessity of good quality biological data and understanding.

Books:

Much of this course will be based on research papers and comprehensive references will be given throughout the course. Four useful books are:

R.M. Anderson and R.M. May, *Infectious Diseases of Humans*, Oxford University Press, 1992. (ISBN 019854040X)

S.P. Ellner and J. Guckenheimer, *Dynamic Models in Biology*, Princeton University Press, 2006 (ISBN 0691125899)

R.M. May and A. McLean, *Theoretical Ecology: Principles and Applications*, Oxford University Press, 2007 (ISBN 0199209995)

M.J. Keeling and P. Rohani, *Modelling Infectious Diseases in Humans and Animals*, Princeton University Press, 2007 (ISBN 0691116172)

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4F7 Brownian Motion

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4f7/)
This module is the same as ST403 Brownian Motion. Students may not register for both.

Lecturer: Tommaso Rosati (Stats)

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 Lectures

Assessment: 85% by 3 hour exam, 15% by assessments

Formal registration prerequisites: None

Assumed knowledge: At least one of:

- ST318 Probability Theory or
- MA359 Measure Theory or
- ST342 Mathematics of Random Events

Useful background:

Synergies: The following module goes well together with Brownian Motion:

- MA482 Stochastic Analysis

Content:

In 1827 the Botanist Robert Brown reported that pollen suspended in water exhibit random erratic movement. This ‘physical’ Brownian motion can be understood via the kinetic theory of heat as a result of collisions with molecules due to thermal motion. The phenomenon has later been related in Physics to the diffusion equation, which led Albert Einstein in 1905 to postulate certain properties for the motion of an idealized ‘Brownian particle’ with vanishing mass: - the path $t \mapsto B(t)$ of the particle should be continuous, - the displacements $B(t+\Delta t)-B(t)$ should be independent of the past motion, and have a Gaussian distribution with mean 0 and variance proportional to Δt.

In 1923 ‘mathematical’ Brownian motion was introduced by the Mathematician Norbert Wiener, who showed how to construct a random function $B(t)$ with those properties. This mathematical object (also called the Wiener process) is the subject of this module. Over the last century, Brownian motion has turned out to be a very versatile tool for theory and applications with interesting connections to various areas of mathematics, including harmonic analysis, solutions to PDEs and fractals. It is also the main building block for the theory of stochastic calculus (see MA482 Stochastic Analysis in term 2), and has played an important role in the development of financial mathematics. Even though it is almost 100 years old, Brownian motion lies at the heart of deep links between probability theory and analysis, leading to new discoveries still today.

Topics discussed in this module include:

- Construction of Brownian motion/Wiener process
- Fractal properties of the path, which is continuous but still a rough, non-smooth function
- Connection to the Dirichlet problem, harmonic functions and PDEs
- The martingale property of Brownian motion and some aspects of stochastic calculus
- Description in terms of generators and semigroups
- Description as a Gaussian process, an important class of models in machine learning
- Some generalizations, including sticky Brownian motion and local times

Books:
Peter Mörters and Yuval Peres, *Brownian Motion*, Cambridge University Press, 2010
Thomas M. Liggett, *Continuous Time Markov Processes - An Introduction*, AMS Graduate studies in Mathematics 113, 2010

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4G4 Introduction to Theoretical Neuroscience

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4g4/)

uk.ac.warwick.sbr.content.ContentDeniedException: Access denied for the source page.

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4G7 Computational Linear Algebra and Optimisation

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4g7/)

Not Running in 2019/20

Status for Mathematics students: List C for MMath. Also listed under Scientific Comupting as CY902

Commitment: 3 one hour lectures per week (one of which will be in the computing lab)

Assessment: 2 hour exam (70%), assignments (30%)

Prerequisites: A good knowledge of a scientific programming language such as C or Fortran is essential. No scripting languages such as matlab or python are permitted. Knowledge of both linear algebra and vector calculus is essential

Leads To:

Content: See module page on CSC site.

Books:

Lloyd Trefethen and David Bau, *Numerical Linear Algebra*, SIAM, 1997

J. Nocedal and S. Wright, *Numerical Optimization*, Springer Verlag 1999

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4H0 Applied Dynamical Systems

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4h0/)
Lecturer: Vassili Gelfreich

Term(s): Term 1

Status for Mathematics students: List C for Math

Commitment: 30 lectures

Assessment: 100% 3 hour examination

Formal registration prerequisites: None

Assumed knowledge:

MA254 Theory of ODEs

MA251 Algebra I: Advanced Linear Algebra:

- Jordan normal form

MA259 Multivariable Calculus:

- Differentiation in more than one dimension, implicit function theorem, divergence theorem

Useful background: Read one of the following two books:

- MW Hirsch, S Smale & RL Devaney, Differential equations, dynamical systems and an introduction to chaos.
- JD Meiss, Differential Dynamical Systems RC Robinson, An introduction to dynamical systems.

Synergies: This module provides a complementary view of dynamical systems theory to others offered by the department. It concentrates on continuous time and aspects relevant to physics and biology. If you want a well rounded training in dynamical systems theory you are recommended to take one of the others plus this one.

Content: This course will introduce and develop the notions underlying the geometric theory of dynamical systems and ordinary differential equations. Particular attention will be paid to ideas and techniques that are motivated by applications in a range of the physical, biological and chemical sciences. In particular, motivating examples will be taken from chemical reaction network theory, climate models, fluid motion, celestial mechanics and neuronal dynamics.

The module will be structured around the following topics:

- Review of basic theory: flows, notions of stability, linearization, phase portraits, etc
- `Solvable' systems: integrability and gradient structure, applications in celestial mechanics and chemical reaction networks
- Invariant manifold theorems: stable, unstable and center manifolds
- Bifurcation theory from a geometric perspective
- Compactification techniques: flow at infinity, blow-up, collision manifolds
- Chaotic dynamics: horsehoes, Melnikov method and discussion of strange attractors
- Singular perturbation theory: averaging and normally hyperbolic manifolds

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4H4 Geometric Group Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4h4/)
Lecturer: Karen Vogtmann

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 3-hour exam 85%, coursework 15%

Formal registration prerequisites: None

Assumed knowledge: Group theory, Euclidean and hyperbolic geometry, Fundamental group and covering spaces. These subjects are covered by Warwick courses MA136 Introduction to Abstract Algebra, MA243 Geometry and MA3F1 Introduction to Topology. Note that MA249 Algebra II: Groups and Rings and MA260 Norms, Metrics and Topologies or MA222 Metric Spaces will also be assumed as they are prerequisites for MA3F1 Introduction to Topology.

Useful background: Any course in algebra, geometry or topology, in particular,

- MA251 Algebra I: Advanced Linear Algebra
- MA3K4 Introduction to Group Theory or MA442 Group Theory (taken last year)

Synergies:

The following modules go well together with Geometric Group Theory:

- MA3H6 Algebraic Topology
- MA3H5 Manifolds
- MA3D9 Geometry of Curves and Surfaces
- MA3E1 Groups and Representations
- MA475 Riemann Surfaces
- MA448 Hyperbolic Geometry
- MA473: Reflection Groups

Content: This module is an introduction to the field of geometric group theory. The basic premise of this field is that topological and geometric methods can be applied to the study of finitely generated groups by studying actions of these groups on various spaces. For example restrictions on the topology and curvature of the space can have strong algebraic consequences for groups that act "properly." Although this basic idea can be traced back a century or more, the subject exploded in the 1980s with work of Thurston and Gromov, and has become a major area of current research. Prominent roles in geometric group theory are played by low dimensional topology and hyperbolic geometry, but it has points of contact with and borrows techniques from a wide range of mathematical subjects.

Learning outcomes: Familiarity with classes of groups commonly studied in geometric group theory, the spaces they act on and ability to use features of these spaces to extract information about the groups.

Books:

Löh, *Geometric Group Theory, An Introduction* : Universitext, Springer (2017)
P. de la Harpe, *Topics in Geometric Group Theory* : Chicago lectures in mathematics, University of Chicago Press (2000)
M. Bridson, A. Haefliger, *Metric Spaces of Non-Positive Curvature* : Grundlehren der Math. Wiss. No. 319, Springer (1999)
C. Druţu, M. Kapovich, *Geometric Group Theory* : Colloquium publications, Vol. 63, American Mathematics Society (2018)
A. Casson and S. Bleiler, *Automorphisms of Surfaces after Thurston*. Cambridge University Press (1988)

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4H7 Atmospheric Dynamics

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4h7/)
Lecturer:

Term(s): Not running 2020/21

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 3 hour exam (100%)

Prerequisites:

Maths/Physics students are required to have two of the following: MA231 Vector Analysis, PX253 PDEs, PX244 Introduction to Fluids

Mathematics students are required to have exposure to physical conservation laws such as momentum and energy and the differential equations that describe them. This means fluids or physics courses from the Warwick Physics department, MA3D1, or A-level Physics or Mechanics A.

Leads To:

Content: Topics would include:

Vertical motion and the role of moisture:

- Atmospheric stability: Dry and saturated adiabatic lapse rates
- Water vapour: Relative humidity, evaporation and condensation

Mechanics in a rotating frame (linear theory):

- Pressure gradients and their origins.
- Coriolis force, geostrophic wind.
- Stability and waves in a rotating frame.
- Stability and waves due to stratification.

Circulation on a global scale (nonlinear theory):

- Prevailing winds, jet streams, synoptic scale motion.
- Air masses, fronts, cyclones and accompanying weather patterns

Mesoscale and microscale motion:

- The planetary boundary layer.
- Ekman layers.
- Thunderstorm initiation.

Books:

J.C. McWilliams, *Fundamentals of Geophysical Fluid Dynamics*, CUP (2006).

B. Cushman-Roisoin, *Introduction to Geophysical Fluid Dynamics*, Prentice-Hall (1994).

Additional resources:

John M. Wallace and Peter V. Hobbs, *Atmospheric science: an introductory survey* (2nd ed), Academic Press, 2006.

Roland Stull, *Meteorology For Scientists And Engineers: A Technical Companion Book To C. Donald Ahrens' Meteorology Today*.

Additional Resources

Archived Pages: 2011 2016 2017 2018

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4H8 Ring Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4h8/)

Lecturer: Marco Schlichting

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 3 hour exam (100%): 4 out of 5 problems, first is compulsory – 40%, the remaining problems are – 20% each

Formal registration prerequisites: None

Assumed knowledge: MA3D5 Galois Theory, MA377 Rings and Modules

Useful background: helpful but not strictly necessary: p-adic numbers, quadratic forms

Synergies: The material of Ring Theory has many basic and more advanced links with Non-commutative Algebra and Algebraic Geometry, Algebraic Number Theory and Group Theory

Content:

- Review of some Galois theory and some of MA377 Rings and Modules
- Quaternion algebras and cyclic algebras
- Central simple Algebras and the Brauer group
- Computation of the Brauer group of local fields and of the rational numbers

Aims: The goal is to understand one of the major results of 20th century pure mathematics: The Albert-Brauer-Hasse-Noether Theorem. The theorem is at the interface of number theory and non-commutative algebra with ramifications to algebraic geometry, representation theory and K-theory; it incapsulates generalisations of various reciprocity laws (quadratic, cubic, etc) and has lead to the modern formulation of class field theory (though we won't have time to go into much of that in the module).

Up to a nilpotent ideal, (possibly non-commutative) Artinian rings, for instance, finite dimensional algebras over a field, are a finite product of matrix rings of division rings. Recall that division rings are those rings in which every non-zero element has a multiplicative inverse. We will learn ways of constructing many examples of division rings. Under a suitable operation, the "set" of finite dimensional division algebras with centre a field F forms a group, called the Brauer group Br(F) of F. We will study and compute the Brauer group of finite fields, the real and complex numbers, p-adic numbers, and most notably of the rational numbers thereby providing a complete classification of division algebras over these fields. Along the way, we will learn about local fields and quadratic forms which are ubiquitous in mathematics. Module homomorphisms f:M->N applied to an element x of M will be written in the traditional manner as f(x).

Books: There will be complete lecture notes for the preparation of which I have used the following books:

Benson Farb, R. Keith Dennis: *Noncommutative Algebra* (Graduate Texts in Mathematics), ISBN: 038794057X

Richard Pierce: *Associative Algebras*. Graduate Texts in Mathematics, 88. Springer-Verlag, New York-Berlin, ISBN: 0-387-90693-2

Philippe Gille, Tamas Szamuely: *Central Simple Algebras and Galois Cohomology*. Cambridge University Press, Cambridge, ISBN: 978-1-316-60988-0

Kersten, Ina: Brauergruppen von Körpern.(German) [Brauer groups of fields] Aspects Math., D6, ISBN:3-528-06380-7

Additional Resources

Archived Pages: 2016 2017 2018

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

MA4H9 Modular Forms

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4h9/)
Lecturer: Samir Siksek

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 lectures, plus a willingness to work hard at the homework

Assessment: 100% examination

Formal registration prerequisites: None

Assumed knowledge:

- MA257 Introduction to Number Theory
- MA3B8 Complex Analysis

Useful knowledge: Core first and second year modules

Synergies: This module complements the following:

- MA426 Elliptic Curves

Leads To: Ph.D. studies in number theory and algebraic geometry

Content: The course's core topics are the following:

- The modular group and the upper half-plane
- Modular forms of level 1 and the valence formula
- Eisenstein series, Ramanujan's Delta function
- Congruence subgroups and fundamental domains, modular forms of higher level
- Hecke operators
- The Petersson scalar product, old and new forms
- Statement of multiplicity one theorems
- The *L*-function of a modular form
- Modular symbols

Books:

F. Diamond and J. Shurman, *A First Course in Modular Forms*, Graduate Texts in Mathematics 228, Springer-Verlag, 2005. (Covers everything in the course and a great deal more, with an emphasis on introducing the concepts that occur in Wiles' work)

J.-P. Serre, *A Course in Arithmetic*, Graduate Texts in Mathematics 7, Springer-Verlag, 1973. (Chapter VII is a short but beautifully written account of the first part of the course which is good introductory reading)

W. Stein, *Modular Forms, A Computational Approach*, Graduate Studies in Mathematics, American Mathematical Society, 2007. (Emphasis on computations using the open source software package Sage)

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules

MA4J0 Advanced Real Analysis

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j0/)
Lecturer: Filip Rindler

Term(s): Term 2

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 3 hour exam (100%)

Formal registration prerequisites: None

Assumed knowledge:

- MA3G7 Functional Analysis I: Banach spaces, Lebesgue spaces, dual spaces, linear operators.
- MA359 Measure Theory: General measures, Lebesgue measure & integral, properties of Lebesgue integral, convergence theorems.

Useful background:

- MA3G8 Functional Analysis II: More familiarity with Banach spaces and their main theorems.
- MA433 Fourier Analysis: Fourier transform & its properties.

Synergies:

- MA4A2 Advanced PDEs: Applications & motivation and graduate courses in Analysis.

Content: The module builds upon modules from the second and third year like MA222 Metric Spaces, MA359 Measure Theory and MA3G7 Functional Analysis I to present the fundamental tools in Harmonic Analysis and some applications, primarily in Partial Differential Equations. Some of the main aims include:

- Setting up a rigorous calculus of rough objects, such as distributions.
- Studying the boundedness of singular integrals and their applications.
- Understanding the scaling properties of inequalities.
- Defining Sobolev spaces using the Fourier Transform and the connections between the decay of the Fourier Transform and the regularity of functions.

Outline:

- Distributions on Euclidean space.
- Tempered distributions and Fourier transforms.
- Singular integral operators and Calderon-Zygmund theory.
- Theory of Fourier multipliers.
- Littlewood-Paley theory.

Books:

- Friedlander, G. and Joshi, M. : *Introduction to the Theory of Distributions,* 2nd edition, Cambridge University Press, 1998.

- Duoandikoetxea, J. : *Fourier Analysis - American Mathematical Society*, Graduate Studies in Mathematics, 2001.

- Muscalu C. and Schlag, W. : *Classical and Multilinear Harmonic Analysis*, Cambridge Studies in advanced Mathematics, 2013.

- Folland, G. *Real Analysis: Modern Techniques and their Applications*, Wiley 1999.

- Grafakos, L. : *Classical Fourier Analysis* - Springer 2008.

- Grafakos, L.: *Modern Fourier Analysis* - Springer 2008.

- Stein, E.M.: *Singular Integrals and Differentiability Properties of Functions and Differentiability Properties of Functions* - Princeton University Press, 1970.

Additional Resources

MA4J1 Continuum Mechanics

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j1/)
Lecturer: Thomas Hudson

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 100% 3 hour written examination

Formal registration prerequisites: None

Assumed knowledge: This module assumes knowledge of various aspects of first and second year core maths material. Modules from other departments may also cover the necessary background. We list where the relevant material can be found for Maths and joint degree students.

- MA106 Linear Algebra: an understanding of vectors and matrices, including the ability to compute eigenvalues and eigenvectors
- MA134 Geometry and Motion or MA259 Multivariable Calculus: knowledge of multivariable calculus, including partial derivatives, divergence, curl and accompanying integral theorems
- MA133 Differential Equations or MA113 Differential Equations A: an ability to compute explicit solutions to simple ODEs
- MA131 Analysis or MA137 Mathematical Analysis or MA259 Multivariable Calculus: an appreciation of continuity and the ability to take limits

Useful background: MA3D1 Fluid Dynamics or MA3J4 Mathematical Modelling with PDE will serve as useful background to the modelling aspect of this module. Some background on the theory of ODEs and PDEs would also be useful, as covered in MA254 Theory of ODEs and MA3G1 Theory of PDEs

Synergies: The third year modules listed above would go well alongside this module. Fourth year modules which would also synergise well are:

- MA4L2 Statistical Mechanics
- MA4L0 Advanced Topics in Fluids
- MA4M2 Mathematics of Inverse Problems

Leads to: The following modules have this module listed as **assumed knowledge** or **useful background:**

Content: The modelling and simulation of fluids and solids with significant coupling and thermal effects is an important area of study in applied mathematics and engineering. Necessary for such studies is a fundamental understanding of the basic principles of continuum mechanics and thermodynamics. This course, which will closely follow the text "A First Course in Continuum Mechanics'' by Andrew Stuart, is a clear introduction to these principles.

The outline will be as follows: we will begin with a review of tensor algebra and calculus, followed by mass and force concepts, kinematics, and then balance laws. We will then proceed to derive some commonly used models governing isothermal fluids and solids, consisting of systems of partial differential equations (PDEs). If time permits we will also explore the thermal case.

Book(s):

Oscar Gonzalez, Andrew Stuart, *A First Course in Continuum Mechanics*, Cambridge University Press, 2008.

Additional Resources

MA4J2 Three-Manifolds

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j2/)

Not Running in 2019/20

Lecturer:

Term(s):

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 85% by 3-hour examination 15% coursework

Prerequisites: MA222 Metric Spaces and MA3F1 Introduction to Topology

Leads To:

Content:
1) Surfaces, handlebodies, I-bundles, polyhedral
2) Hauptvermutung, Heegaard splittings, S^3, T^3, PHS
3) Reducibility, Alexander's Theorem, knot complements, submanifolds of R^3
4) Fundamental group, incompressible surfaces, surface bundles
5) Tori and JSJ decomposition, circle bundles
6) Seifert fibered spaces
7) Loop theorem
8) Normal surfaces
9) Sphere theorem
10) Discussion of geometrization conjecture

Other possible topics:
Poincare conjecture, Fox's reimbedding theorem, space forms spherical, euclidean, hyperbolic, eg dodecahedral space, Thurston's eight geometries, Dehn fillings topologically, algebraically, geometrically, eg fillings of the trefoil, figure eight, non-Haken manifolds, three views of PHS (following Gordon).

Aims: An introduction to the geometry and topology of three-dimensional manifolds, a natural extension of MA3F1 Introduction to Topology

Objectives: By the end of the module the student should be:
Familiar with the basic examples (the three-sphere, the three-torus, knot components...).
Able to compute the fundamental group of a three-manifold M from a selection of presentations of M.
Familiar with the sphere and torus decomposition.
Able to state the loop theorem and use it (for example, to prove that knot components are aspherical).

Books:
Three-dimensional Topology by Andrew J Casson
The Theory of Normal Surfaces by Cameron Gordon
Notes on Basic 3-Manifold Topology by Allen Hatcher
3-Manifolds by John Hempel
Classical Tessellations and Three-Manifolds by José María Montesinos

MA4J3 Graph Theory

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j3/)
Lecturer: Vadim Lozin

Term(s): Term 1

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 100% 3 hour examination

Formal registration prerequisites: None

Assumed knowledge: None

Useful background:

- MA241 Combinatorics

Synergies:

- MA241 Combinatorics
- CS254 Algorithmic Graph Theory
- MA4M8 Theory of Random Graphs

Leads To:

Content: Graph theory is a rapidly developing branch of mathematics that finds applications in other areas of mathematics as well as in other fields such as computer science, bioinformatics, statistical physics, chemistry, sociology, etc. In this module we will focus on results from structural graph theory. The module should provide an overview of main techniques with their potential applications. It will include a brief introduction to the basic concepts of graph theory and it will then be structured around the following topics:

Structural graph theory:

- Graph decompositions
- Graph parameters

Extremal graph theory:

- Ramsey's Theorem with variations
- Properties of almost all graphs

Partial orders on graphs:

- Minor-closed, monotone and hereditary properties
- Well-quasi-ordering and infinite antichains

Aims: To introduce students to advanced methods from structural graph theory.

Objectives: By the end of the module the student should be able to:

- State basic results covered by the module
- Understand covered concepts from graph theory
- Use presented graph theory methods in other areas of mathematics
- Apply basic graph decomposition techniques

Books:

Bollobás, Béla (2004), *Extremal Graph Theory*, New York: Dover Publications, ISBN 978-0-486-43596-1
Diestel, Reinhard (2005), *Graph Theory* (3rd ed.), Berlin, New York: Springer-Verlag, ISBN 978-3-540-26183-4

Additional Resources

MA4J4 Quadratic Forms

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j4/)

Not Running in 2019/20

Lecturer:

Term(s):

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 3-hour examination (100%)

Prerequisites: MA251 Algebra I, MA249 Algebra II,
Desirable: MA3D5 Galois Theory, MA377 Rings and Modules

Leads To: PhD studies in Number Theory, Algebraic Geometry and Algebraic K-theory

Content:
Quadratic and symmetric bilinear forms over fields
The Witt group W(F) of a field F, chain lemma, cancellation and presentation of W(F)
Classification of quadratic forms over Q, R, finite fields and algebraically closed fields
Stable classification of symmetric bilinear forms over the integers
Formally real fields, signatures, sums of squares, torsion in W(F), transfer
Extension to Dedekind domains, Milnor's exact sequence

Aims:
Quadratic forms are homogeneous polynomials of degree 2 in several variables. They appear in many parts of mathematics where one reduces the classification of certain objects to the classification of quadratic forms. This happens for instance in algebra (quaternion algebras), in manifold theory (cohomology intersection form), in Lie theory (Killing form), in lattice theory (e.g., sphere packing problems), in number theory (sums of squares formulas, quadratic reciprocity) etc. The aim of this module is to understand the classification of quadratic forms over fields (e.g., field of rational numbers, finite fields) and certain rings (e.g., the integers) and to understand the relationship between properties of quadratic forms and properties of the fields in question.

Objectives: By the end of the module the student should be able to:
Understand the use of Witt groups in the classification of quadratic forms
Compute Witt groups in easy examples

Decide whether two given quadratic forms (over Q, R, F_q etc) are equivalent
Relate properties of fields to properties of quadratic forms and vice versa

Books:

Kazimierz Szymiczek, *Bilinear Algebra: An Introduction to the Algebraic Theory of Quadratic Forms*. 1997. xii+486 pp. ISBN: 90-5699-076-4
(Elementary text with lots of exercises, covers part of the module)

John Milnor, Dale Husemoller, *Symmetric Bilinear Forms*. 1973. viii+147 pp
(Great text, covers everything in the module but no exercises)

T Y Lam, *Introduction to Quadratic Forms over Fields*. 2005. xxii+550 pp. ISBN: 0-8218-1095-2
(Covers everything in the module and much more with lots of exercises)

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4J5 Structures of Complex Systems

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j5/)

Lecturer: Markus Kirkilionis

Term: Term 1

Status for Mathematics students: List C

Commitment: 30 lectures

Assessment: 80% 3 hour examination 20% Project

Formal registration prerequisites: None

Assumed knowledge:

MA398 Matrix Analysis and Algorithms:

- Methodological foundations in linear algebra and matrix algorithms as well as hands-on experience in programming

ST112 Probability B:

- Basic probability theory
- Random variables

Useful background:

ST202 Stochastic Processes:

- Markov processes and Markov chains

MA241 Combinatorics:

- Foundations of graph theory

MA252 Combinatorial Optimisation:

- Algorithms in graph theory and NP-hard problems

Synergies: The following modules go well together with Structures of Complex Systems:

- MA4E7 Population Dynamics: Ecology and Epidemiology
- MA4M1 Epidemiology by Example
- MA4M4 Topics in Complexity Science

Leads to: The following modules have this module listed **assumed knowledge** or **useful background:**

Content:
Part A: Complex Structures

Graphs, the language of relations:

- Introduction to graph theory
- Degree distributions, their characteristics, examples from real world complex systems (social science, infrastructure, economy, biology, internet)
- Introduction to algebraic and computational graph theory

Evolving graph structures:

- Stochastic processes of changing graph topologies
- Models and applications in social science, infrastructure, economy and biology
- Branching structures and evolutionary theory

Graphs with states describing complex systems dynamics:

- Stochastic processes defined on vertex and edge states
- Models and applications in social science and game theory, simple opinion dynamics
- Opinion dynamics continued

Graph applications:

- Graphs and statistics in social science
- Graphs describing complex food webs
- Graphs and traffic theory

Extension of graph structures:

- The general need to describe more complex structures, examples, introduction to design
- Hypergraphs and applications
- Algebraic topology and complex structures

Part B: Complex Dynamics:

Agent-based modelling:

- Introduction to agent-based modelling
- Examples from social theory
- Agent-based modelling in economy

Stochastic processes and agent-based modelling:

- Markov-chains and the master equation
- Time-scale separation
- The continuum limit (and ‘inversely’ references to numerical analysis lectures)

Spatial deterministic models:

- Reaction-diffusion equations as limit equations of stochastic spatial interaction
- Basic morphogenesis
- The growth of cities and landscape patterns

Evolutionary theory I:

- Models of evolution
- Examples of complex evolving systems, biology and language
- Examples of complex evolving systems, game theory

Evolutionary theory II:

- Basic genetic algorithms

- Basic adaptive dynamics
- Discussion and outlook

Aims:

- To introduce mathematical structures and methods used to describe, investigate and understand complex systems
- To give the main examples of complex systems encountered in the real world
- To characterize complex systems as many component interacting systems able to adapt, and possibly able to evolve
- To explore and discuss what kind of mathematical techniques should be developed further to understand complex systems better

Objectives: By the end of the module the student should be able to:

- Know basic examples of and important problems related to complex systems
- Choose a set of mathematical methods appropriate to tackle and investigate complex systems
- Develop research interest or practical skills to solve real-world problems related to complex systems
- Know some ideas how mathematical techniques to investigate complex systems should or could be developed further

Books: There are currently no specialized text books in this area available, but all the standard textbooks related to the prerequisite modules indicated are relevant.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4J6 Mathematics and Biophysics of Cell Dynamics

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma4j6/)
Lecturer: Nigel Burroughs

Term: Not running 2020/21

Status for Mathematics students: List C

Commitment: 30 lectures and weekly assignments

Assessment: 3 hour examination (100%)

Prerequisites:
Previous experience with a couple of Dynamical Systems, PDEs, probability theory/stochastic processes, continuum mechanics or physical principles such as elasticity and energy would be beneficial; students from (3rd and 4th year) Mathematics or Physics with backgrounds covering some of these areas should find the course accessible. Statistics students with both experience with PDEs and stochastic processes should also find it accessible. Given the diversity of techniques used in the course, **do not worry if you haven't got them all, techniques will be covered.** MA256 Introduction to Systems Biology or MA390 Topics in Mathematical Biology provide some useful background in modelling. Probability A/B (ST111/2) content will be assumed whilst ST202 Stochastic Processes provides additional useful background for the probabilistic aspects of the course. MA250 Introduction to partial differential equations provides some background in PDEs. Programming: Optional examples with a programming component will be available which will require MatLab or another high-level language.

Leads To:

Content:

1. Spatial systems and organisation principles. Cell adhesion, protein (Turing) patterns and diffusion driven instability.

2. Molecule diffusion and search times. Diffusion along DNA (1D), in membranes (2D) and in 3D.

3. Polymerisation underpinning motion and work. Microtubule dynamics (dynamic instability and catastrophes) and actin.

4. Molecular motors. the flashing ratchet.

5. Cell movement. Actin gels and pushing beads.

6. Cell division if time permits. Chromosome self-organisation processes.

Aims:
How cells manage to do seeming intelligent things and respond appropriately to stimulus has generated scientific and philosophical debate for centuries given that they are just a 'bag' of chemicals. This course will attempt to offer some answers using state-of-the-art mathematical/physics models of fundamental cell behaviour from both bacteria and mammals. A number of key models have emerged over the last decade dealing with spatial-temporal dynamics in cells, in particular cell movement, but also in developing crucial understanding of the basic cellular architecture governing dynamic processes. We will thus explore a number of biological phenomena to illustrate fundamental biological principles and mechanisms, including for example molecular polymerisation to perform work. This course will take a mathematical modelling viewpoint, developing both modelling techniques but also essentials of model analysis. We will draw on a large body of mathematical areas from both deterministic- dynamical systems methods (20%), (continuum) mechanics (20%), and probabilistic arenas - Fokker Planck equations, diffusions (60%); indicated percentages are approximate and may vary from year to year. We will thus draw on a wide variety of techniques to best address the issues, techniques that will be covered in the course.

Objectives:
By the end of the module the student should be able to:
Develop spatial-temporal models of biological phenomena from basic principles
Understand the basic organisation and physical principles governing cell dynamics and structure
Characterise the dynamics of simple (stochastic) models of biological polymers (actin, tubulin)
Construct and solve optimisation problems in biological systems, e.g. for a diffusing protein to find a target binding site
Reproduce models and fundamental results for a number of cell behaviours (division, actin gels)

Books:
There are currently no specialised text books in this area available.

Additional Resources

2011 2015 2018

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA595 Topics in Stochastic Analysis

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma595/)

Not Running in 2015/16

Lecturer:

Term(s):

Status for Mathematics students: List D

Commitment: 30 lectures

Assessment: 3 hour exam 100%

Prerequisites: Knowledge of basic stochastic calculus including stochastic differential equations driven by Brownian motion will be assumed. Measure theory and functional analysis are basic tools and familiarity with basic concepts of differentiable manifolds is likely to be needed. Useful preparatory courses include: MA482 Stochastic Analysis, MA460 Differential Geometry.

Leads To:

Content: The natural state space for stochastic differential equations is a smooth manifold. Even if that manifold is a Euclidean space, if the equation has a more interesting structure than that of just additive noise it induces differential geometric structures which help to identify the behaviour of the solutions (the ``volatility'' can often determine a Riemannian metric for example, whose curvature affects the long time behaviour of solutions). On the other hand the solution to the equation can be considered as a map from path space on some R^m, i.e, Wiener space, to the space of the manifold, and this can be analysed by techniques of infinite dimensional calculus, in particular those known as Malliavin Calculus.

The precise content of the course will be decided after consulting those who expect to come to the lectures. If you are intending to come it might help if you could contact me sometime in Term 1. Of course if you have not done so you will be very welcome to come! but you will then have much less influence on the content.

MA 460 looks as if it will be a near perfect course introducing much of the differential geometry which arises in the theory. The first part, at least, is strongly recommended for anyone who wishes to continue in this area either as a researcher or as a practitioner.

Aims:

Objectives:

Books: The following contain useful material for the course:

Rogers, L. C. G.; Williams, David; *Diffusions, Markov processes, and martingales. Vol. 2. It™ calculus*. Reprint of the second (1994) edition. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2000. xiv+480 pp. ISBN: 0-521-77593-0 60J60
Elworthy, David; *Geometric aspects of diffusions on manifolds. École d'Eté de Probabilités de Saint-Flour XV-XVII*,1985-87, 277-425, Lecture Notes in Math., 1362, Springer, Berlin, 1988.
Bell, Denis R; *The Malliavin calculus.* Reprint of the 1987 edition. Dover Publications, Inc., Mineola, NY, 2006.
Hsu, Elton P. *Stochastic analysis on manifolds.* Graduate Studies in Mathematics, 38. American Mathematical Society, Providence, RI, 2002.

Additional Resources

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

MA4K8 MA4K9 Projects

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/ma469/)
Organisers: Charles Elliott, Edward Hill

Term(s): Terms 1-2

Status for Mathematics students: Core for 4th Year G103

Assessment: See below

The fourth-year Project module comes in two flavours:

- **MA4K8 Maths-in-Action (MiA-Projects):**
  Description: This module is suitable for students going on to careers (such as in data analysis or quantitative analysis in finance) in which developing mathematics will be a vital skill as well as students intending to pursue further mathematical studies and research. A primary purpose of these projects is further development of communication skills in speaking and writing. The projects involve a deeper understanding of how mathematics underpins a particular topic in the modern world and then communicating this understanding in the form of a presentation, a written popular science article, and a written scholarly report at the MMath level. Your Maths-in-Action projects will show how some of the mathematics you have learnt at Warwick

affects contemporary life and technology.
Aims: The broad aims are: to develop your ability to communicate mathematics to diverse audiences and to give you a deeper appreciation of how mathematics underpins the modern world. Doing a Maths-in-Action project will teach you the art of scholarship and is also an opportunity to engage in your own mathematical research activity. It will help you to acquire a variety of presentation skills.
Resources: Maths-in-Action Resources page
Moodle page: https://moodle.warwick.ac.uk/course/view.php?id=52270

- **MA4K9 Research (R-Projects):**
  Description: These projects are valuable for students intending to pursue further mathematical studies such as research degrees. It is also suitable for students going on to careers (such as in data analysis or quantitative analysis in finance) in which developing mathematics will be a vital skill. Finally, it is for anyone wishing to experience the joy of mathematical study at the frontiers of research.
  Aims: The primary aim of the Research Project is to give you experience of mathematics as it is pursued close to the frontiers of research, not just as a spectator sport but as an engaging, evolving activity in which you yourself can play a part.
  Resources: R-Projects Resource page
  Moodle page: https://moodle.warwick.ac.uk/course/view.php?id=52271

16 June 2023 Q&A session: Recording available here

IMPORTANT: **Please note the Important Dates sections below for the R-Projects and the MiA Projects. Deadlines must be strictly adhered to!**

It is your responsibility to make sure you are registered for the correct version of the project on eMR!

In addition, all MMath students **must** register their project choice on Moodle by 23:59 on
Sunday, 30 October 2022.

Registration will open on Saturday, 01 October 2022

MATHS-in-ACTION PROJECTS

Assessment: The Maths-in-Action Projects are assessed on:

- Scholarly Report (60% of the module credit).
- Popular Article (20% of the module credit).
- Poster and Presentation (15% of the module credit).
- Progress Report (5% of the module credit).

Themes for 2022/23:

Algebra and Topology in Biology,
Climate,
Collective dynamics,
Data, Information and Complexity,
Epidemiology,
Geometry in nature and data,
Inverse problems,
Machine learning,
Opinion dynamics,
Quantum computing,
Traffic and autonomous driving.

It is important that you spend some time exploring each theme before making your choices. Past experience shows that rushing into a choice based on title alone is a bad idea.

MiA Project Important Dates: The submission deadlines below are strict and marks will be deducted for late submissions!

Further information about various submissions and meetings will be posted on the News Items on the Maths-in-Action Resources page and the Moodle page. You should check it regularly.

- *Introduction*: **Week 1**. The organiser will give a brief overview of the MMaths projects, as part of the options fair or in a separate (virtual) meetup.
- Short open meeting for general discussion, and Q & A: **2:00 pm Wednesday, 19 October, 2022** (Week 3).
- *Registration*: You must register your choice of theme on Moodle by **Sunday, 30 October 2022** (the end of Week 4). Note, this is not the same as module registration.
- Meeting to discuss Progress Reports and other issues: **2:00 pm Wednesday, 7 December, 2022** (Week 10).
- *Progress Report*: You must submit the Progress Report via Moodle by **12 noon on Tuesday 17 January** (Week 2), **2023.**
- Meeting to provide feedback and to answer further questions: **2:00 pm Wednesday, 25 January 2023** (Week 3).
- *Submission of Poster/Presentation:* You must submit **an electronic copy of your poster and presentation** via Moodle by **12 noon on Friday, 17 February, 2023** (Week 6). This deadline is the same for every student, regardless of your presentation session.
- *Presentations*: Presentations will take place on Wednesday afternoons in Term 2 in weeks 8 and 9. Every student has to participate in each day. Further details can be found on the resources page.
- *Submission of Scholarly Reports and Popular Article*: You must submit **one electronic copy (as pdf) of the Scholarly Report and the Popular Article** on Moodle by **3pm on Tuesday, 4 April, 2023**.
  Please contact the Taught Programmes Office ugmathematics@warwick.ac.uk for any questions concerning the submission procedure.

RESEARCH PROJECTS

Assessment: The Research Projects are assessed on the basis of

- A short progress report in Term 2 (5% of the module credit)
- A written dissertation (80% of the module credit)
- An oral presentation and defence of dissertation (15% of the module credit)

Themes and supervision: The R-Project can be any area of mathematics offered by permanent staff. Before you register for a Research Project, you must first take the following steps:

1. Find a member of staff willing to supervise you;
2. Agree on a theme suited to your mathematical background and interests, and to your supervisor's expertise;
3. Negotiate a title and brief for your project, and discuss its aims and objectives. It is normal for this to need renegotiating as the project evolves and final titles sometimes differ from the title originally registered;

A list of the Research project themes offered by staff members can be found here. It might also be useful to look at the research interests of permanent staff.

If you have your own ideas about a theme for an R-project, feel free to ask any permanent member of staff whether they would be willing to be your supervisor, but remember, staff are under no obligation to supervise an R-project, and are, in any case, discouraged from supervising more than two a year.

Research Project Important Dates: The dates below are strict and marks will be deducted for late submissions!

- *Introduction*: **Week 1,**. The organiser will give a brief overview of the MMaths projects, as part of the options fair or in a separate (virtual) meetup.
- *Registration:* You must register your project by **Sunday, 30 October 2022** (the end of Week 4). Note, this is not the same as module registration. Rather, this activity is registering the project title and supervisor on Moodle.
- *Progress Report:* You must submit the Progress Report via Moodle by **12 noon on Tuesday 17 January** (Week 2), **2023**. Note, this form will be reviewed by your supervisor, so you need to begin discussing the report before the deadline, preferably by the end of Term 1.
- *Dissertation:* You must submit **one electronic copy (as pdf)** on Moodle by **3pm on Tuesday, 4 April, 2023.**
  Please contact the Taught Programmes Office ugmathematics@warwick.ac.uk for any questions concerning the submission procedure.
- *Oral examination:* Oral examinations will happen at the beginning of term 3. It is the student's responsibility to organise a date and venue with first and second markers, either virtually or face-to-face, to be agreed by those involved.

Additional Resources - Research Projects

Archived Pages: 2014 2018

Additional Resources - Math-in-Action Projects

Archived Pages: 2014 2018

Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

CO905 Stochastic Models of Complex Systems

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/co905/)

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Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

CO907 Quantifying Uncertainty and Correlation in Complex Systems

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/co907/)

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Year 1 regs and modules
G100 G103 GL11 G1NC

Year 2 regs and modules
G100 G103 GL11 G1NC

Year 3 regs and modules
G100 G103

Year 4 regs and modules
G103

Exam information
Core module averages

ST4 Modules

(https://warwick.ac.uk/fac/sci/maths/currentstudents/ughandbook/year4/st4xx/)

Name	Description	Last Updated
Integrated Masters Projects Allocations	Integrated Masters Project Allocations 23-24 completion	16/05/23
ST401	Stochastic Methods in Finance	06/10/22
ST402	Risk Theory (Last teaching in 2023/24. This module is replaced by ST348 Risk Theory from 2024/25 onwards.)	12/05/23
ST403	Brownian Motion	06/10/22
ST404	Applied Statistical Modelling	06/10/22
ST405	Bayesian Forecasting and Intervention with Advanced Topics	06/10/22
ST406	Applied Stochastic Processes with Advanced Topics	06/10/22
ST407	Monte Carlo Methods	06/10/22
ST409	Medical Statistic with Advanced Topics	06/10/22
ST410	Designed Experiments with Advanced Topics	06/10/22
ST411	Dynamic Stochastic Control (suspended in 22/23)	03/05/23
ST412	Multivariate Statistics with Advanced Topics	06/10/22
ST413	Bayesian Statistics and Decision Theory with Advanced Topics	06/10/22
ST414	Advanced Topics in Statistics (suspended in 22/23)	15/09/22
ST415	Statistics Masters Dissertation	06/10/22
ST417	Topics in Applied Probability (suspended in 22/23)	15/09/22
ST418	Statistical Genetics with Advanced Topics	06/10/22
ST419	Advanced Topics in Data Science	06/10/22
ST420	Statistical Learning and Big Data	06/10/22
ST421	Data Science Masters Dissertation	15/09/22