Opinion Dynamics and Price Formation

The University of Warwick

Centre for Complexity Science

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Introduction

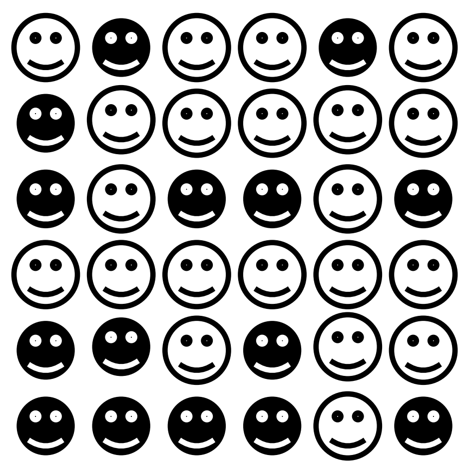
1. Standard Voter Model
2. Heterogeneous Voter Model
3. VM in Networks
4. Social Pressure in Coevolving VM

BASIC IDEAS

-Binary state of opinion: $n_i(t) = \begin{cases} 1 \\ 0 \end{cases}$

-Activation Poisson Process → Activation Rate :: λ_i

-Choice of Copied Neighbor: $P\{j|i\}$



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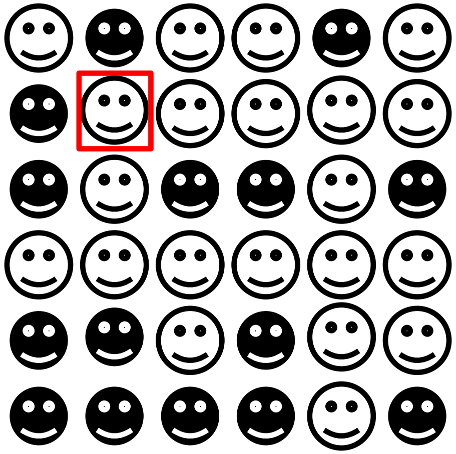

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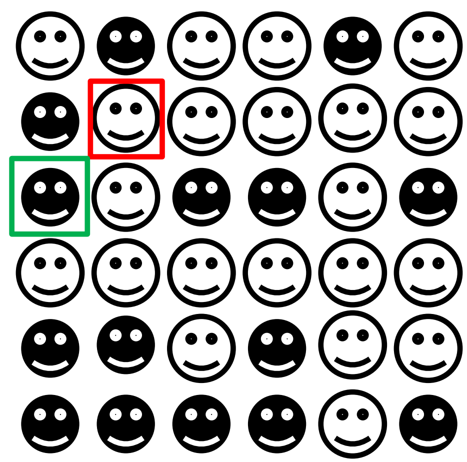
•Generic Network:

$$P\{j|i\} = \frac{a_{ij}}{k_i}$$

•MF Random Network:

$$P\{j|i\} = \frac{1}{N-1}$$



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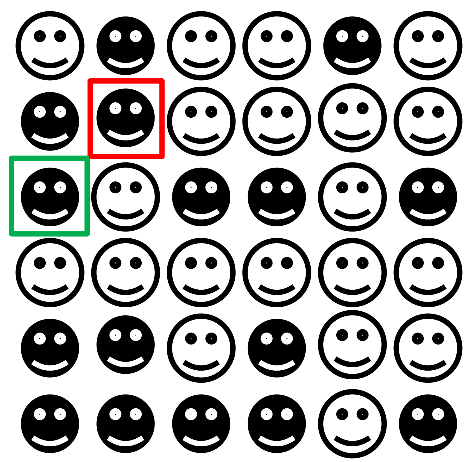

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$$n_i(t+dt) = n_i(t)[1-\xi_i(t)] + \eta_i\xi_i(t)$$

$$\xi_i(t) = \begin{cases} 1 & \text{with prob. } \lambda_i dt, \\ 0 & \text{with prob. } 1-\lambda_i dt \end{cases} \qquad \eta_i(t) = \begin{cases} 1 & \text{with prob. } \sum_{j\neq i} P\{j|i\} n_j(t), \\ 0 & \text{with prob. } 1-\sum_{j\neq i} P\{j|i\} n_j(t) \end{cases}$$

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-Ensemble Average Evolution: $\rho_i(t) = \langle n_i(t) \rangle_{\vec{n}}$

$$\frac{d\rho_i}{dt} = \lambda_i \left(\sum_{j \neq i} P\{j|i\} \rho_j(t) - \rho_i \right)$$

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-Conservation Laws:

$$\phi(j) = \sum_{i=1}^{N} \phi(i) P\{j|i\} \longrightarrow \sum_{i=1}^{N} \frac{\phi(i)}{\lambda_i} \rho_i(t) = const$$

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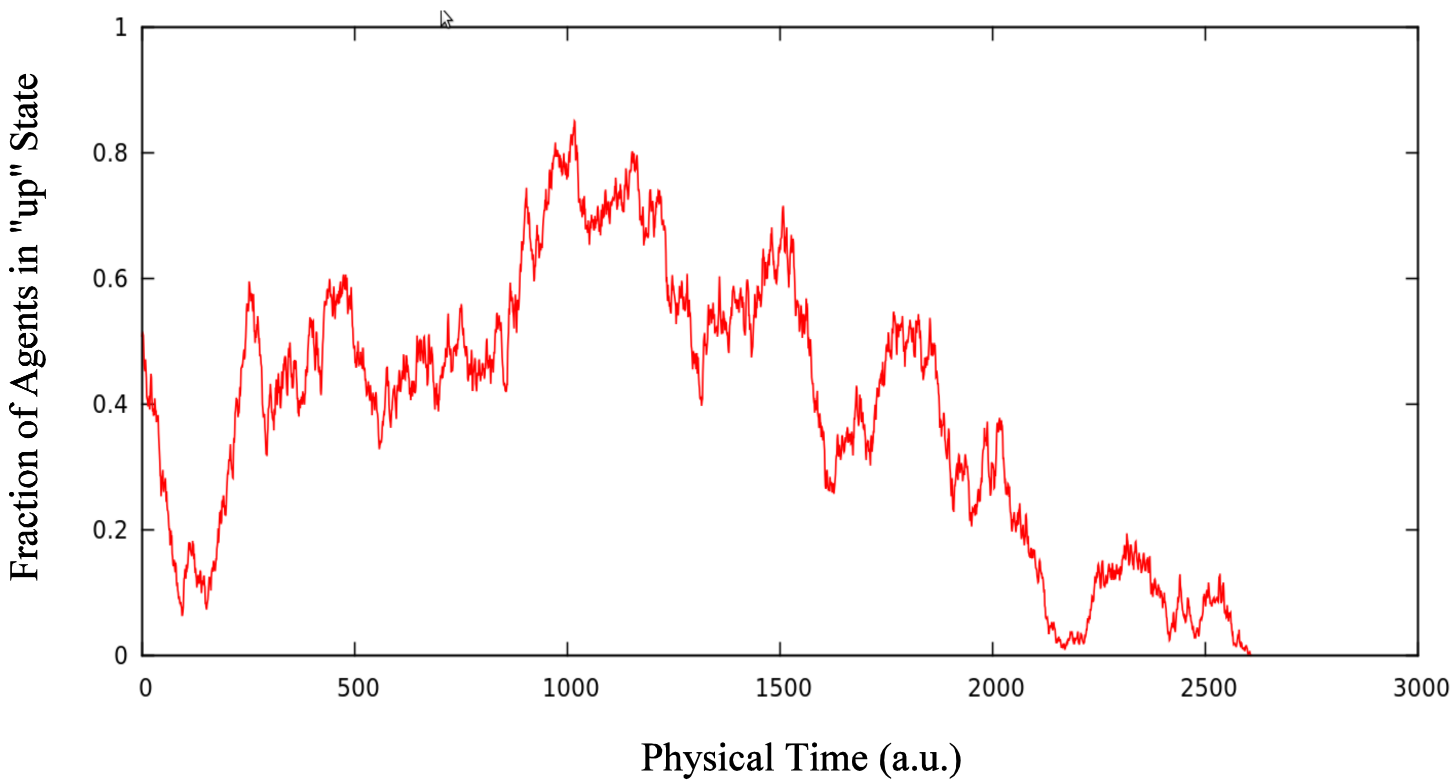
-Conservation Laws:

$$\sum_{i=1}^{N} \frac{\phi(i)}{\lambda_i} \rho_i(t) = const \longrightarrow P_1 = \frac{\sum_{i=1}^{N} \frac{\phi(i)}{\lambda_i} \rho_i(t=0)}{\sum_{i=1}^{N} \frac{\phi(i)}{\lambda_i}}$$

BASIC IDEAS

-Typical path of "magnetization": quasi-diffusion to one absorbing state (consensus)



REFERENCE

M.Boguñá and G. Mosquera-Doñate, "Follow the Leader: Herding Behavior in Heterogeneous Populations", *Physical Review E.* **91** (2015).

ASSUMPTIONS

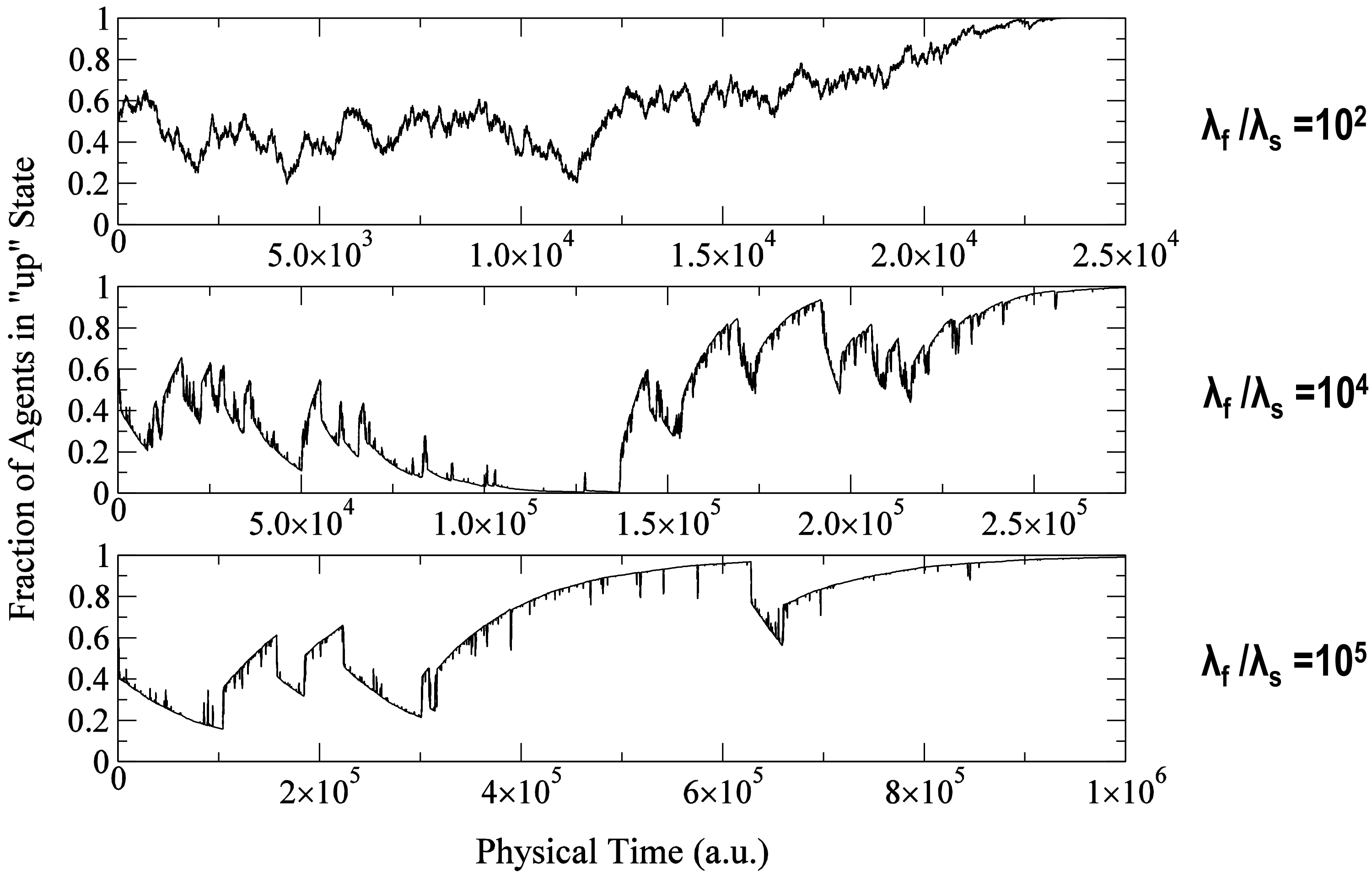
-Heterogeneity in Activation Rates ⟶ Two Species:
- λ_f (fast) : minority
- λ_s (slow) : majority

-Separated Time-Scales: ⟶ $\lambda_f >> \lambda_s$

-Hierarchy in Neighbor Choice ⟶ $P\{j|i\} = \frac{f(\lambda_j)}{\sum_{k \in N_G(i)} f(\lambda_k)}$

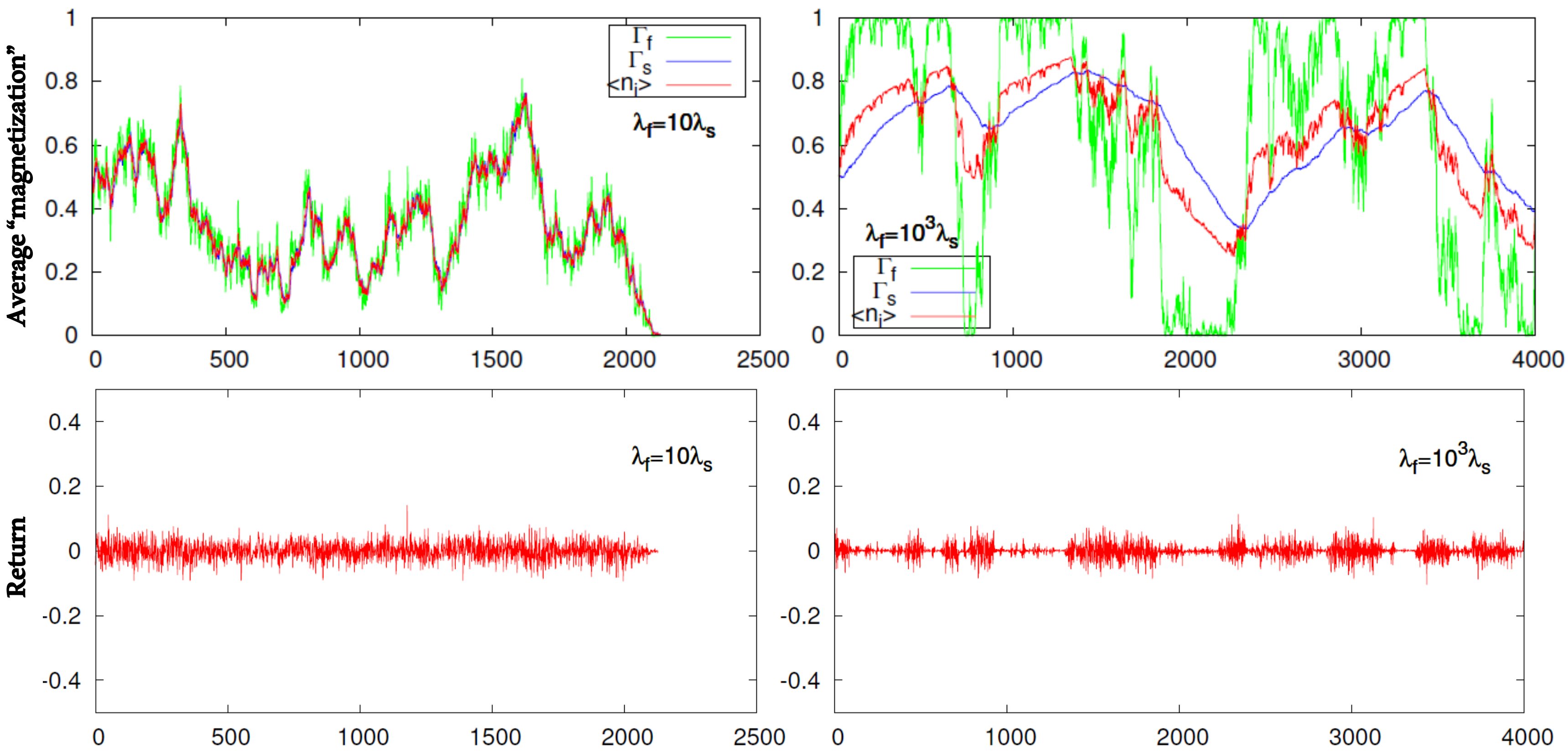
$$f(\lambda_i) = \lambda_i^{\sigma} \quad \text{with } \sigma \geq 1$$

SIMULATIONS



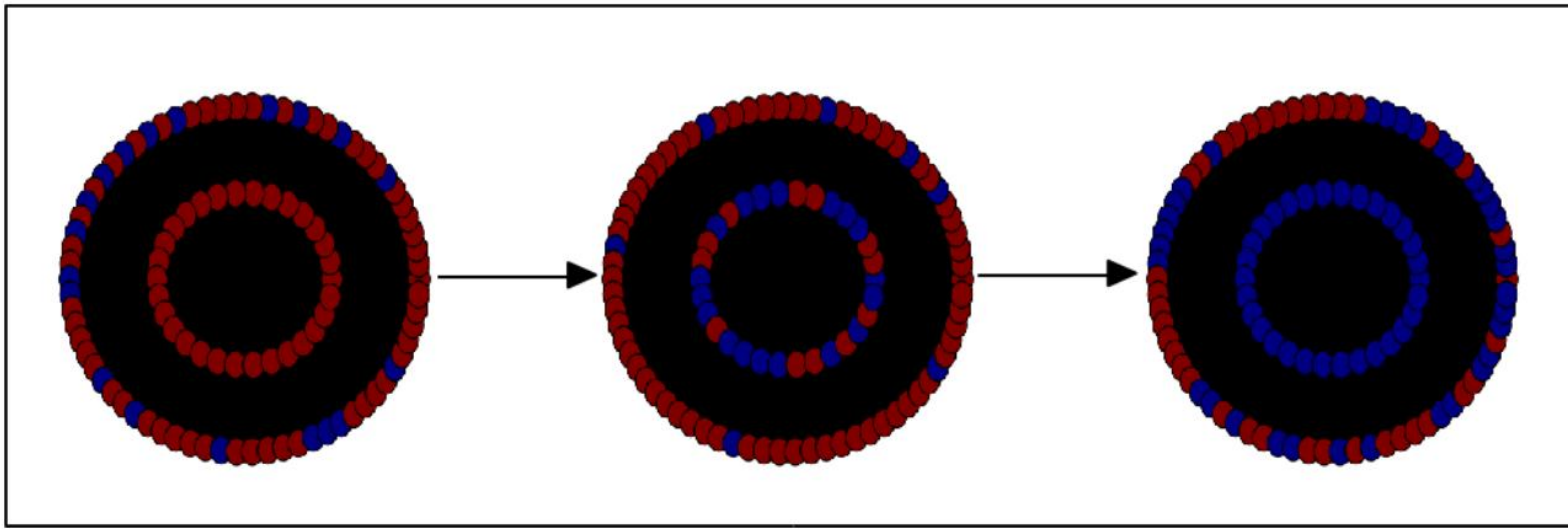
SIMULATIONS

$$\Gamma(t) \equiv \frac{1}{N}\sum_{i\in G} n_i(t) \qquad \Gamma_f(t) \equiv \frac{1}{N_f}\sum_{i\in fast} n_i(t) \qquad \Gamma_s(t) \equiv \frac{1}{N_s}\sum_{i\in slow} n_i(t)$$



$N_f=300 \quad N_s=700$

REPOLARIZATION OF THE LEADING CORE



PHASE TRANSITION AND BISTABILITY

- Focker-Planck Effective Potential for Fast Dynamics:
- Quasi-Constant Approximation: ρ_S=1/2

$$V_{eff} = \left(1 - \frac{N_f}{2k_{fs}}\right) \ln \left\{ \frac{(k_{fs}+1) - k_{fs}(1-2\rho_f)^2}{k_{fs}+1} \right\} \quad k_{fs} \equiv \frac{\lambda_f}{\lambda s} \frac{N_f}{N_s}$$

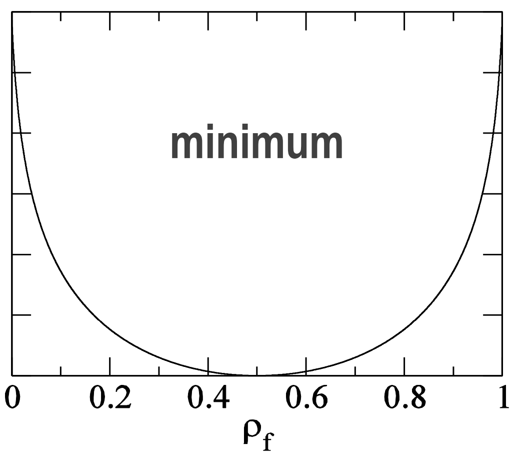
↓

extremum in ρ_f=1/2 :

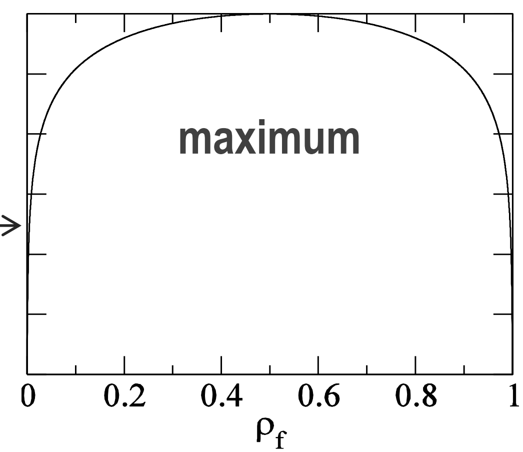
↓

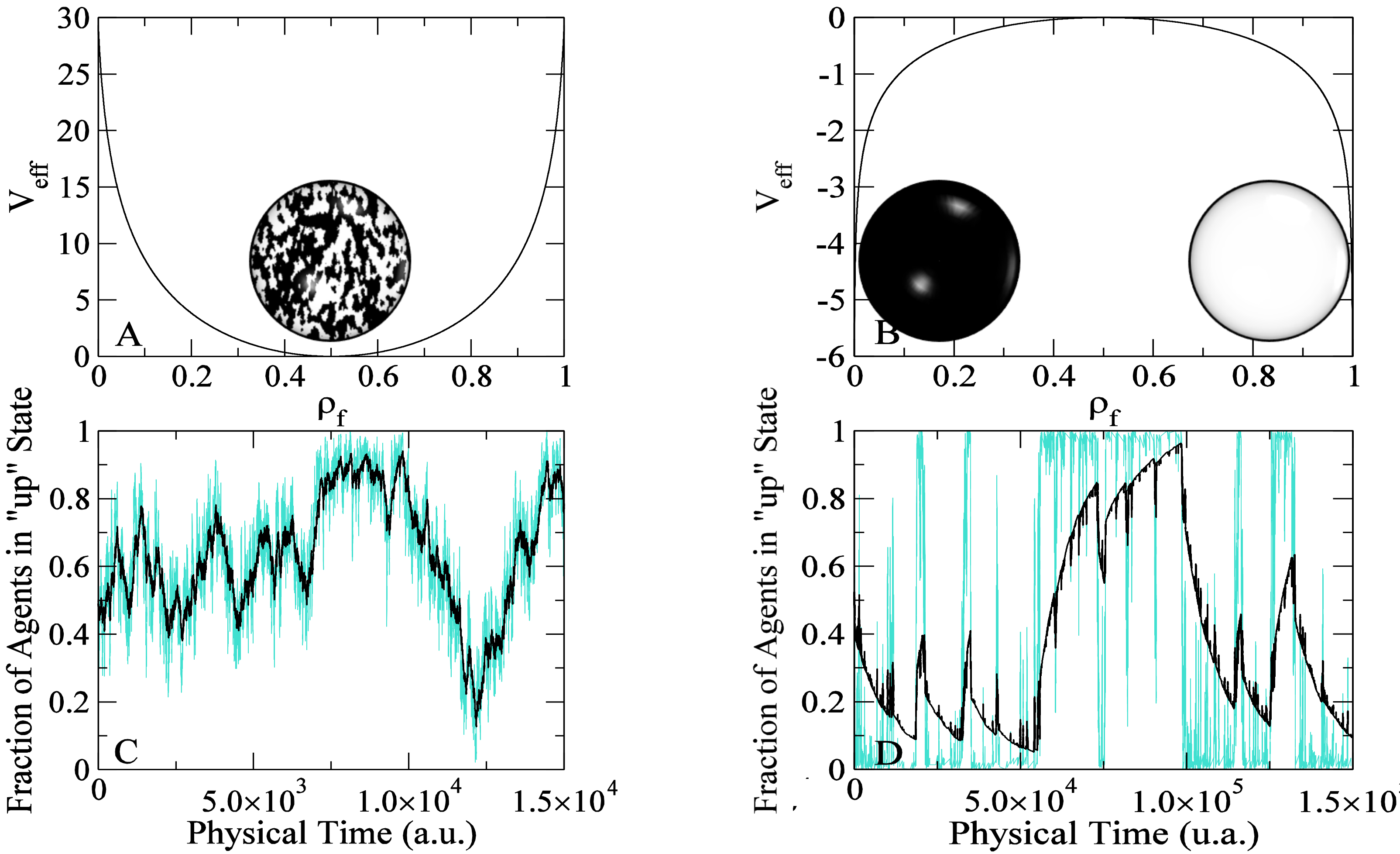
$$2\frac{f(\lambda_f)}{f(\lambda_s)} > N_s$$

true → minimum



false → maximum





-N_f=500 ; K_{fs}=25

K^c_{fs}=250

-N_f=500 ; K_{fs}=2500

STOCHASTIC σ-PROCESS

-Stochastic Updating Process for σ ⟶ $f(\lambda_i) = \lambda_i^\sigma \quad \text{with } \sigma \geq 1$

-Poisson updating ⟶ realizations of σ(t) ~ $U(\sigma_{min}, \sigma_{max})$

-Inhomogeneous Periods ⟶ Shorter convulse (bimodal) phases?

$$T_\sigma(t) = \frac{1}{\lambda_\sigma(t)} = \frac{k_\sigma}{\sigma(t)} \qquad k_\sigma > 0$$

STOCHASTIC σ-PROCESS

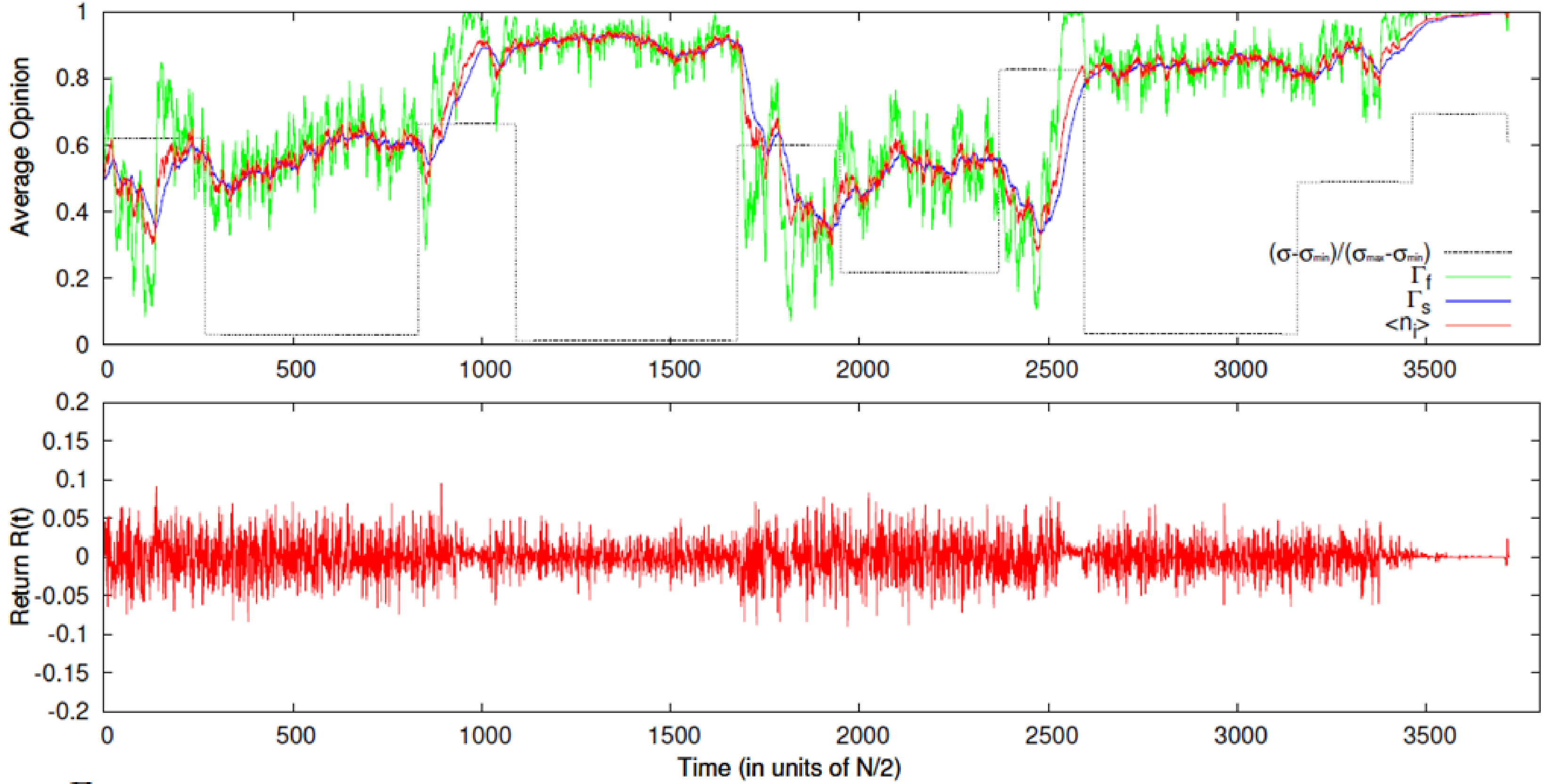


Figure 4: Realization of the σ-process model with $N_f = 300$, $N_s = 700$, $\lambda_f = 100\lambda_s$, $\sigma_{min} = 0.5$, $\sigma_{max} = 1.5$ and $k_\sigma = 150(N_f + N_s)$.

STOCHASTIC σ-PROCESS

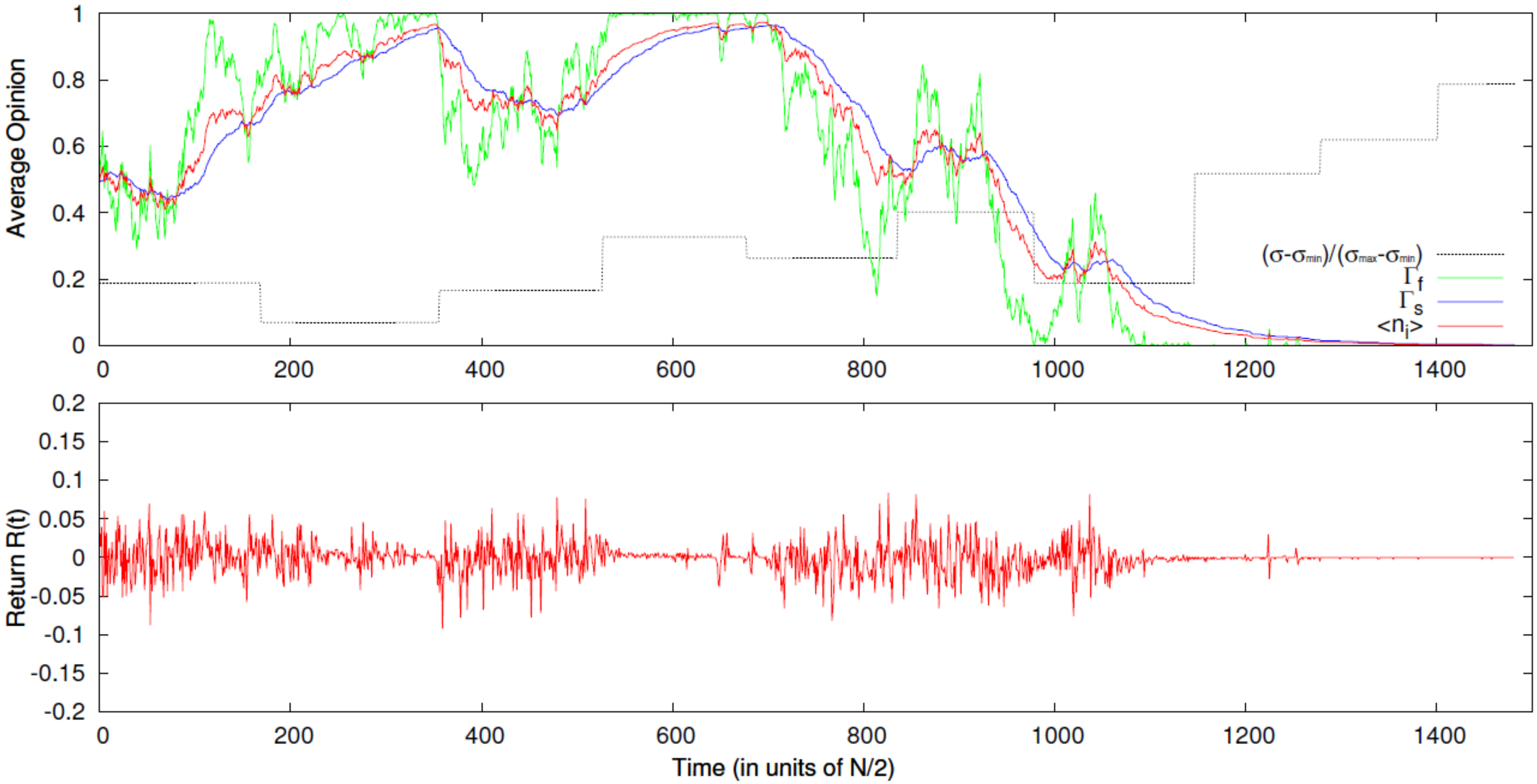
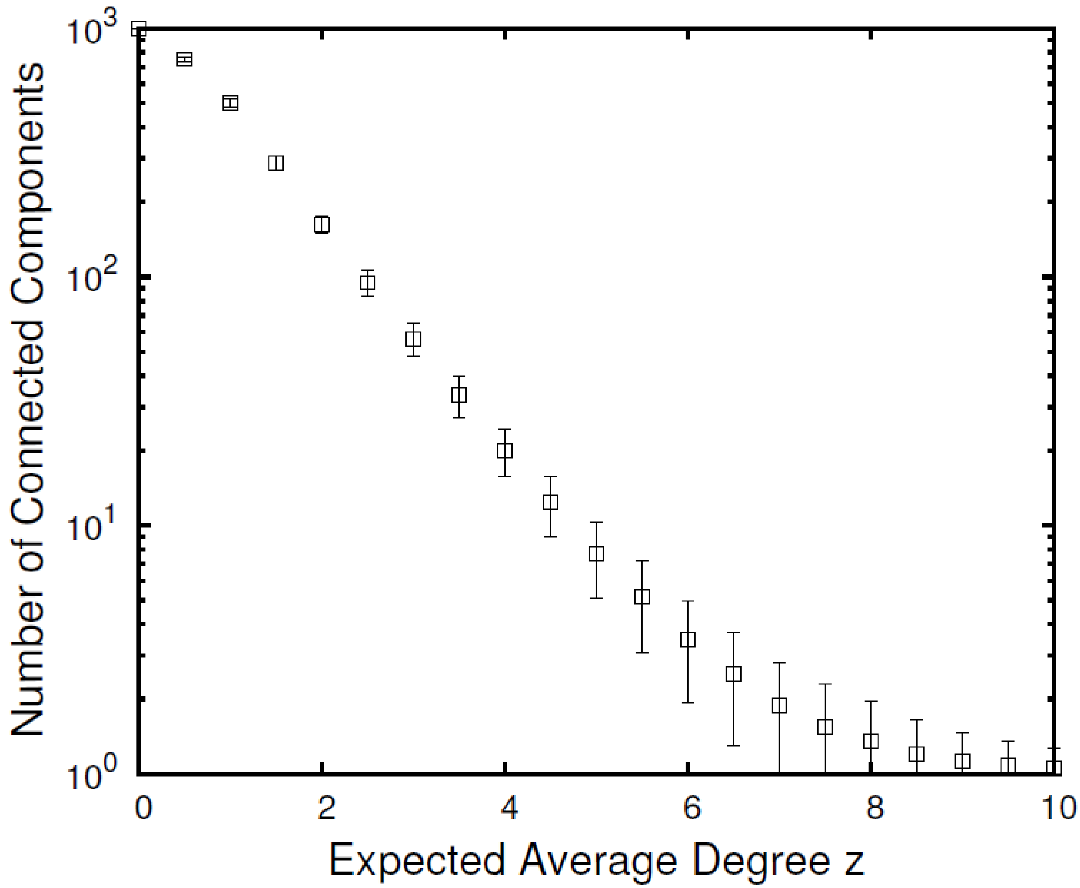


Figure 5: Realization of the σ-process model with $N_f = 300$, $N_s = 700$, $\lambda_f = 100\lambda_s$, $\sigma_{min} = 1$, $\sigma_{max} = 2$ and $k_\sigma = 100(N_f + N_s)$.

ERDÖS-RÉNYI RANDOM GRAPHS

-Edge Probability → $p = \frac{z}{N-1}$



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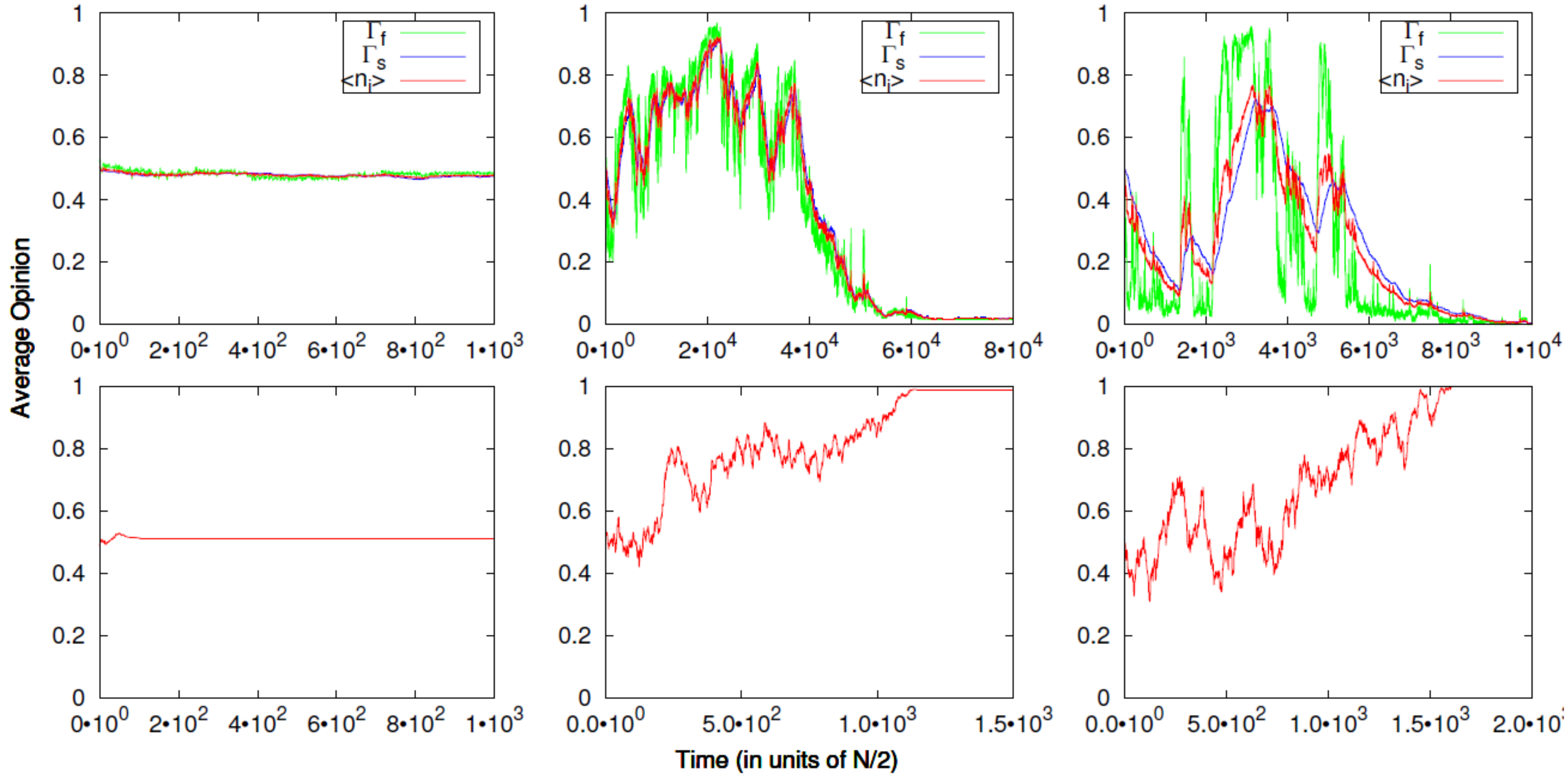


Figure 6: Realizations of the Voter Model on Erdős-Rényi random graphs with N=1000 agents. Top plots use the Heterogeneous Population model with $\lambda_f = 1000\lambda_s$ and $N_f = 300$, whereas bottom plots use homogeneous population in terms of activation rates. Right, center and left columns represent networks with z=8, z=4 and z=1 respectively.

PREFERENTIAL ATTACHMENT GRAPHS

-Barabási – Albert model ⟶ $$p_i = \frac{k_i}{\sum_j k_j}$$

-Topology-based Hierarchy ⟶ $$P(j|i) = \frac{{k_{EP_j}}^{\sigma}}{{k_{EP_j}}^{\sigma} + {k_{EP_i}}^{\sigma}}$$

$$\sigma \in \mathbb{R}$$

PREFERENTIAL ATTACHMENT GRAPHS

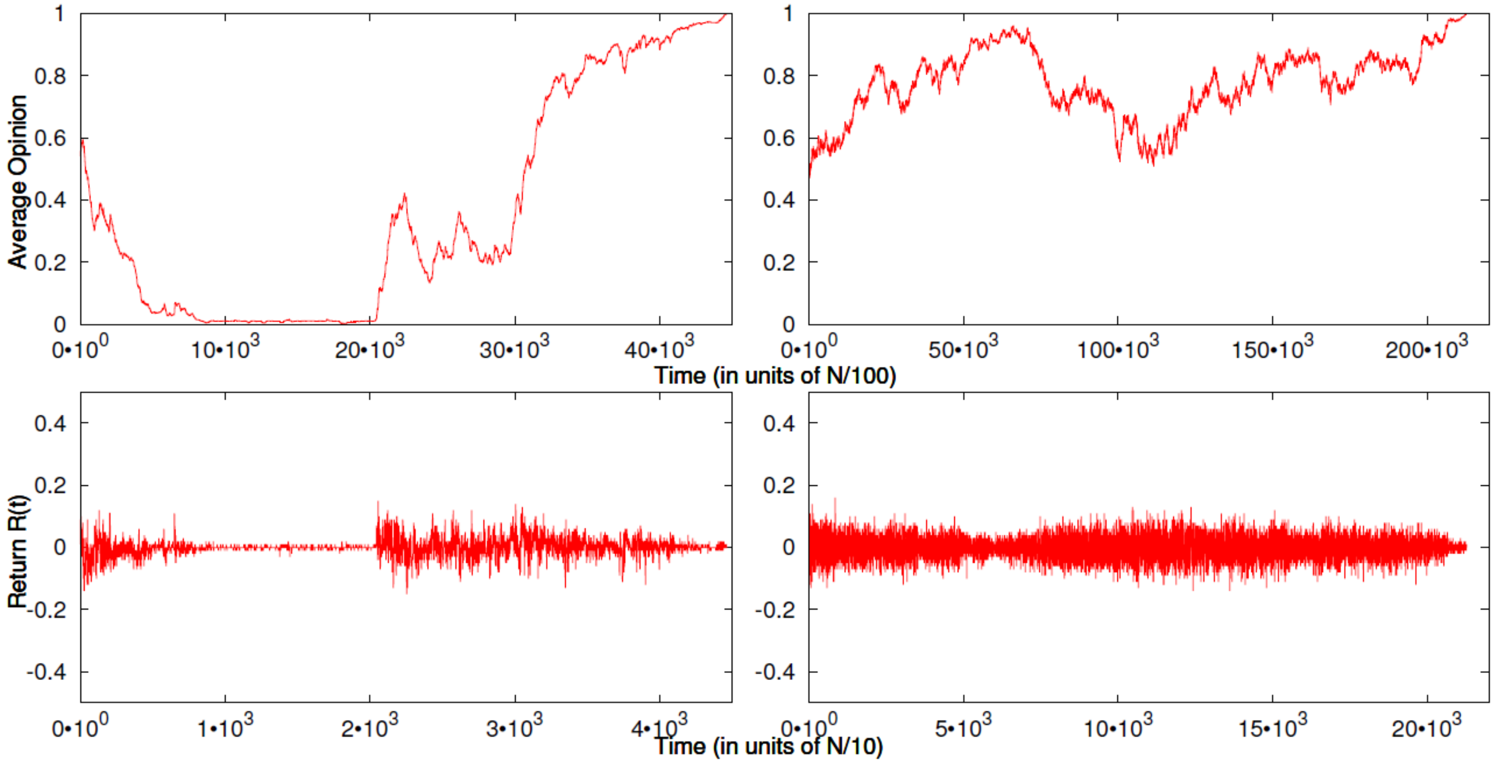


Figure 8: Average instantaneous opinion of the system (top) and associated return $R(t)$ (bottom) for two realizations of the preferential attachment voter model. Simulation parameters are $N = 1000$ and $m = 1$ (number of edges for each new node) for both cases, and $\sigma = 1$ (left) $\sigma = 0$ (right)

PREFERENTIAL ATTACHMENT GRAPHS

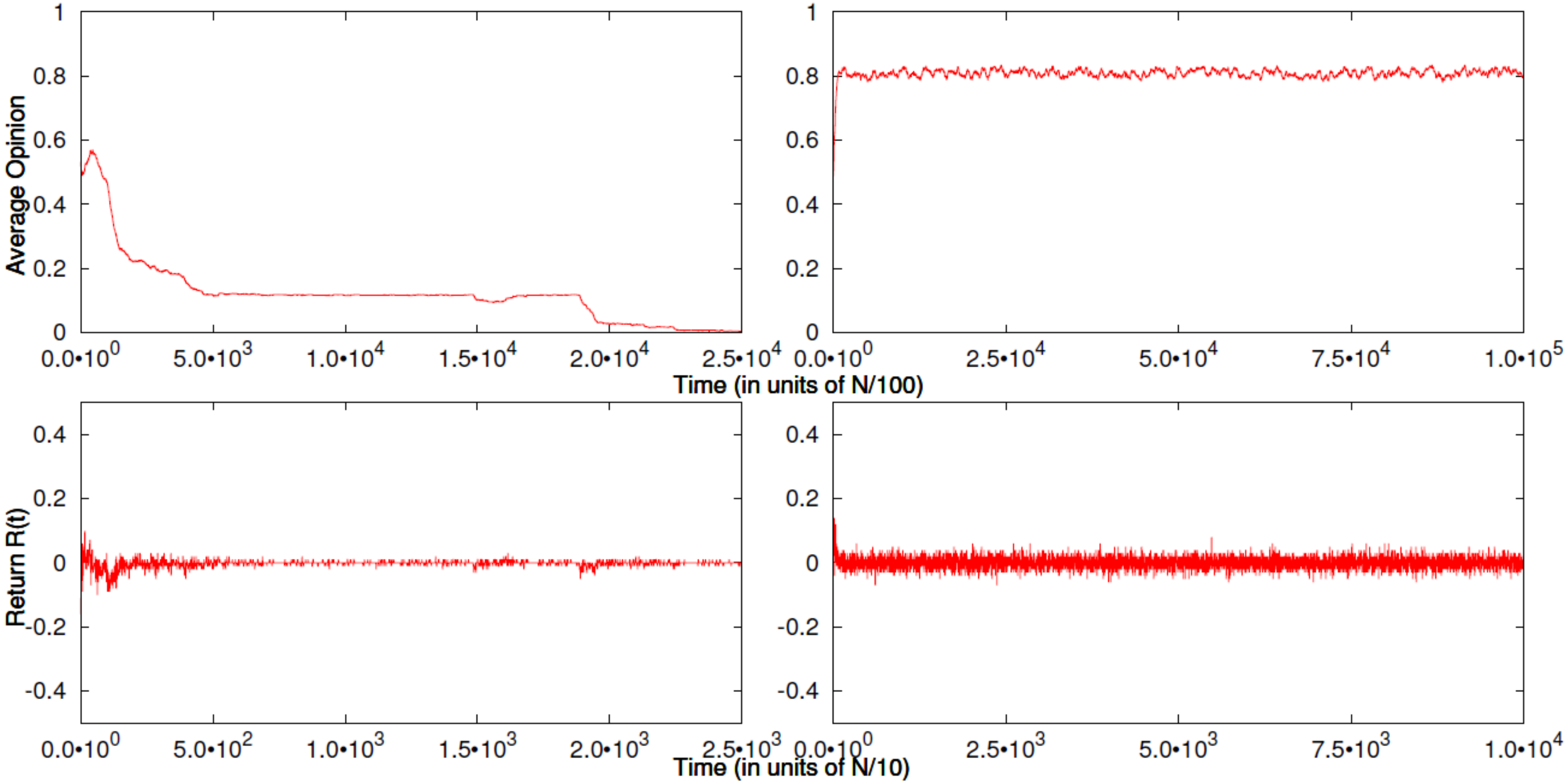
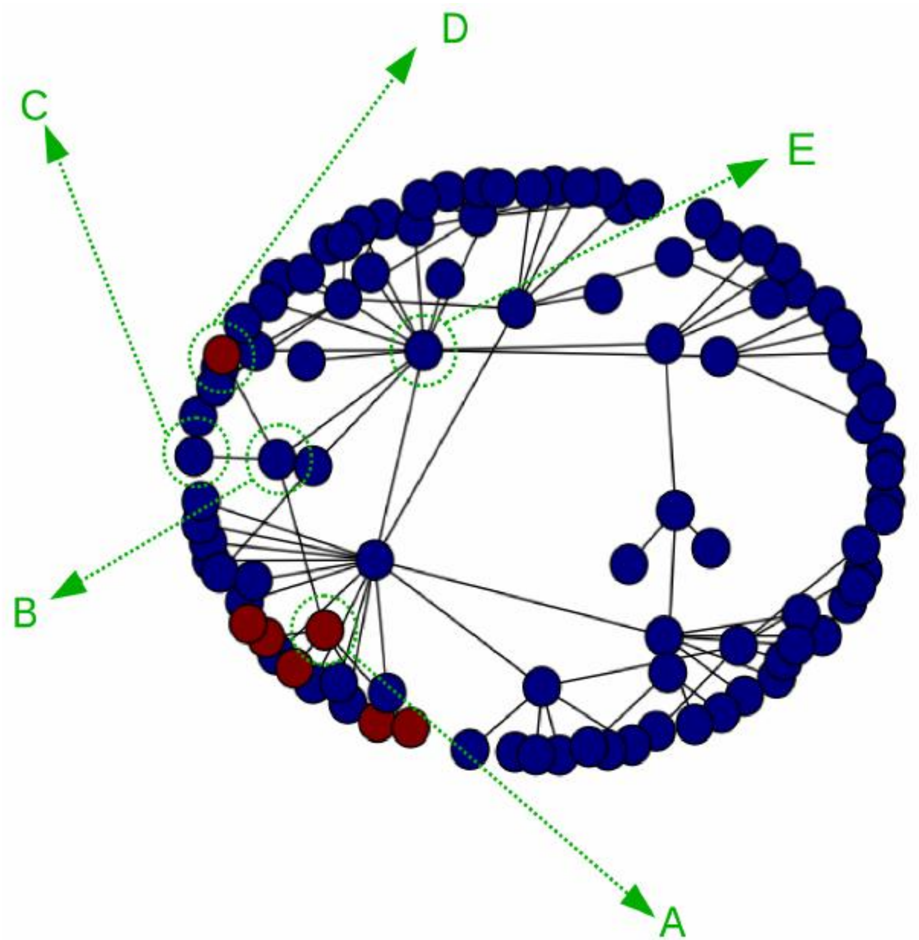


Figure 9: Average instantaneous opinion of the system (top) and associated return $R(t)$ (bottom) for two realizations of the preferential attachment voter model. Simulation parameters are $N = 1000$ and $m = 1$ (number of edges for each new node) for both cases, but $\sigma = 3$ (left) and $\sigma = 1000$ (right)

PREFERENTIAL ATTACHMENT GRAPHS

$$\lim_{\sigma\to\infty} P(j|i) = 1 \qquad \lim_{\sigma\to\infty} P(i|j) = 0, \qquad \text{for } d_j > d_i.$$



REVISITING THE σ-PROCESS

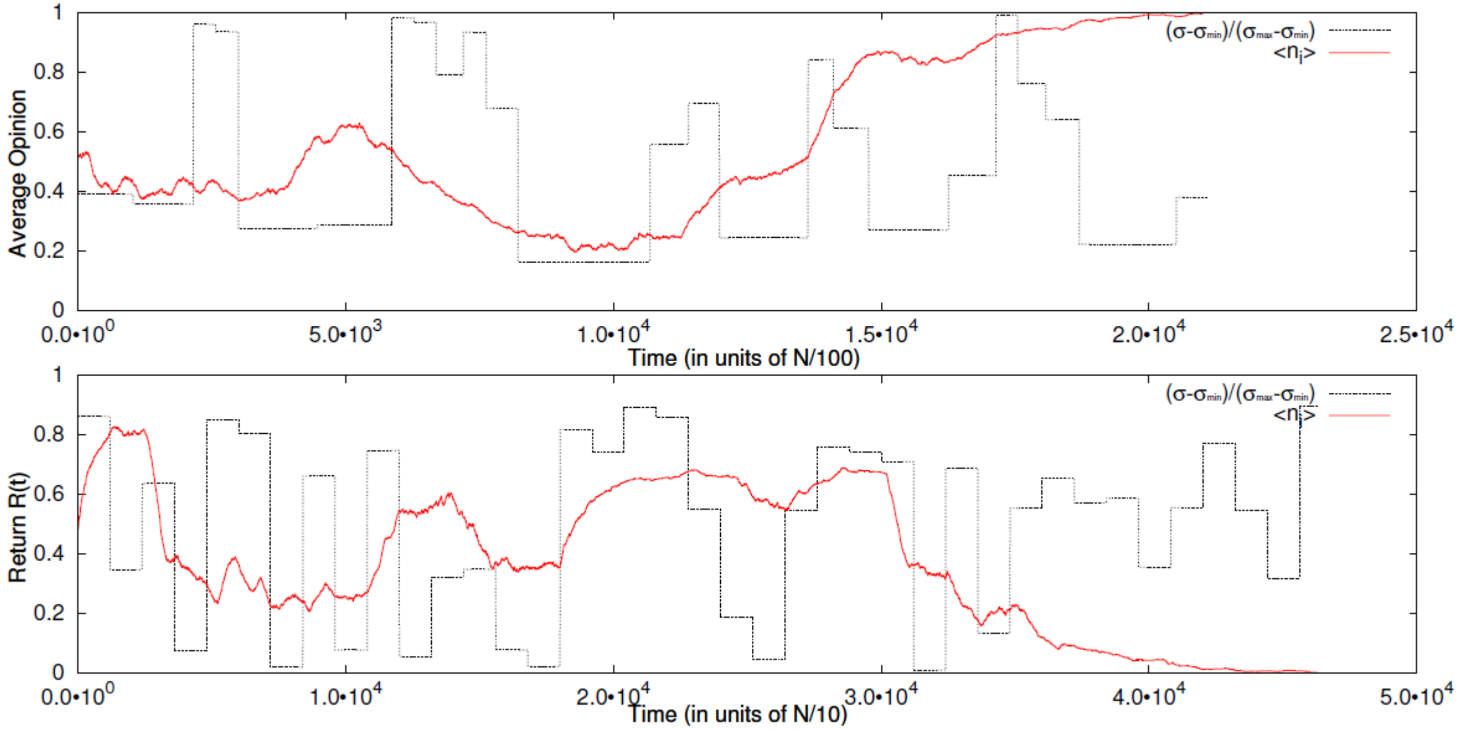


Figure 11: Average instantaneous opinion of the system for two realizations of the preferential attachment voter model with a σ-process. Simulation parameters are $N = 1000$ and $m = 1$ (number of edges for each new node) for both cases. Top plot has a stochastic $\lambda_\sigma(t)$ and bottom plot has a fixed $\lambda_\sigma = 6n$.

SOCIAL PRESSURE VOTER MODEL

$$n_i(t+dt) = n_i(t)[1-\xi_i(t)] + \eta_i\xi_i(t)$$

$$\xi_i(t) = \begin{cases} 1 & \text{with prob. } \lambda_i dt, \\ 0 & \text{with prob. } 1-\lambda_i dt \end{cases}$$

$$\eta_i(t) = \begin{cases} 1 & \text{with prob. } P_1^i(t) \\ 0 & \text{with prob. } 1-P_1^i(t) \end{cases}$$

$$P_1^i(t) = S_i(t)\langle n_j(t)\rangle_{j\in\nu_i} + (1-S_i(t))\mathcal{U}_i(t)$$

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$$\mathcal{U}_i(t) \sim U(0,1) \quad // \quad \mathcal{U}_i(t) = 1 - \sum_{j\in\nu_i} n_j(t)$$

$$\langle n_j(t)\rangle_{j\in\nu_i} = \sum_{j\neq i} P\{j|i\} n_j(t) \qquad P\{j|i\} = \frac{a_{ij} f(k_j^{out})}{\sum_k a_{ki} f(k_k^{out})}$$

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$$S_i(t) = f(k_i^{in}(t), k_i^{out}(t), \textstyle\sum_{j\in\nu_i} k_j^{out}(t)) \in [0,1]$$

$$S_i(t) = 1 - e^{-x}, \quad \text{where} \quad x = k_i^{in}(t)\frac{\sum_{j\in\nu_i} k_j^{out}(t)}{C + k_i^{out}(t)}$$

PRICE FORMATION AND NODE EARNINGS

-Price – Magnetization Coupling:

$$P(t) = \frac{K}{N}\sum_i n_i(t) \quad \text{where} \quad K \in [0, \infty)$$

-Log – Normal Random Variable:

$$P(t) = e^{\mu(t)+\sigma(t)Z} \quad \text{where} \quad Z \sim N(0,1)$$

$$\mu(t) = f(\vec{n}, G) \quad \sigma(t) = g(\vec{n}, G)$$

-Continuous Double Auction: Order Book Statistical Models

-Node Earnings $E_i(t+1) = E_i(t) \pm P(t+1)$

-Order Size, Activity Rate and Influence $\lambda_i \sim exp(\frac{k_i^{out}}{\lambda}), \quad \lambda \in \mathbb{R}^+$

COEVOLVING NETWORKS

-Inhomogeneous Dynamics Random Graph:

-Earnings-based Hierarchical Attachment:

$$P_{ij}(t) = \Psi(E_i(t), E_j(t))$$

$$\Psi(E_i(t), E_j(t)) = \mathbb{I}_{E_i < E_j} \frac{E_i - Ej}{\max_{i,j \in G} \Psi(E_i, E_j)}$$

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-Exogenous Approach: $E_i(t) \sim BM(\mu, \sigma) \quad \forall i \in G$

-Endogenous Approach: Social Pressure + Earnings + Coevolving Network