On evolving network models and their influence on opinion formation

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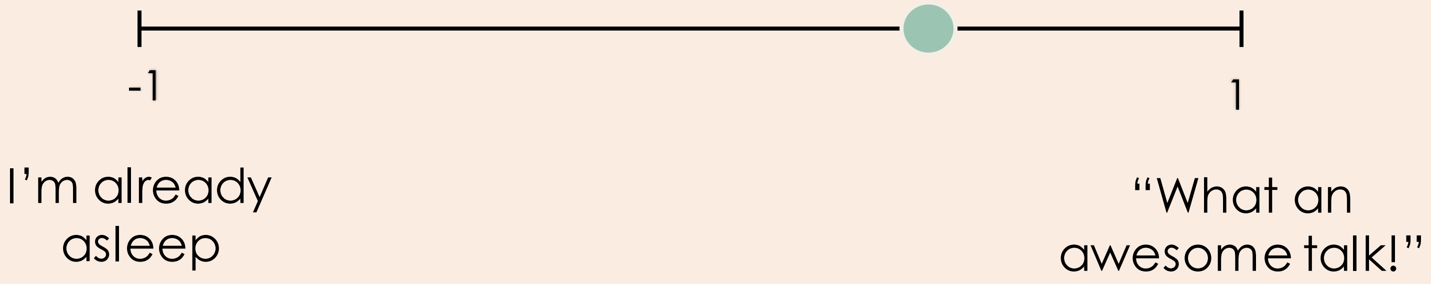
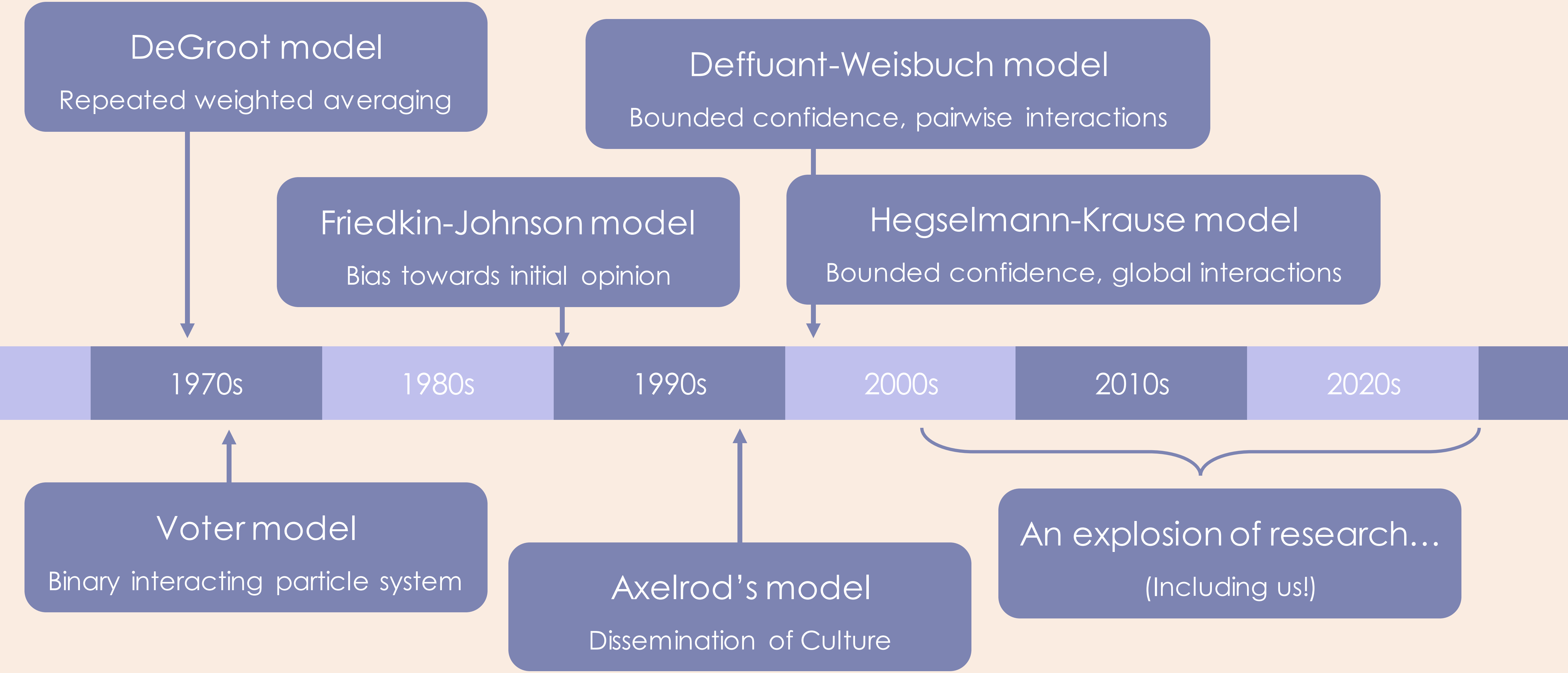
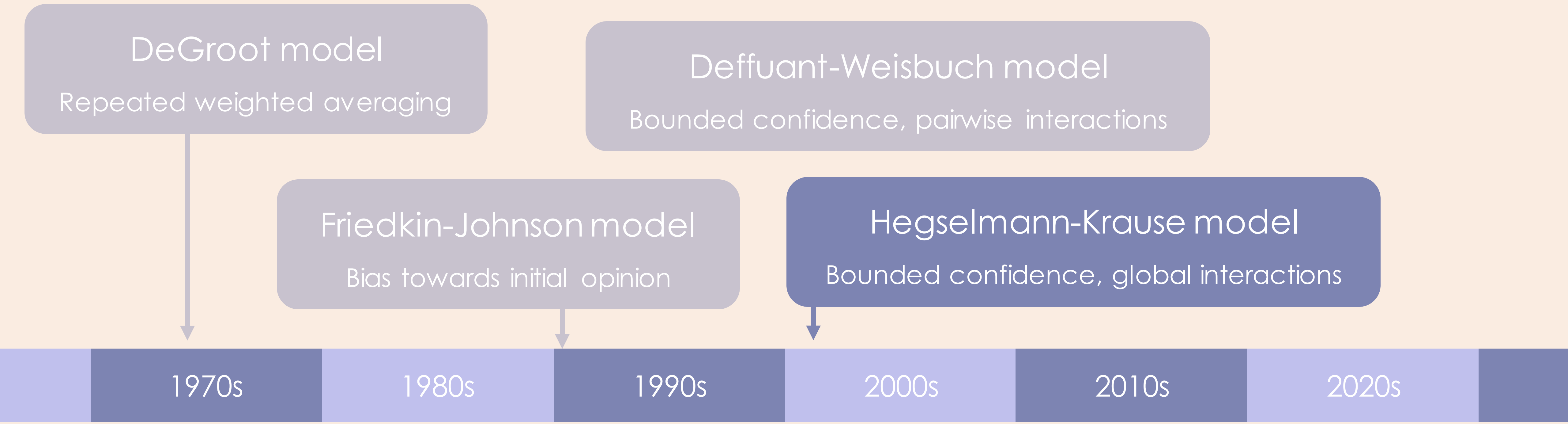
In this talk I will introduce a novel model of opinion dynamics that couples an opinion formation model with a general interaction function to a coevolving social network in which individuals build relationships through continued meaningful interaction.

Opinions





Axelrod's Puzzle: "If people tend to become more alike in their beliefs, attitudes, and behaviour when they interact, why do not all such differences eventually disappear?"

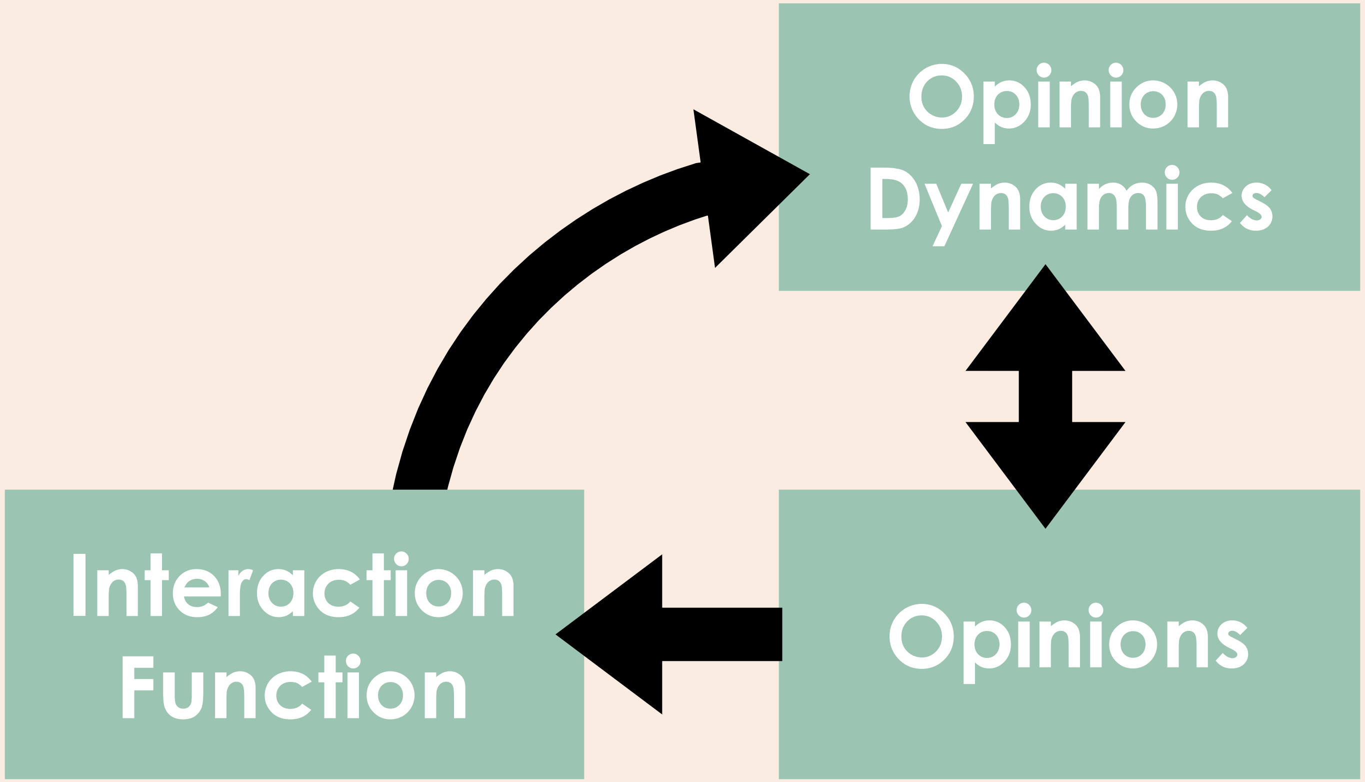


Bounded confidence set:

$$I(x,i) = \{ j : |x_i - x_j| < R \}$$

Bounded confidence dynamics:

$$\frac{dx_i}{dt} = \frac{1}{|I(x,i)|} \sum_{j \in I(x,i)} (x_j - x_i)$$


Opinion
Dynamics
Interaction
Function
Opinions

Bounded confidence dynamics:

$$\frac{dx_i}{dt} = \frac{1}{|I(x,i)|} \sum_{j \in I(x,i)} (x_j - x_i)$$

Define the interaction function:

$$\phi_R(|x_i - x_j|) = \begin{cases} 1 \text{ if } |x_i - x_j| < R \\ 0 \text{ if } |x_i - x_j| \geq R \end{cases}$$

Bounded confidence dynamics:

$$\frac{dx_i}{dt} = \frac{\sum_j \phi_R(|x_i - x_j|)(x_j - x_i)}{\sum_j \phi_R(|x_i - x_j|)}$$

Bounded confidence dynamics:

$$\frac{dx_i}{dt} = \frac{\sum_j \phi_R(|x_i - x_j|)(x_j - x_i)}{\sum_j \phi_R(|x_i - x_j|)}$$

Simplify the normalisation

Bounded confidence dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N}\sum_j \phi_R(|x_i - x_j|)(x_j - x_i)$$

Bounded confidence dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_j \phi_R(|x_i - x_j|)(x_j - x_i)$$

Define a general interaction function:

$$\phi(|x_i - x_j|) : [0,2] \to [0,1]$$

General opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_j \phi(|x_i - x_j|)(x_j - x_i)$$

Bounded confidence dynamics:

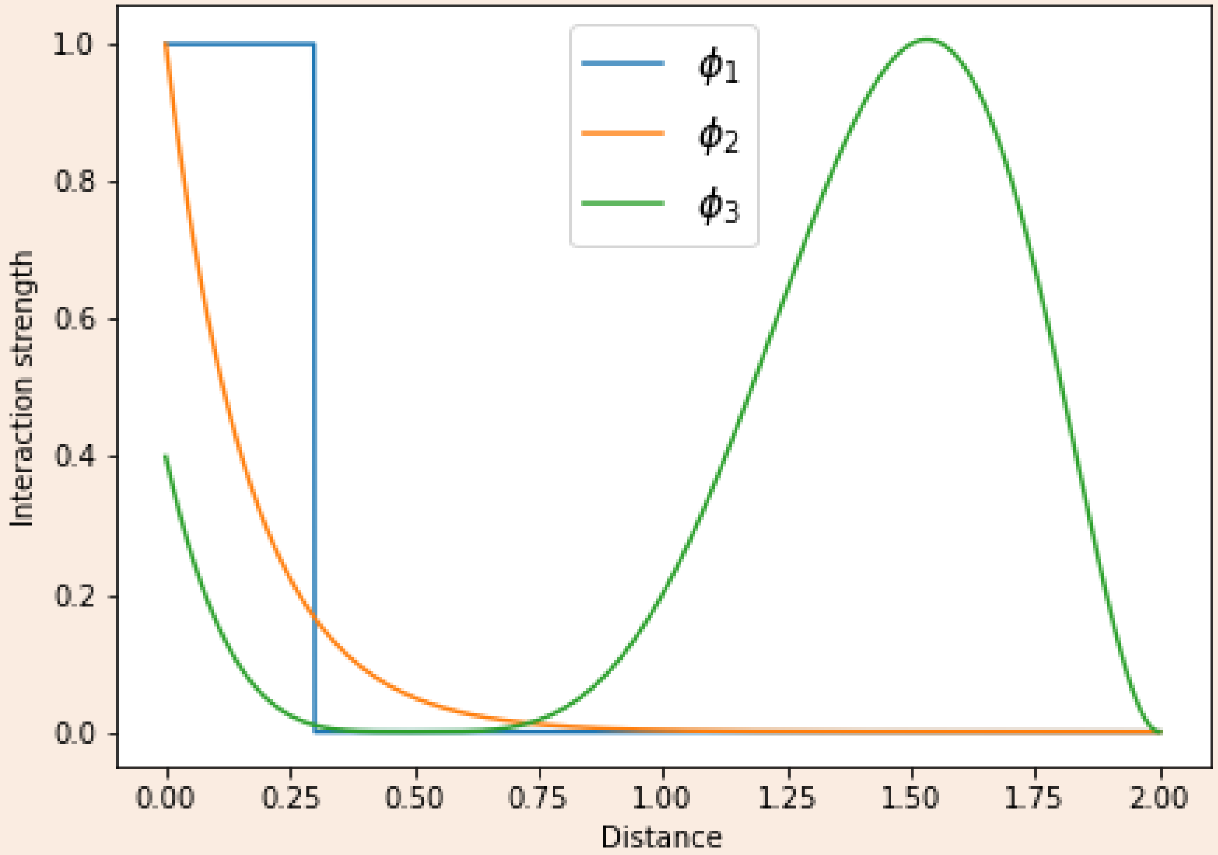
$$\frac{dx_i}{dt} = \frac{1}{|I(x,i)|} \sum_{j \in I(x,i)} (x_j - x_i)$$

General opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_{j} \phi(|x_i - x_j|)(x_j - x_i)$$

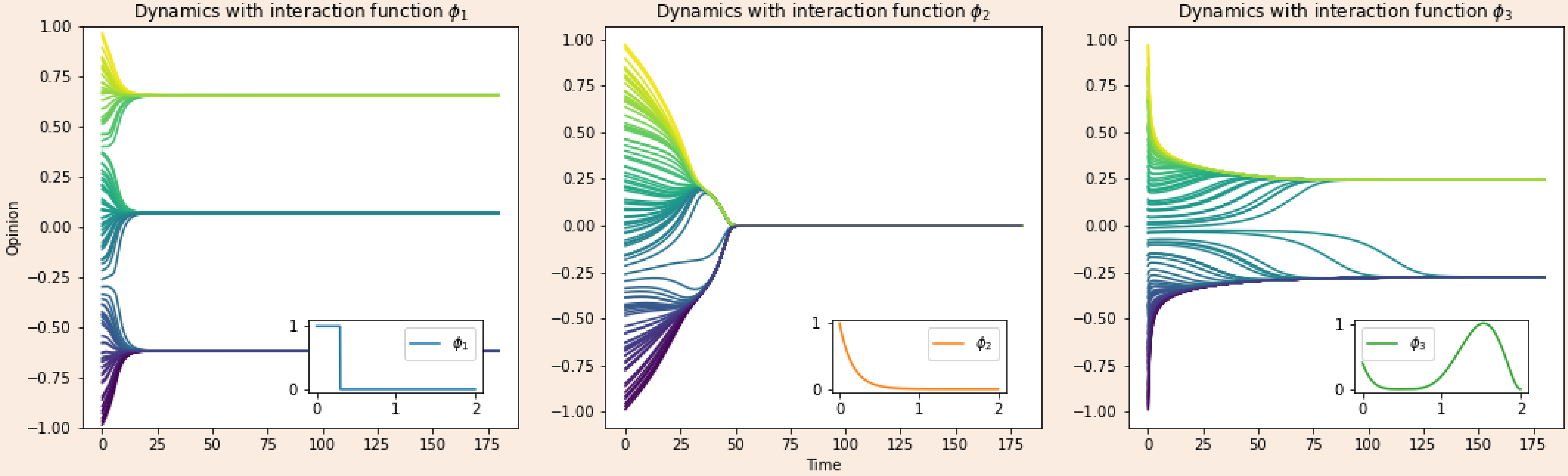
What does ϕ now represent?

- The attention/value you give somebody's opinion?
- How much an opinion influences you?
- How much dissonance/discomfort the difference in opinion creates?
- Some unspecified mix of these effects?



Example interaction functions:

$$\phi_1(r) = \begin{cases} 1 \ if \ r \leq 0.3 \\ 0 \ if \ r \geq 0.3 \end{cases}, \quad \phi_2(r) = e^{-6r}, \ \phi_3(r) = \frac{8}{5}\left(r - \frac{1}{2}\right)^2 (r+1)(r-2)^2$$



Example interaction functions:

$$\phi_1(r) = \begin{cases} 1 \; if \; r \leq 0.3 \\ 0 \; if \; r \geq 0.3 \end{cases}, \quad \phi_2(r) = e^{-6r}, \; \phi_3(r) = \frac{8}{5}\left(r - \frac{1}{2}\right)^2 (r+1)(r-2)^2$$

<u>Definition:</u> The **opinion diameter** is given by:

$$D(t) = \max_{i,j} \left| x_j(t) - x_i(t) \right|.$$

<u>Definition:</u> The population reaches **consensus** if:

$$\lim_{t \to \infty} D(t) = 0.$$

General opinion dynamics:

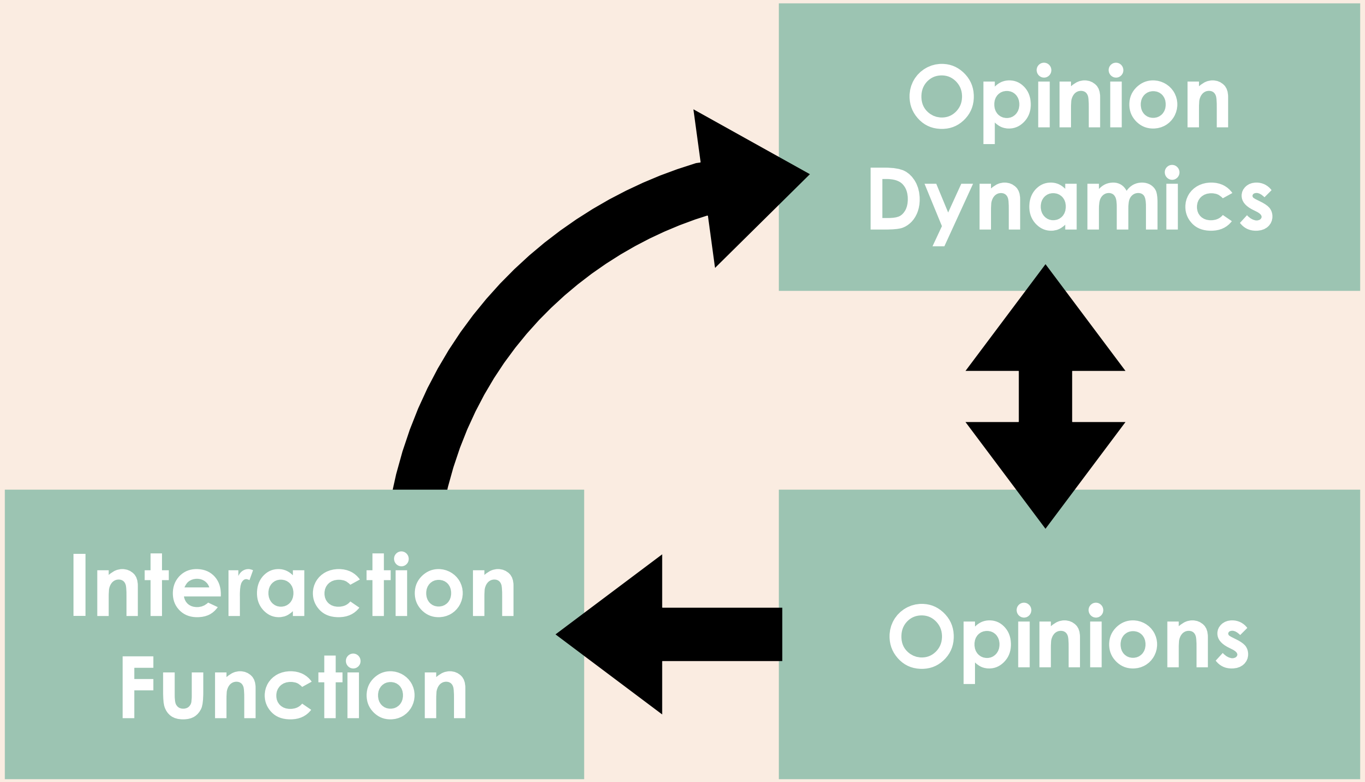
$$\frac{dx_i}{dt} = \frac{1}{N} \sum_j \phi(|x_i - x_j|)(x_j - x_i)$$

<u>Proposition:</u> $x_i(t) \in [-1,1]$ for all $t \geq 0$. Additionally, $D(t)$ converges to some value in $[0,2]$ as $t \to \infty$.

<u>Proposition</u>: For any $\epsilon > 0$ there exists a time t^* at which, for all pairs of individuals i and j

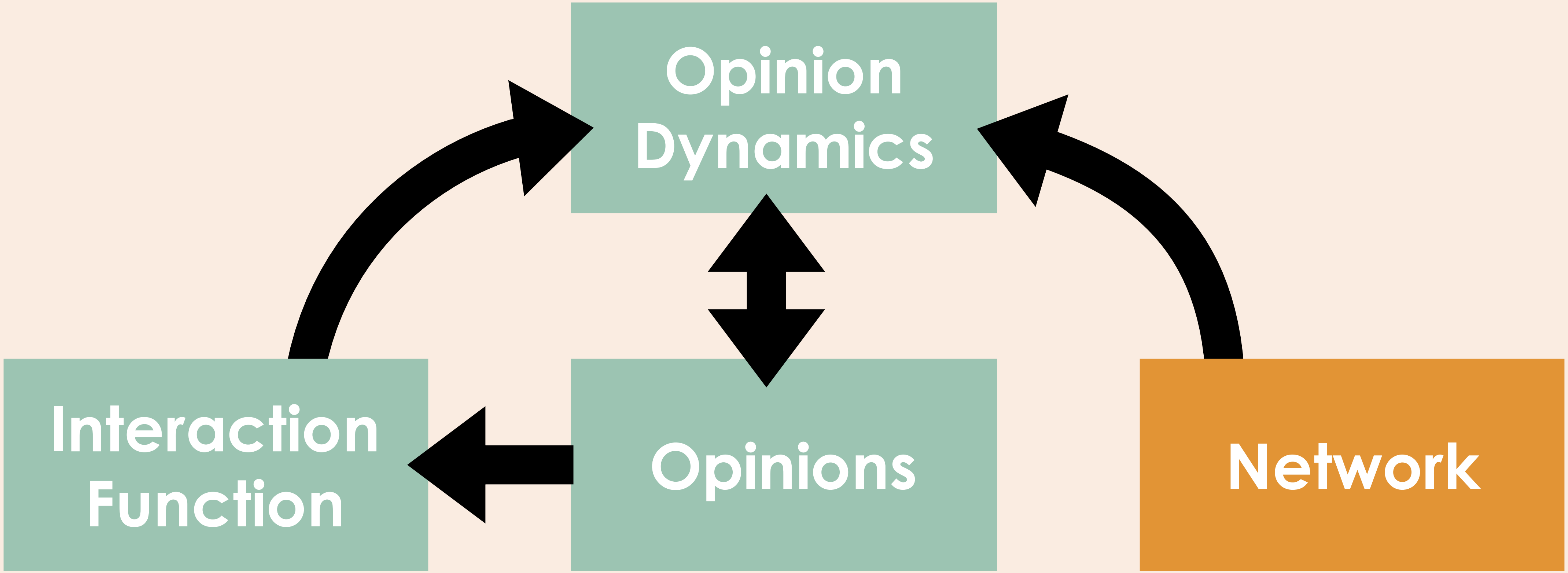
$$\phi\left(\left|x_j(t^*) - x_i(t^*)\right|\right) \left|x_j(t^*) - x_i(t^*)\right|^2 < \epsilon.$$

<u>Proposition</u>: If there exists a constant $c > 0$ such that $\phi(r) > c$ for all $r \in [0,2]$ then **consensus is guaranteed** for any $x(0)$.



So far…

- Seen some history of opinion dynamics
- Constructed a **general model**
- Observed both **consensus** and **polarisation**.


Opinion Dynamics
Interaction Function
Opinions
Network

General opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N}\sum_j \phi(|x_i - x_j|)(x_j - x_i)$$

Introduce a network: w

Network opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N}\sum_j w_{ij}\,\phi(|x_i - x_j|)(x_j - x_i)$$

Network opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_j w_{ij}\, \phi(|x_i - x_j|)(x_j - x_i)$$

Account for network in normalisation

$$k_i = \sum_j w_{ij}$$

Network opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{k_i} \sum_j w_{ij}\, \phi(|x_i - x_j|)(x_j - x_i)$$

General opinion dynamics:

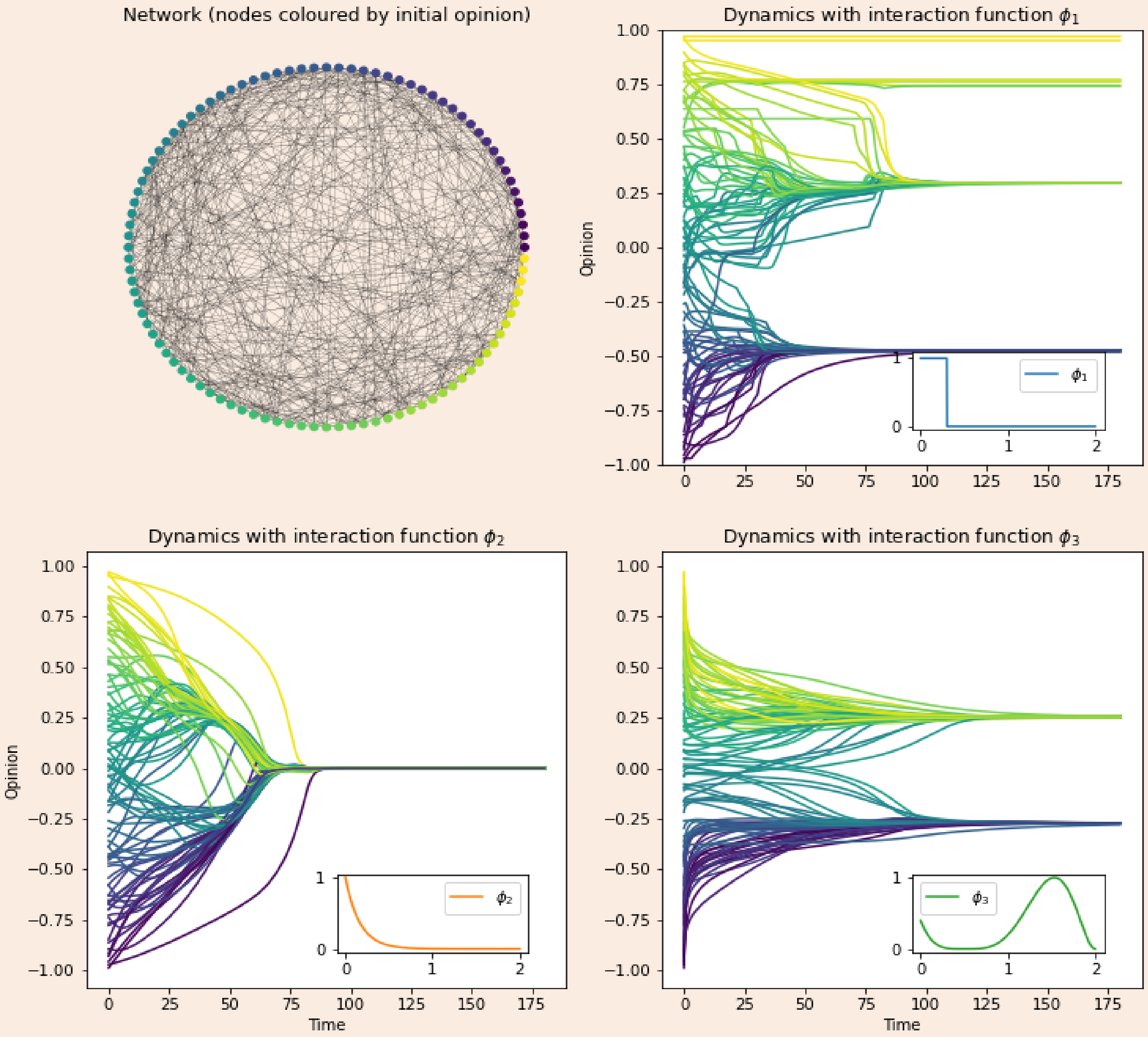
$$\frac{dx_i}{dt} = \frac{1}{N}\sum_j \phi(|x_i - x_j|)(x_j - x_i)$$

Network opinion dynamics:

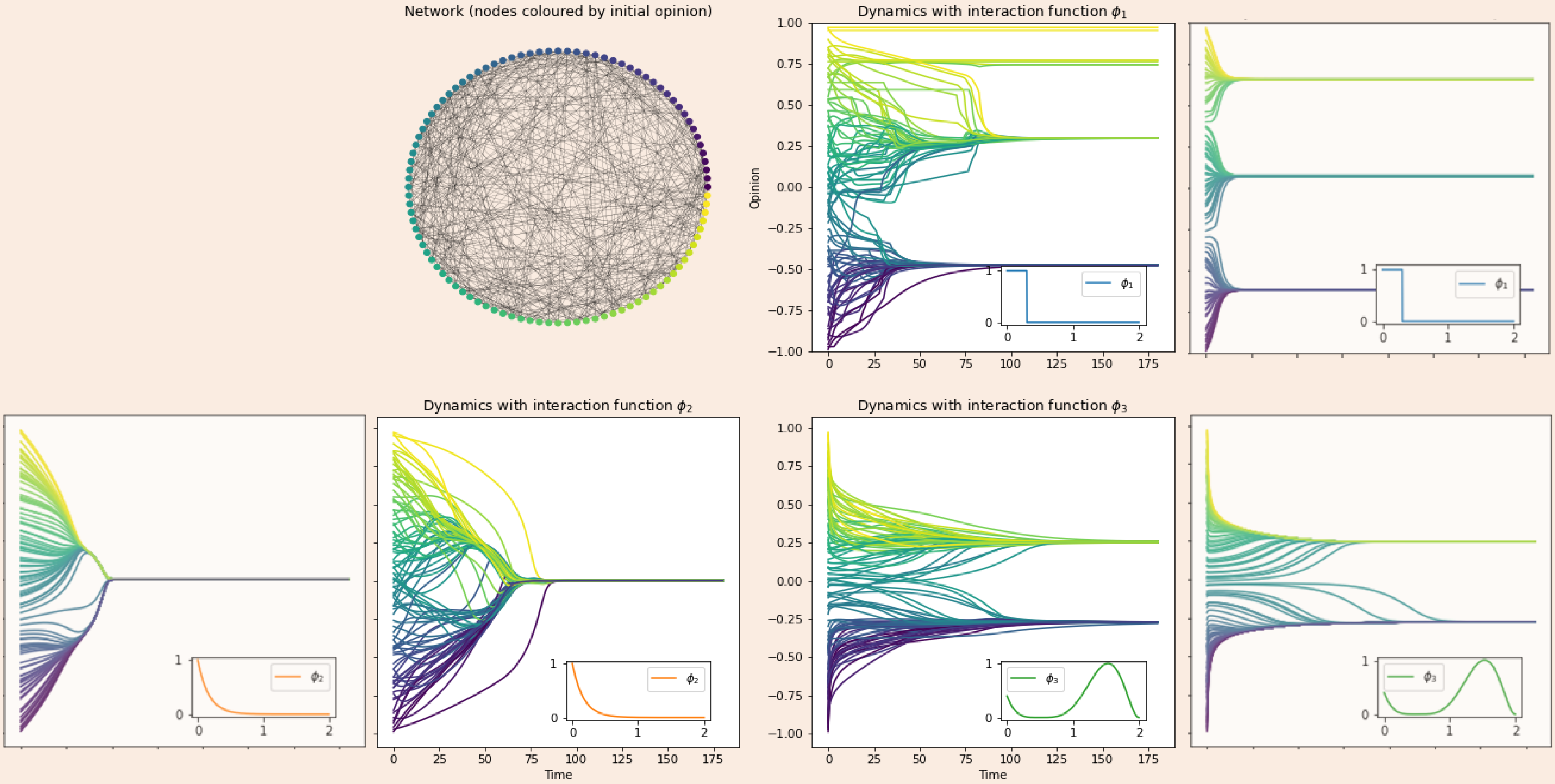
$$\frac{dx_i}{dt} = \frac{1}{k_i}\sum_j w_{ij}\, \phi(|x_i - x_j|)(x_j - x_i)$$

What does w represent?

- Spatial constraints?
- To indicate expertise?
- Social relationships (e.g. trust, confidence)?



Network (nodes coloured by initial opinion)
Dynamics with interaction function ϕ_1
Dynamics with interaction function ϕ_2
Dynamics with interaction function ϕ_3
Opinion
Time
ϕ_1
ϕ_2
ϕ_3


Network (nodes coloured by initial opinion)
Dynamics with interaction function ϕ_1
Opinion
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
25
50
75
100
125
150
175
ϕ_1
Dynamics with interaction function ϕ_2
Time
ϕ_2
Dynamics with interaction function ϕ_3
Time
ϕ_3

Definition: The opinion diameter is given by:

$$D(t) = \max_{i,j} \left| x_j(t) - x_i(t) \right|.$$

Definition: The population reaches consensus if:

$$\lim_{t \to \infty} D(t) = 0.$$

Network opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{k_i} \sum_j w_{ij}\, \phi\left(\left|x_i - x_j\right|\right)\left(x_j - x_i\right)$$

Proposition: $x_i(t) \in [-1,1]$ for all $t \geq 0$. Additionally, $D(t)$ converges to some value in $[0,2]$ as $t \to \infty$.

Proposition: For any $\epsilon > 0$ there exists a time t^* at which, for all pairs of individuals i and j

$$w_{ij}\, \phi\left(\left|x_j(t^*) - x_i(t^*)\right|\right) \left|x_j(t^*) - x_i(t^*)\right|^2 < \epsilon.$$

Proposition: If **w is connected** and there exists a constant $c > 0$ such that $\phi(r) > c$ for all $r \in [0,2]$ then **consensus is guaranteed** for any $x(0)$.

Case Study: Bounded Confidence

Bounded confidence on a network:

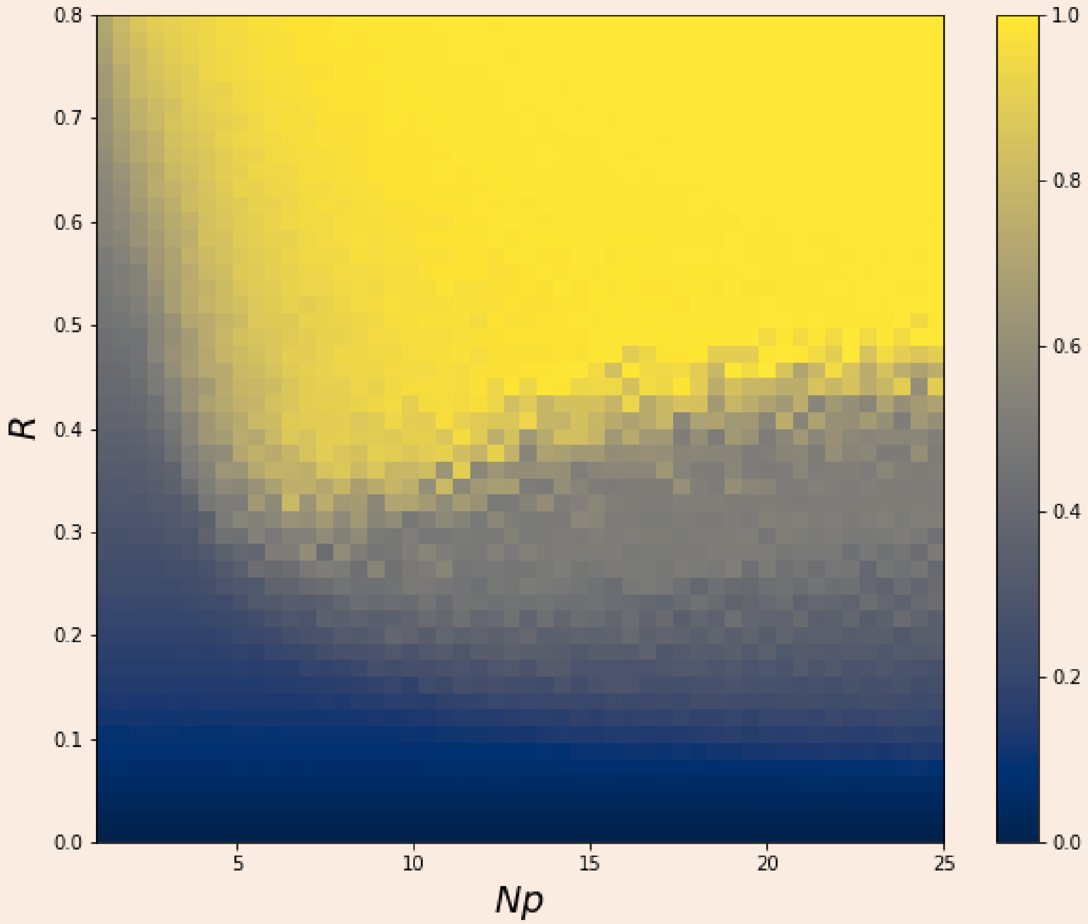
$$\frac{dx_i}{dt} = \frac{1}{k_i}\sum_j w_{ij}\,\phi_R(|x_i - x_j|)(x_j - x_i)$$

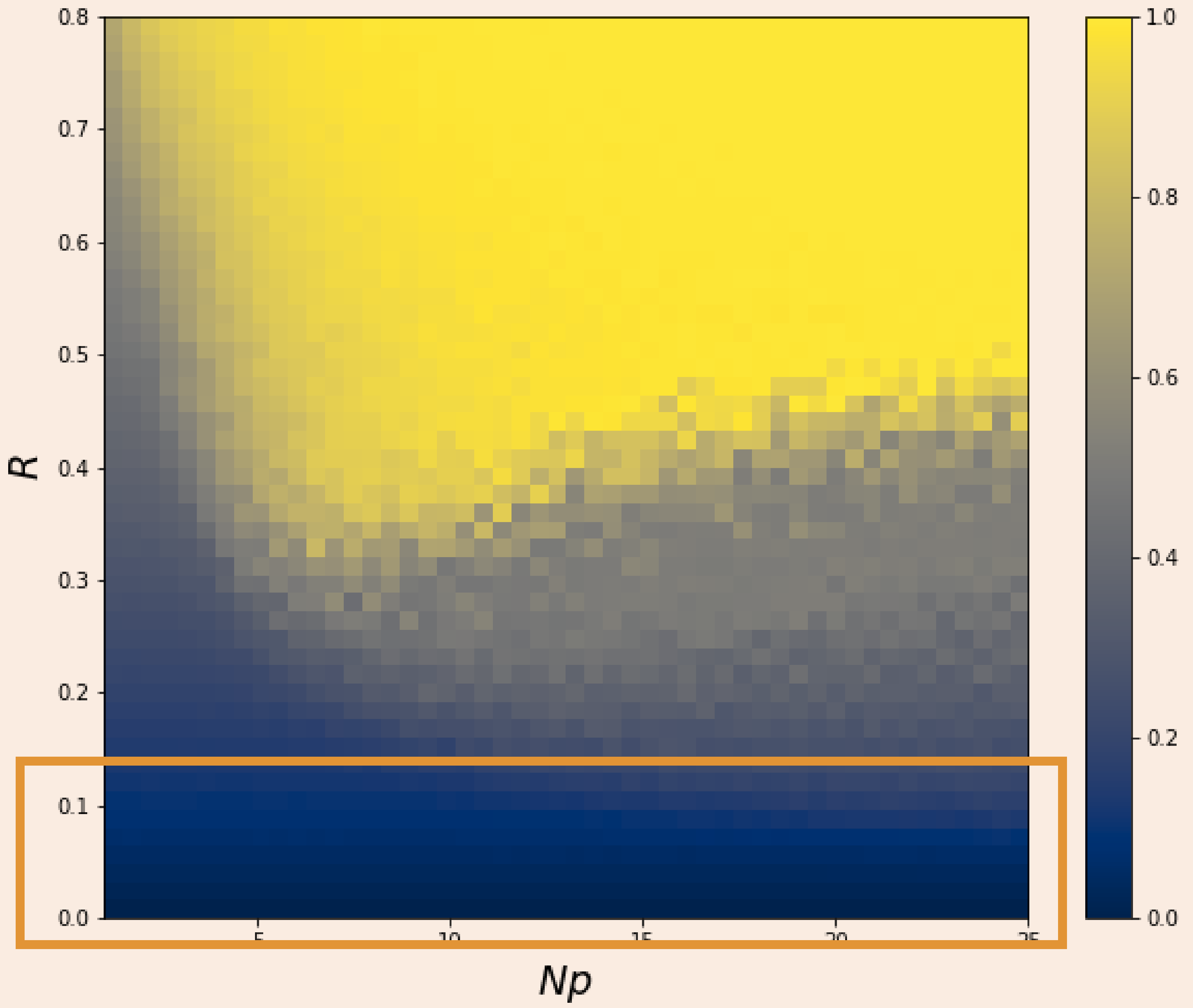
Networks:

- Using Erdos-Renyi random networks with edge probability p.
- The expected number of connections for each node is Np.

Order parameter:

$$Q = \frac{1}{N^2}\sum_{i,j}\phi_R(|x_i - x_j|)$$





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R
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Np
1.0
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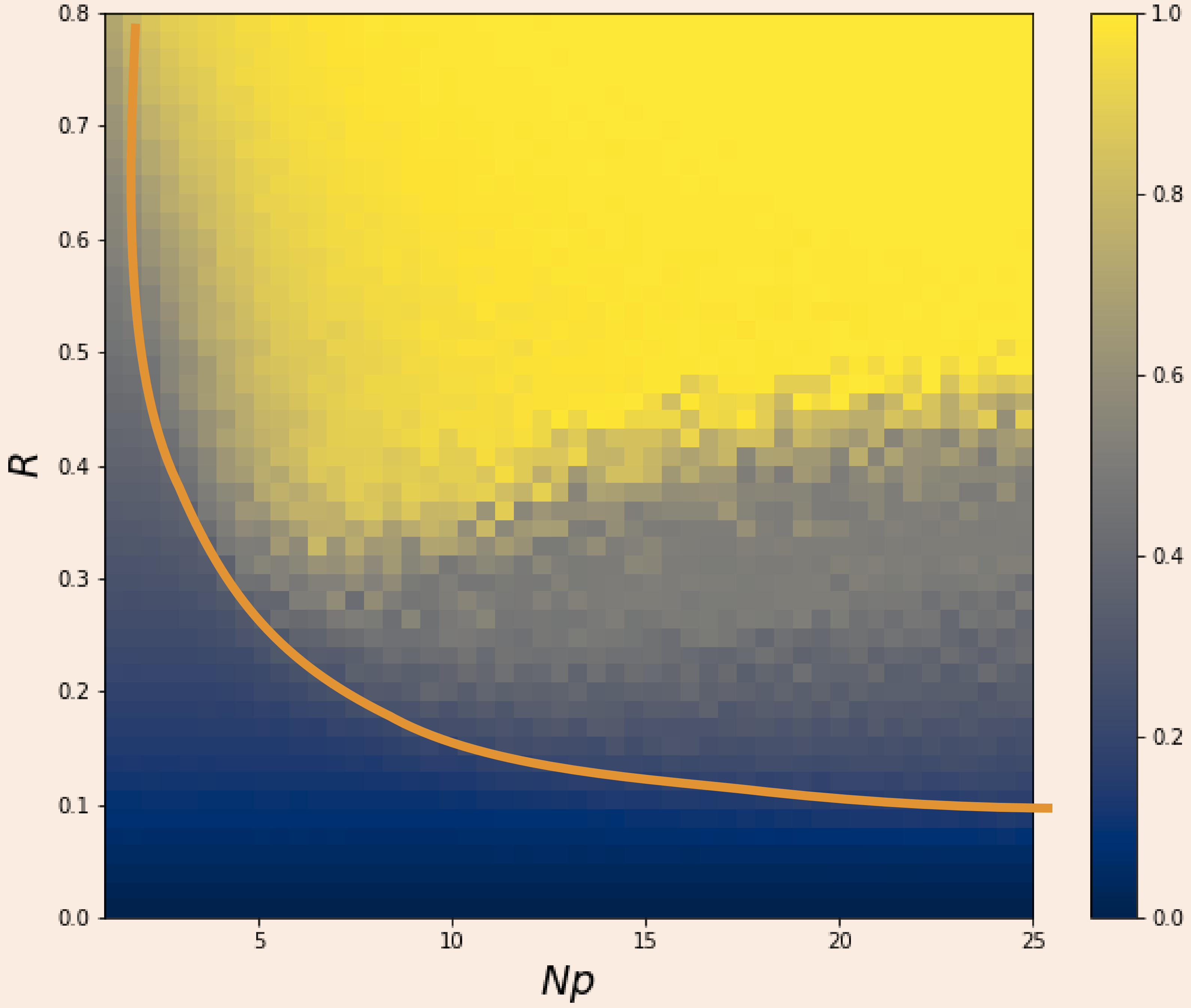
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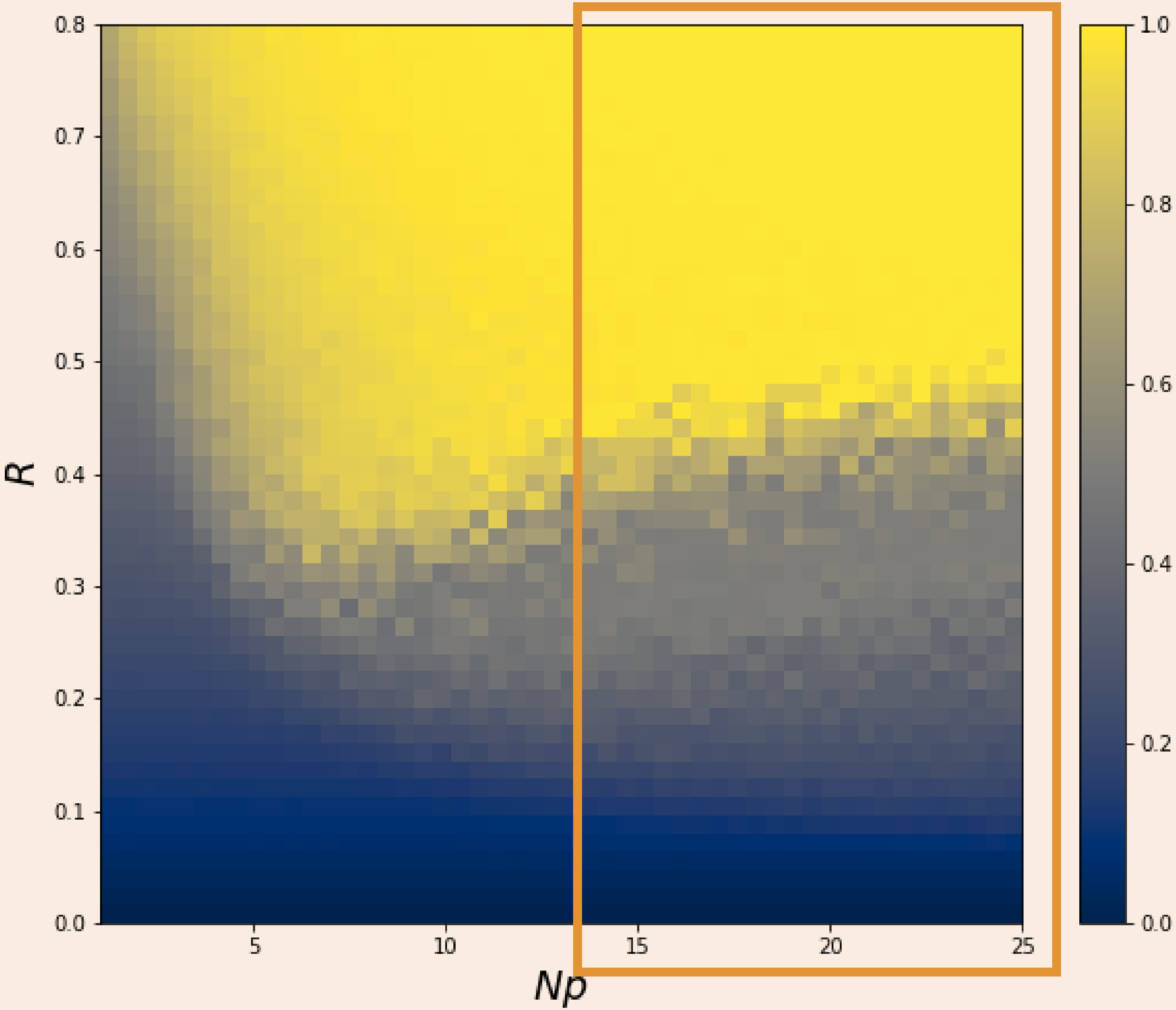

R
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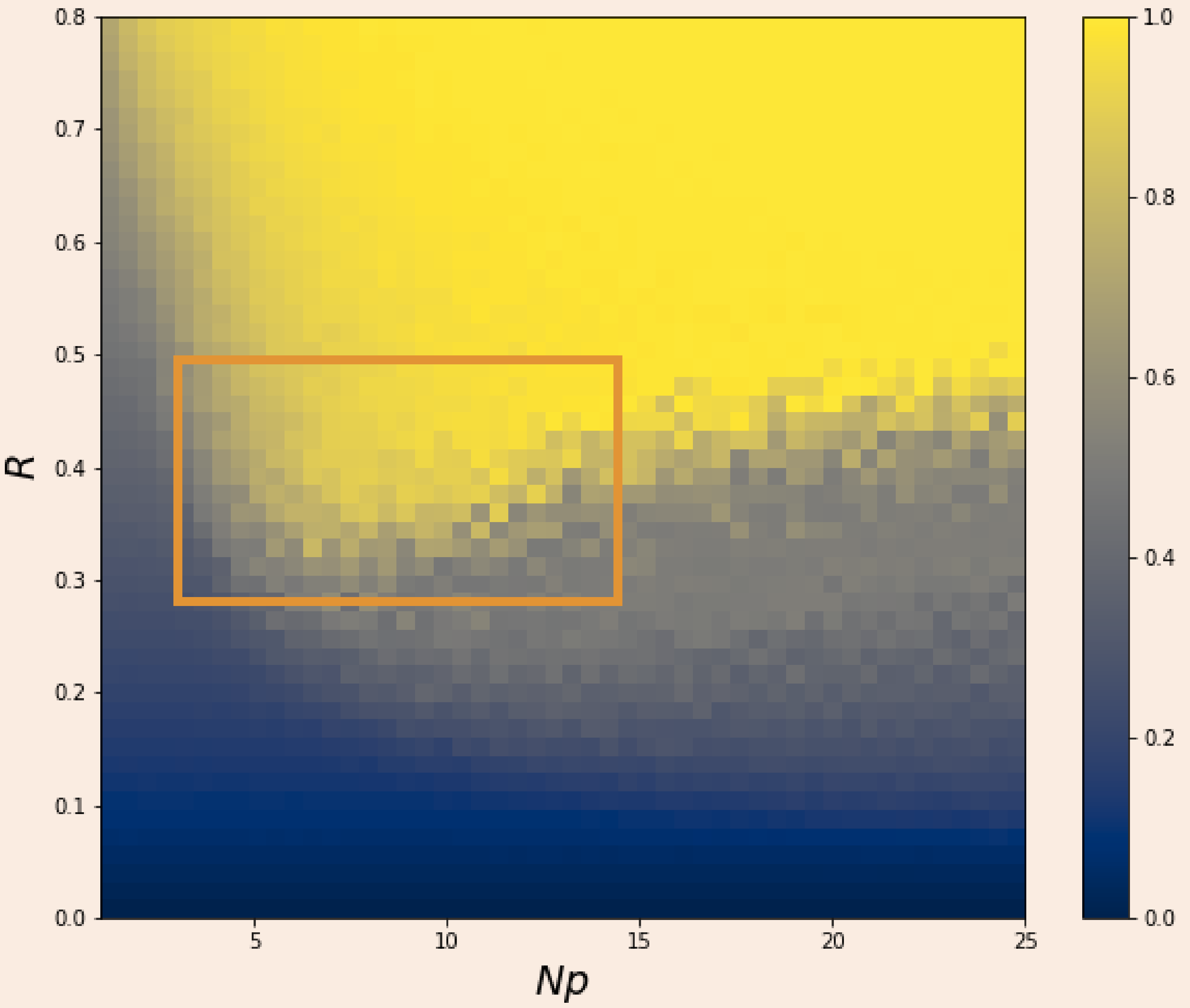

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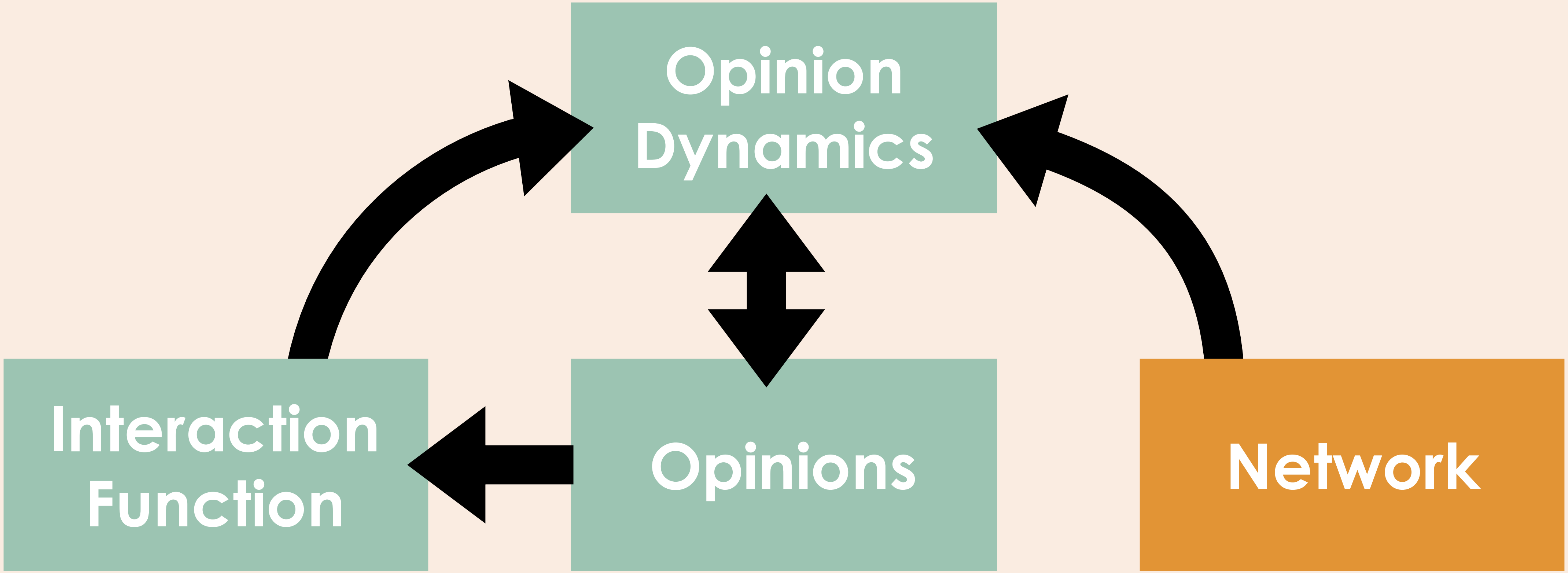
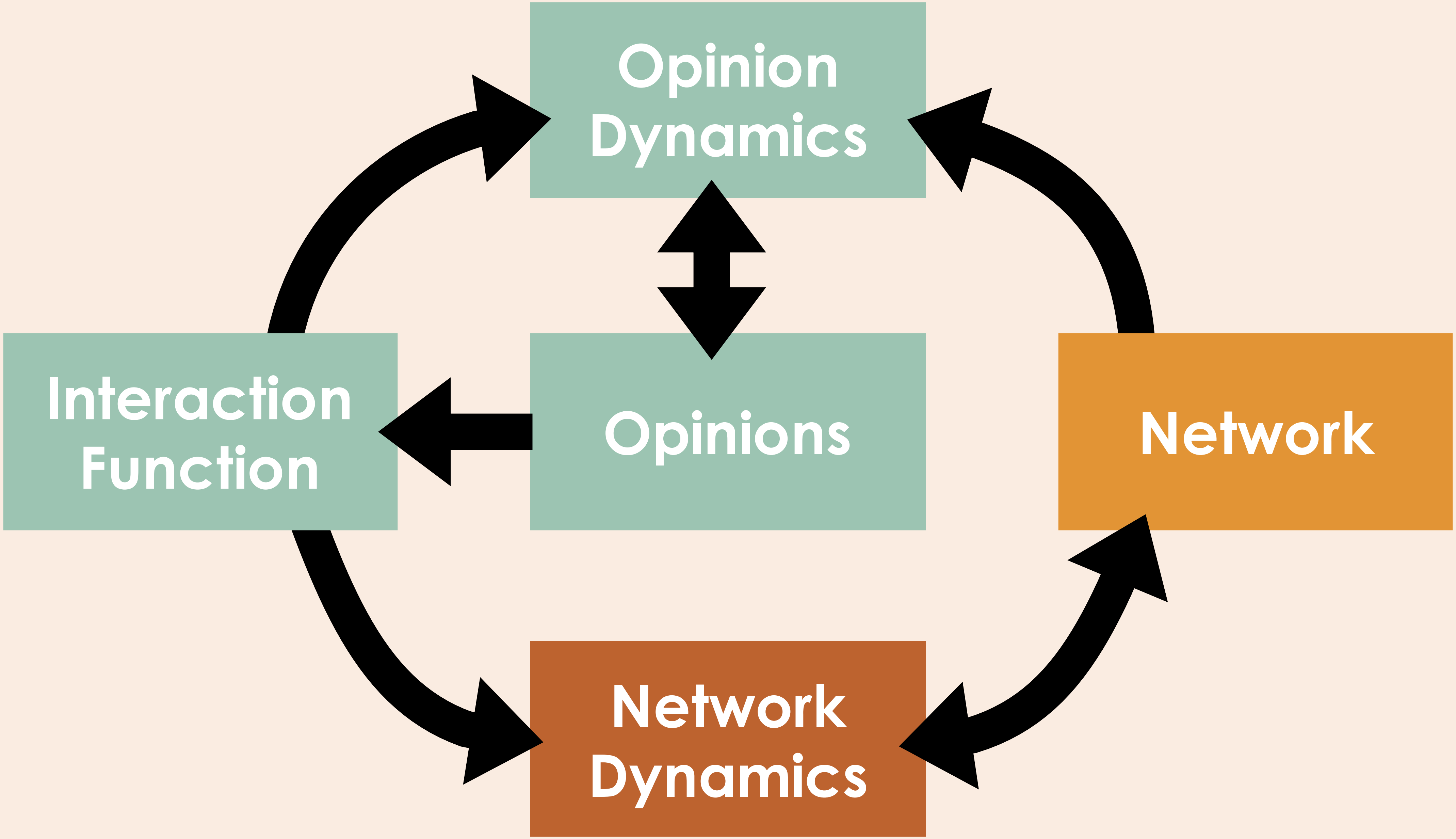


So far…

- Constructed a **general network model**
- Extended analytic results to include a network.
- Investigated the complex **impact of R and p**.


Opinion Dynamics
Interaction Function
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Network Dynamics

In this talk I will introduce a novel model of opinion dynamics that couples an opinion formation model with a general interaction function to a coevolving social network in which individuals build relationships through continued meaningful interaction.

Increasing $w_{ij}(t)$

Not an instantaneous change

Interaction function ϕ

$w(t)$

$$\frac{dx_i}{dt} = \frac{1}{k_i} \sum_{j=1}^{N} w_{ij}\, \phi(|x_j - x_i|)\,(x_j - x_i)$$

$$\frac{dw_{ij}}{dt} = \phi(|x_j - x_i|) \boxed{\text{Growth function}} - \left(1 - \phi(|x_j - x_i|)\right) \boxed{\text{Decay function}}$$

Memory weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}$$

Logistic weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\, w_{ij}\,(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}(1 - w_{ij})$$

Friend-of-a-friend (FOAF) weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\,(w_{ij} + \lambda(W^2)_{ij})(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}$$

Memory weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)(1 - w_{ij}) - (1 - \phi(|x_i - x_j|))\, w_{ij}$$
$$= \phi(|x_i - x_j|) - w_{ij}$$

$$w_{ij}(t) = e^{-t} w_{ij}(0) + \int_0^t e^{s-t}\, \phi(|x_j(s) - x_i(s)|)\, ds$$

Logistic weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\, w_{ij}\, (1 - w_{ij}) - (1 - \phi(|x_i - x_j|))\, w_{ij}(1 - w_{ij})$$
$$= (2\phi(|x_i - x_j|) - 1)\, w_{ij}(1 - w_{ij})$$

Friend-of-a-friend (FOAF) weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\, (w_{ij} + \lambda (W^2)_{ij})(1 - w_{ij}) - (1 - \phi(|x_i - x_j|))\, w_{ij}$$

Memory weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}$$
$$= \phi(|x_i - x_j|) - w_{ij}$$

Logistic weight dynamics:

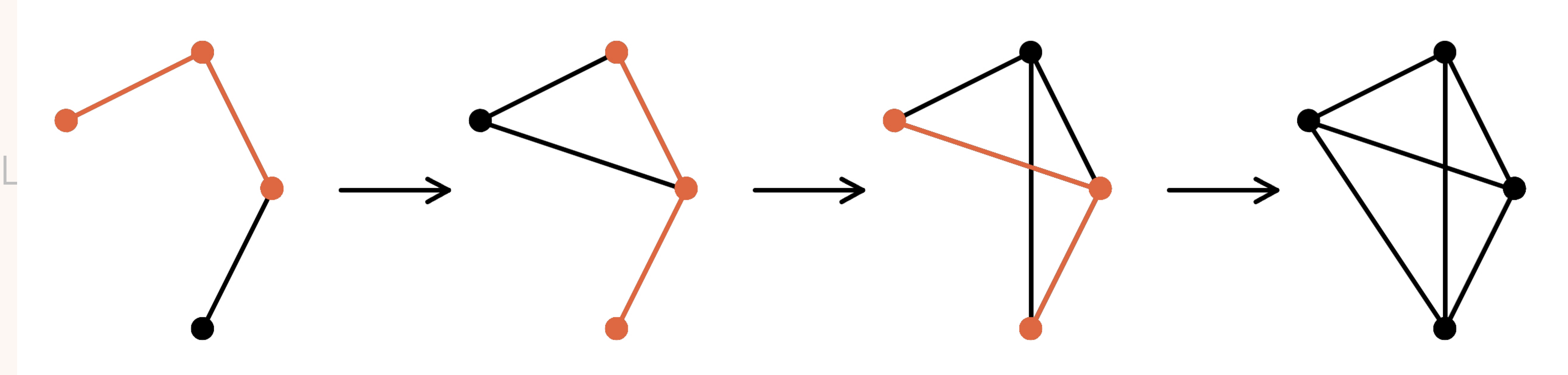
$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\, w_{ij}\,(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}(1 - w_{ij})$$
$$= \left(2\phi(|x_i - x_j|) - 1\right) w_{ij}(1 - w_{ij})$$

Friend-of-a-friend (FOAF) weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\left(w_{ij} + \lambda (W^2)_{ij}\right)(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}$$

Memory weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right) w_{ij}$$

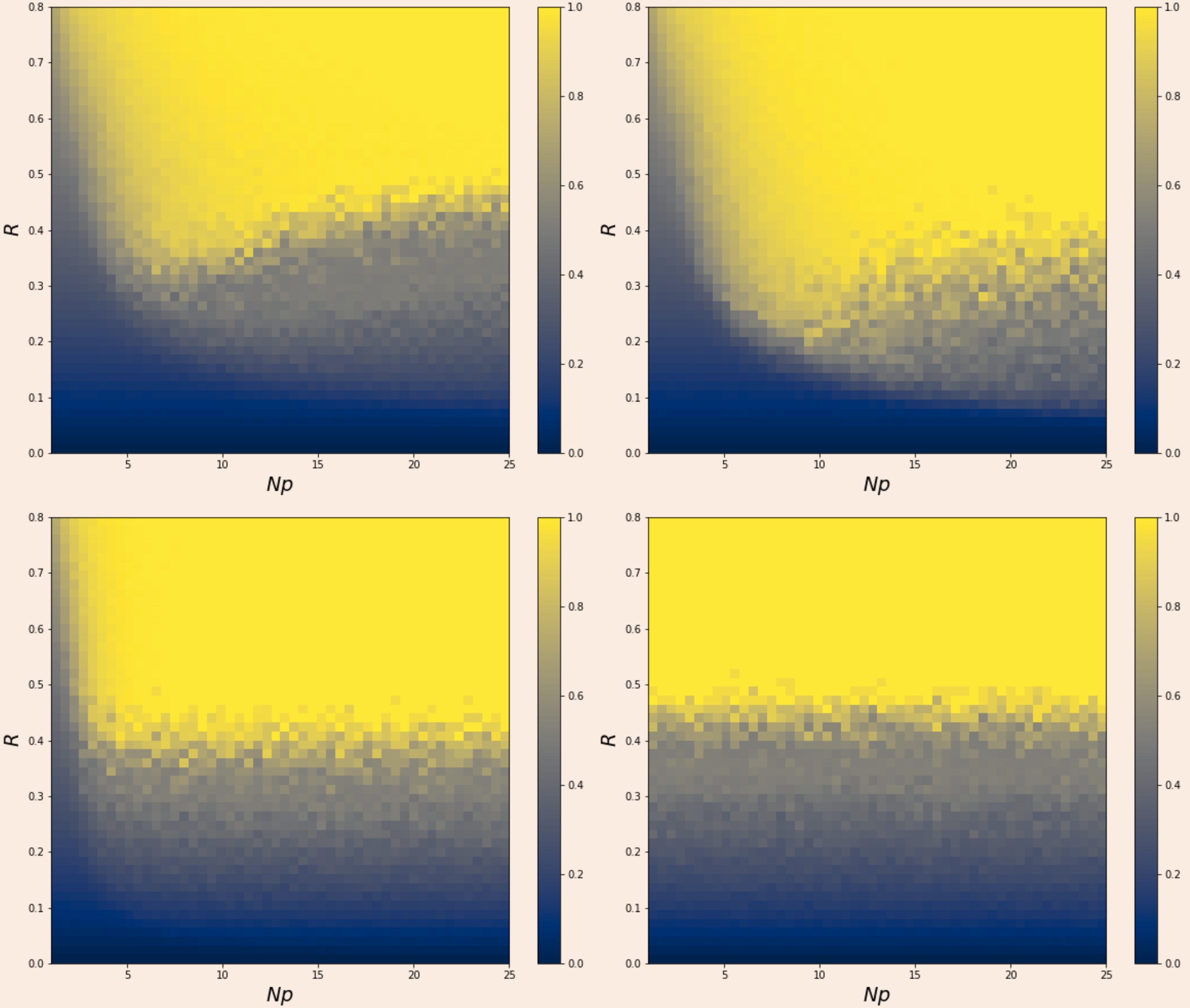


Friend-of-a-friend (FOAF) weight dynamics:

$$\frac{dw_{ij}}{dt} = \phi(|x_i - x_j|)\,\left(w_{ij} + \lambda(w^2)_{ij}\right)(1 - w_{ij}) - \left(1 - \phi(|x_i - x_j|)\right)\, w_{ij}$$

Fixed network

Logistic network dynamics



FOAF network dynamics

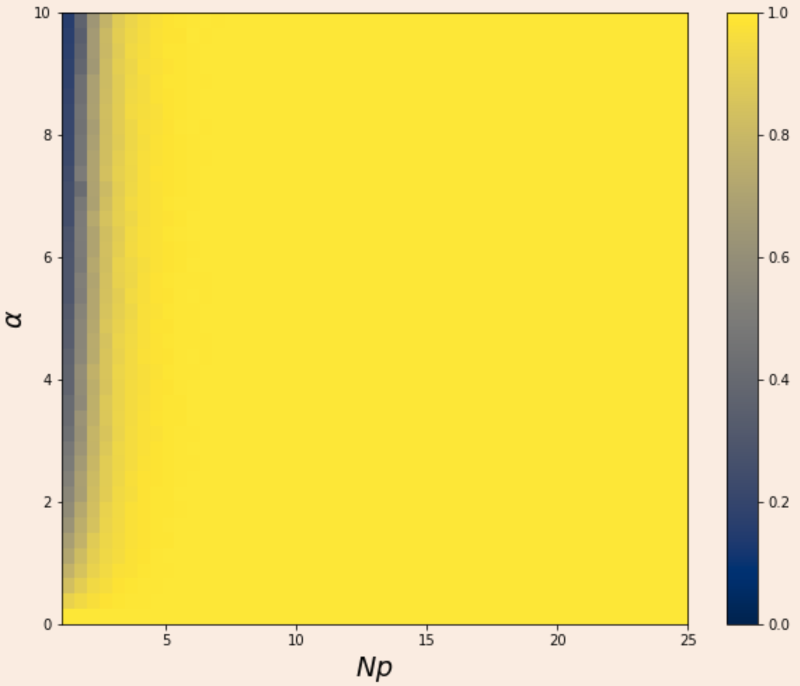
Memory network dynamics

Exponential interaction function:

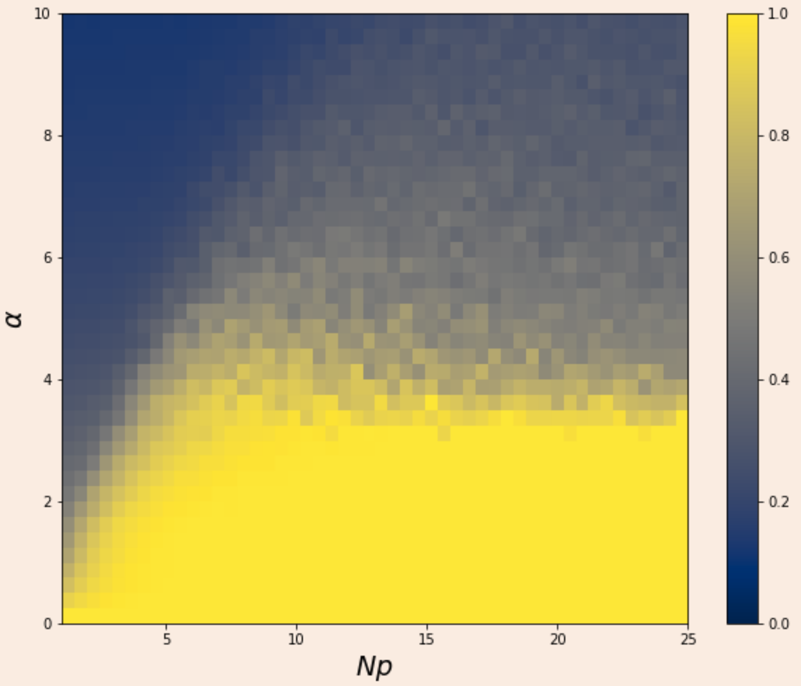
$$\phi^{\alpha}(|x_i - x_j|) = e^{-\alpha r |x_i - x_j|}$$

Case Study: Exponential interaction

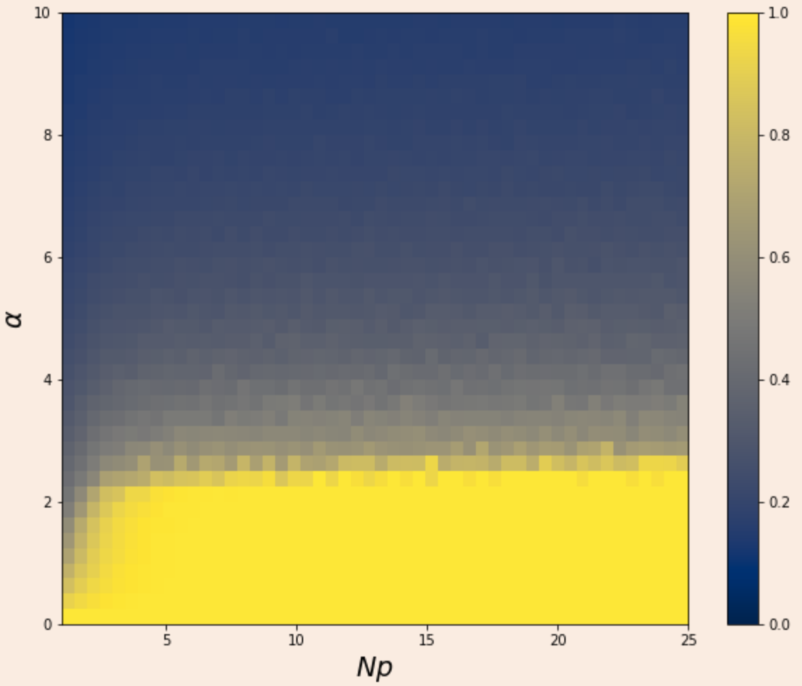
Fixed network

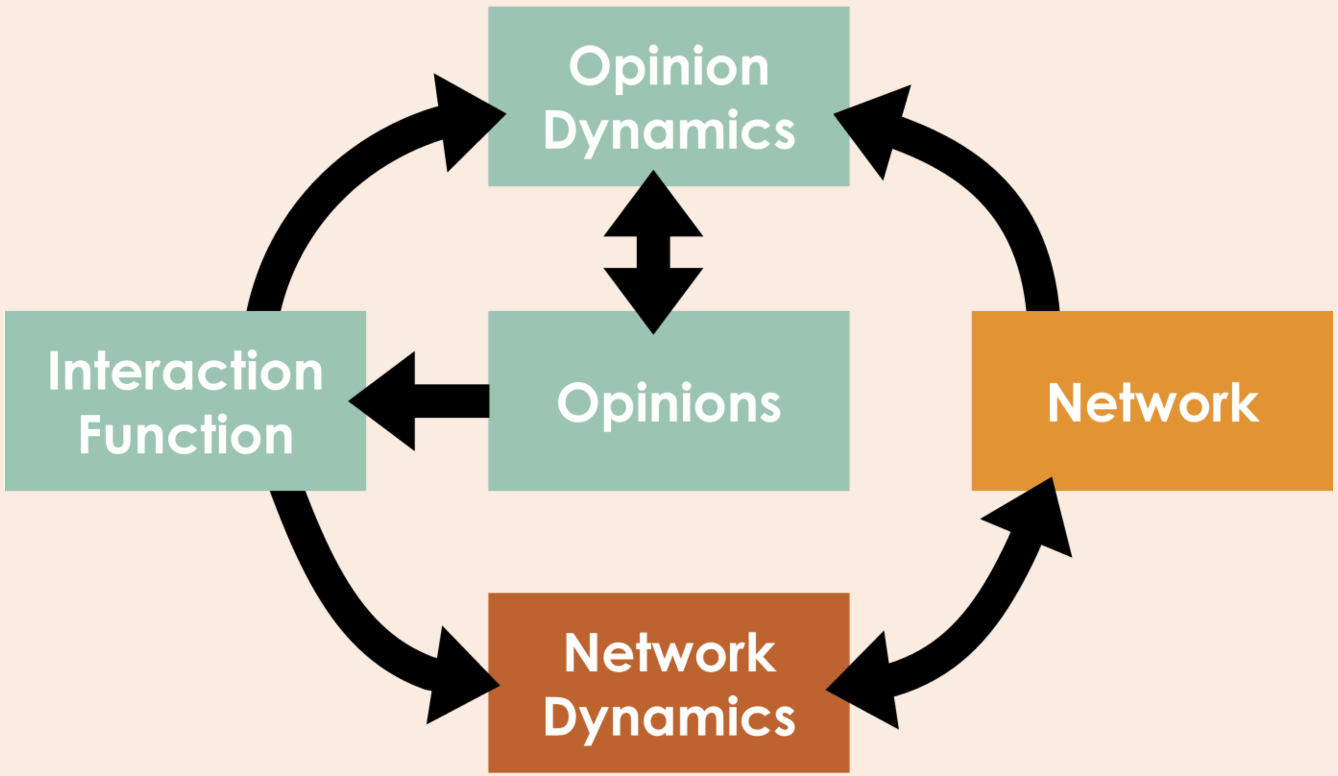


Logistic network dynamics



FOAF network dynamics





Opinion dynamics models capture consensus, polarisation and fragmentation.

Introducing a network creates a complex pattern of behaviours.

A new model where the interaction function balances growth and decay of edge weights.

Network dynamics can both help create consensus and entrench polarised views.

On evolving network models and their influence on opinion formation

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