On evolving network models and their influence on opinion formation

Andrew Nugent, Susana Gomes, Marie-Therese Wolfram
MathSys CDT, University of Warwick   Mathematics Institute, University of Warwick

Webpage

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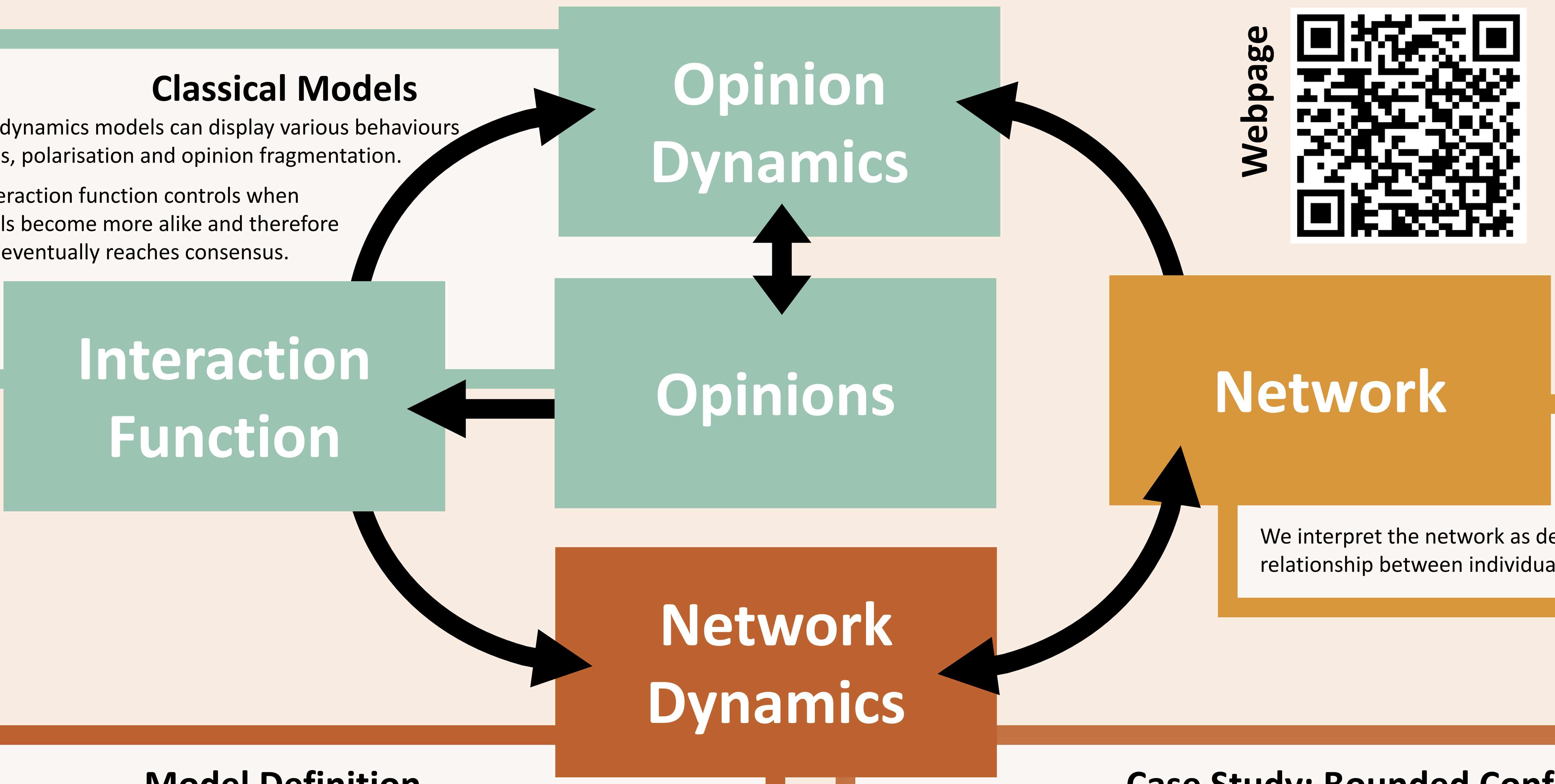
Classical Models

Classical opinion dynamics models can display various behaviours such as consensus, polarisation and opinion fragmentation.

The choice of interaction function controls when pairs of individuals become more alike and therefore if the population eventually reaches consensus.



The network limits communication between individuals, but adds additional structure.

We interpret the network as describing the level of trust and the relationship between individuals.

Model Definition

A continuous time model with opinions in the interval $x_i \in [-1,1]$ and weights $w_{ij} \in [0,1]$.

The interaction function $\phi: [0,2] \to [0,1]$ describes how the distance between individuals' opinions affects their perceived value.

Strength of the relationship between individuals i and j

$$\frac{dx_i}{dt} = \frac{1}{k_i}\sum_j \phi(|x_i - x_j|)\, w_{ij}\,(x_j - x_i)$$

Normalisation by node degree

$$k_i = \sum_j w_{ij}$$

The interaction function balances a growth term and a decay term in the weight dynamics.

Growth and decay terms also ensure weights remain in the interval $[0,1]$.

Memory weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij} = \phi(|x_i - x_j|) - w_{ij}$$

Logistic weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\, w_{ij}(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}(1 - w_{ij}) = \left(2\phi(|x_i - x_j|) - 1\right) w_{ij}(1 - w_{ij})$$

Friend-of-a-friend (FOAF) weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\left(w_{ij} + \lambda (W^2)_{ij}\right)(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}$$

FOAF weight dynamics allow for triadic closure, as demonstrated below.

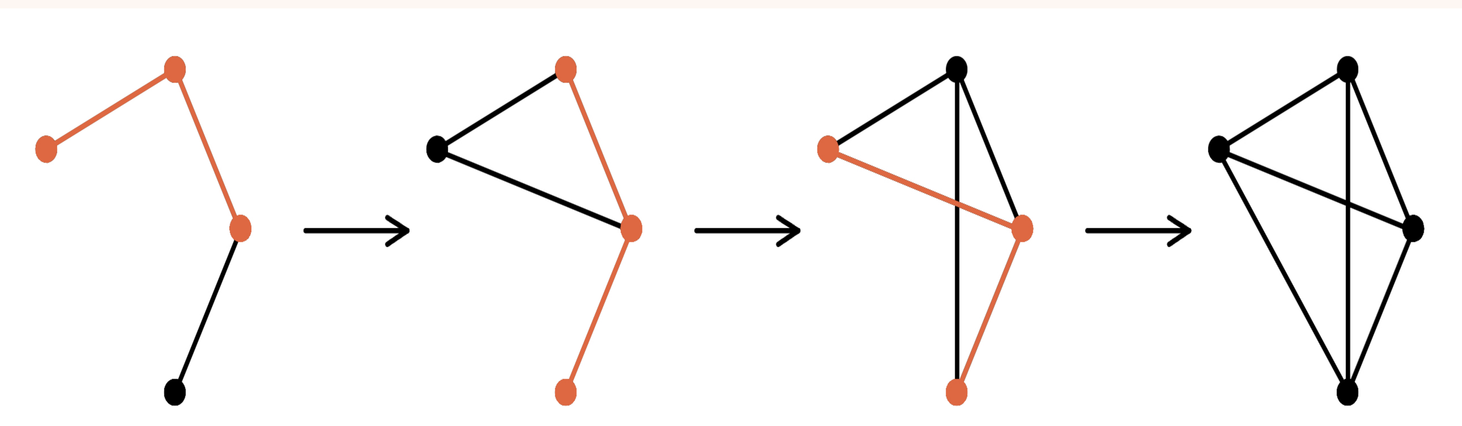


Figure 1: Diagram showing the formation of new edges through triadic closure. Orange nodes and edges indicate the triad that is to be closed.

Extreme Timescales

We now consider a continuous interaction function and investigate the relative timescales of opinion and weight dynamics by multiplying the latter by a factor $\tau > 0$. Extreme timescales correspond to the following limits,

- As $\tau \to 0$ the system approaches the model with weights fixed at their initial values.
- As $\tau \to \infty$ the network is overcome, and the system approaches a new model

$$\frac{dX_i}{dt} = \frac{1}{K_i}\sum_j \phi(|X_i - X_j|)^2\,(X_j - X_i)$$

$$K_i = \sum_j \phi(|X_i - X_j|)$$

Simulations indicate that at intermediate timescales there is a transition between these two extremes.

Case Study: Bounded Confidence

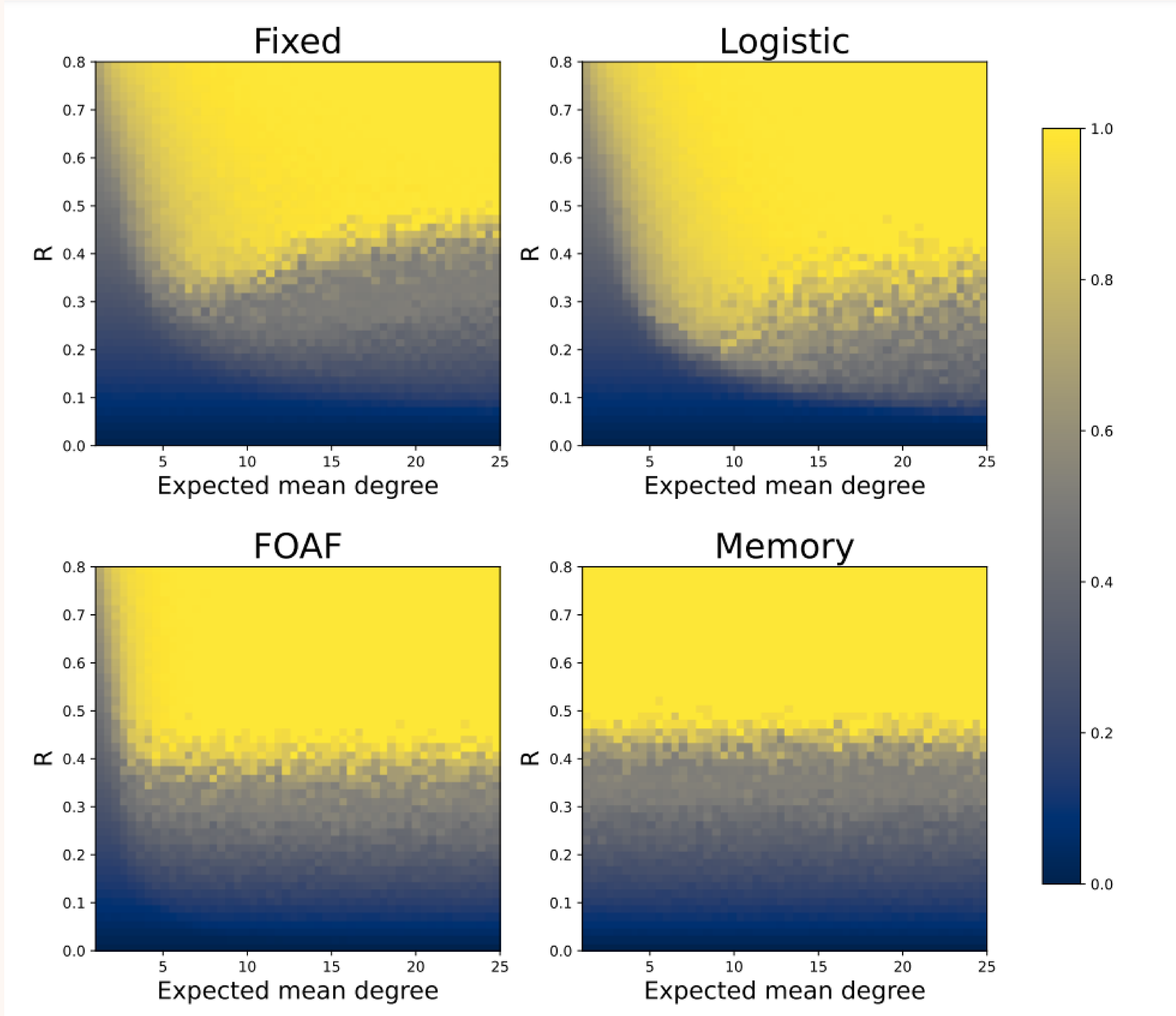
We investigate simulations of the model using the common bounded confidence interaction function, as in the classical Hegselmann-Krause model.

$$\phi(|x_i - x_j|) = \begin{cases} 1 \text{ if } |x_i - x_j| < R \\ 0 \text{ if } |x_i - x_j| \geq R \end{cases}$$

At the end of each simulation the order parameter Q describes opinion spread, with Q^{-1} giving approximately the number of opinion clusters.

$$Q = \frac{1}{N}\sum_{i,j} \phi(|x_i - x_j|)$$

Initial opinions are chosen uniformly at random on $[-1,1]$. Initial networks are Erdos-Renyi random networks with varying mean degrees. The population size $N = 500$.



Caption: Heatmaps showing the order parameter at the end of simulations for each type of weight dynamics. For each cell ten simulations are run. Yellow areas indicate consensus, grey indicates polarisation and blue indicates a high number of opinion clusters.

With fixed weights the relationship between initial mean degree and Q is not monotonic, indicating a potential `ideal' level of connectivity.

As is typical with bounded confidence, higher values of R lead to greater agreement.

As logistic weight dynamics do not create any new edges their effect is minimal.

FOAF weight dynamics have greater impact, largely removing the effect of the initial mean degree. The memory weight dynamics eliminate this effect entirely, although this does not always lead to greater consensus as in some parameter regions the `ideal' connectivity is lost.

Summary

- A new model of opinion and network dynamics in which the interaction function **balances growth and decay terms** for each edge weight.
- Network dynamics can **both create and prevent consensus**, depending on the choice of interaction function and mechanisms for edge creation.
- The **relative timescale** of opinion and network dynamics has a major impact, with extreme timescales mirroring other models.