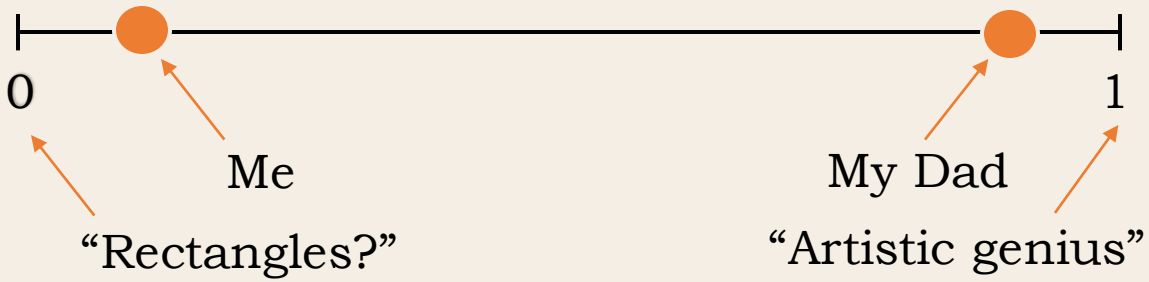
Models of Opinion Formation

Andrew Nugent (a.nugent@warwick.ac.uk)

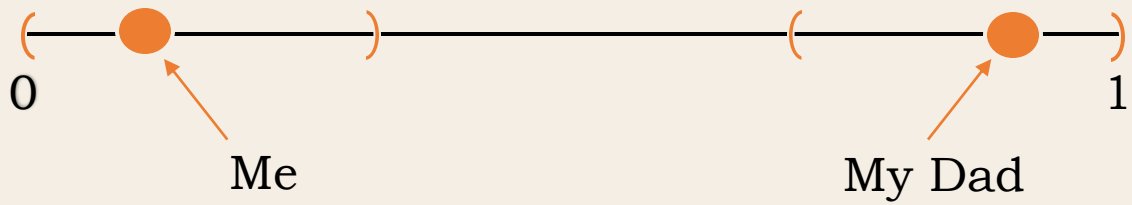
Supervised by Susana Gomes and Marie-Therese Wolfram

What do you think of this painting?



Orange, Red, Yellow Mark Rothko, 1961

Bounded Confidence



Orange, Red, Yellow Mark Rothko, 1961

Bounded Confidence

Individual i's opinion: x_i

Confidence radius: R

Confidence set:

$$I(i,x) = \{ j : |x_i - x_j| < R \}$$

Orange, Red, Yellow Mark Rothko, 1961

Hegselmann Krause Model

Individual i's opinion: x_i

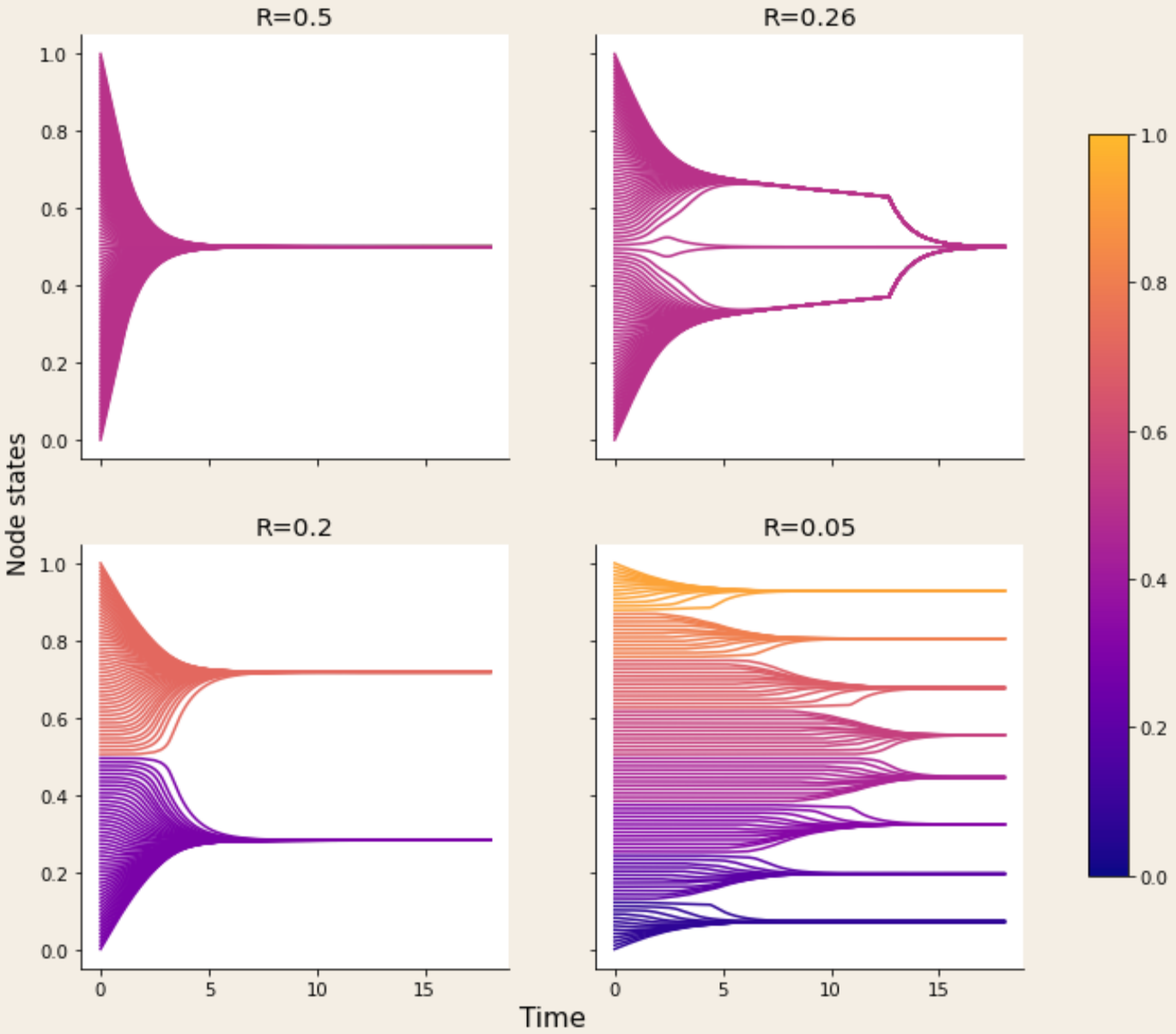
Confidence radius: R

Confidence set:

$$I(i,x) = \{ j : |x_i - x_j| < R \}$$

Opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_{j \in I(i,x)} (x_j - x_i) \ , t \in [0,T]$$



Example Hegselmann-Krause model dynamics.

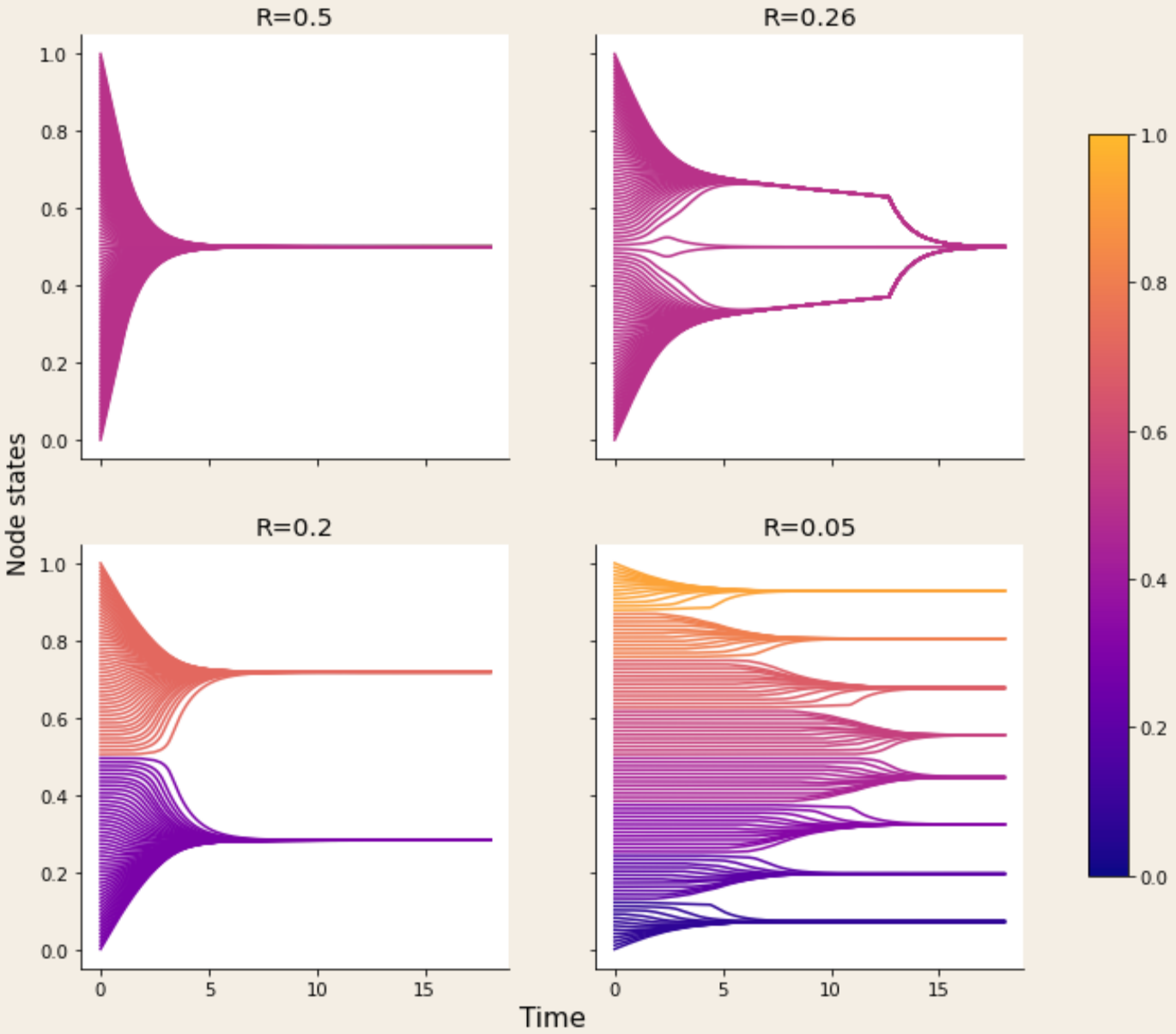
Hegselmann Krause Model

Opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_{j \in I(i,x)} (x_j - x_i) \ , t \in [0, T]$$

Order parameter
(how many opinion clusters form):

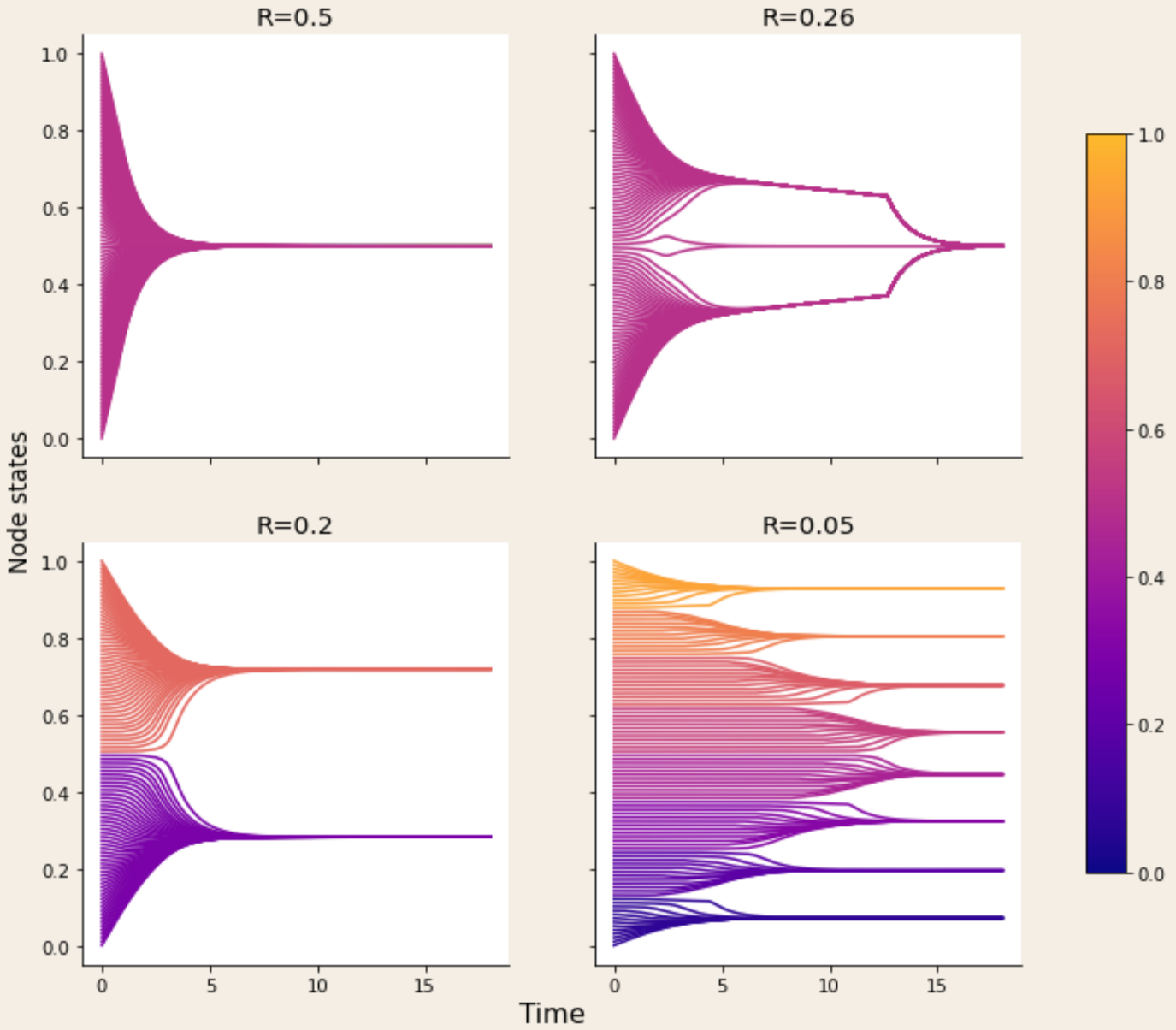
$$Q = \frac{1}{N^2} \sum_{i,j=1}^{N} \mathbf{1}_{\{|x_i(T) - x_j(T)| < R\}}$$



Example Hegselmann-Krause model dynamics.

Extensions to the HK model

1. Individuals interact across a **social network**.
2. Individuals have opinions on **multiple topics**.
3. Individuals' opinions are affected by **noise**.



Example Hegselmann-Krause model dynamics.

Social Network Model

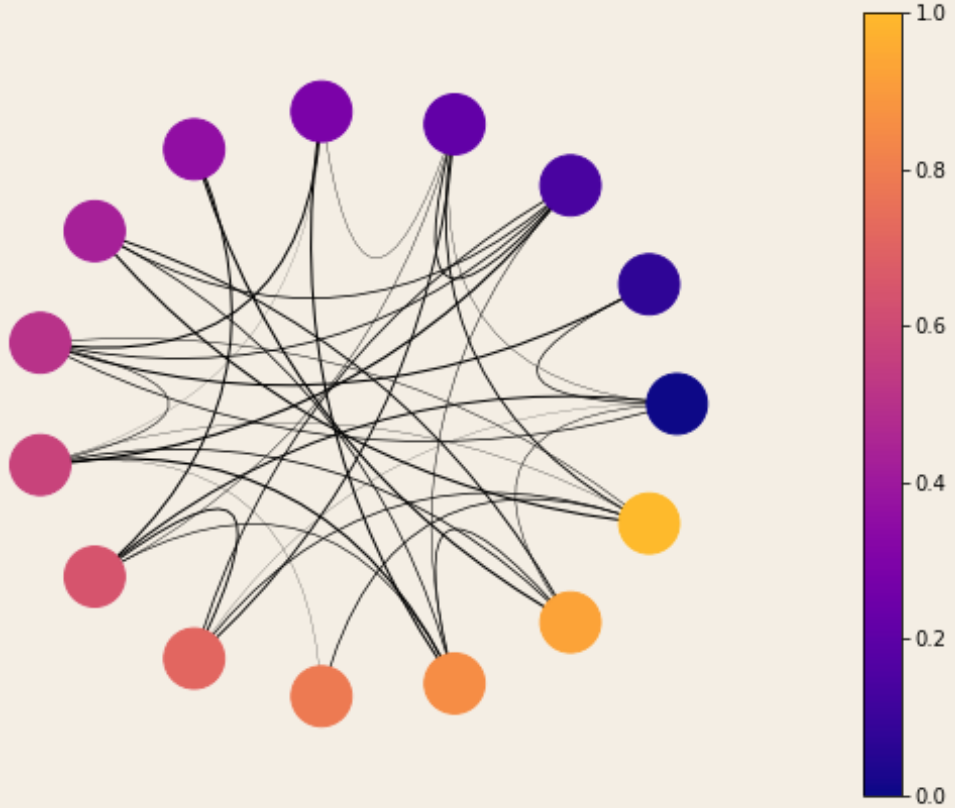
Opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_{j \in I(i,x)} (x_j - x_i) \ , t \in [0, T]$$

Opinion dynamics on a network:

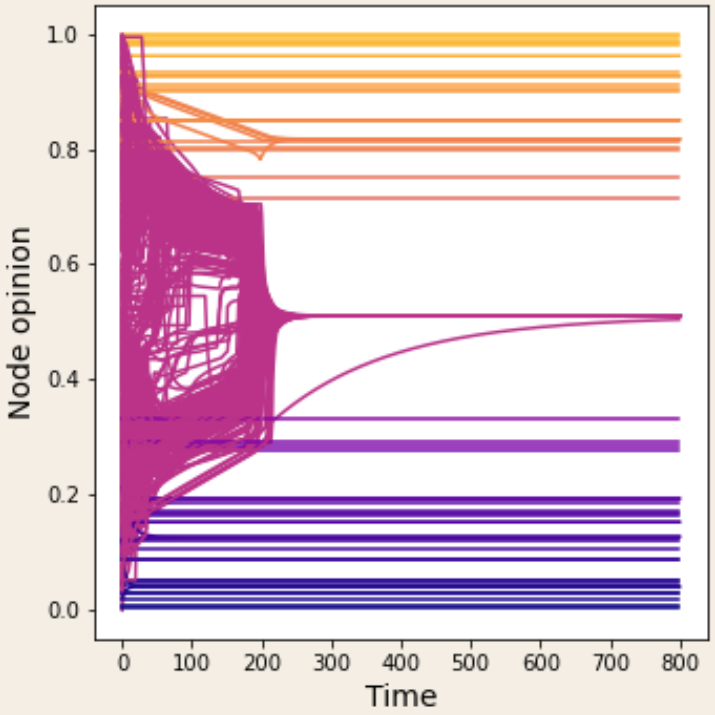
$$\frac{dx_i}{dt} = \frac{1}{k_i} \sum_{j \in I(i,x)} w_{ij}(x_j - x_i) \, ,$$

$$k_i = \sum_{j \in V} w_{ij}$$

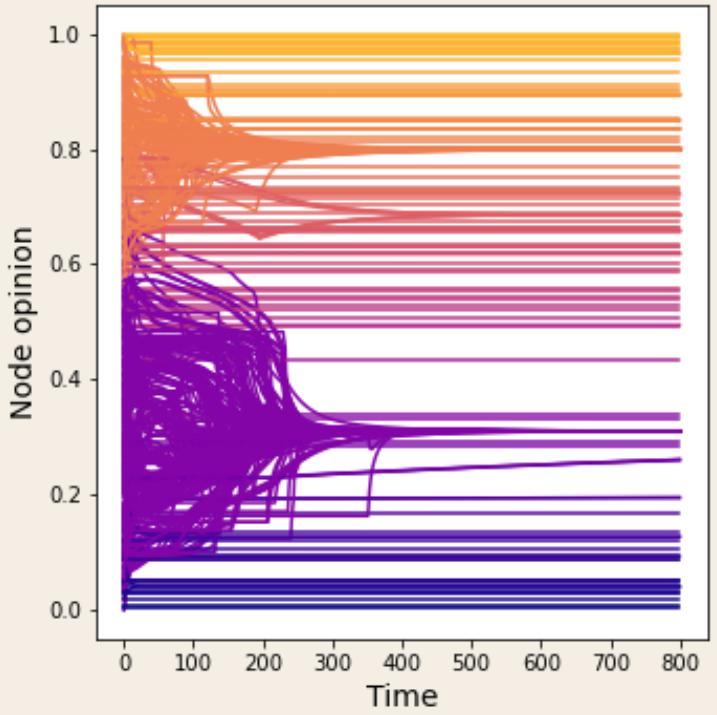


Example weighted Erdős-Rényi random network with nodes coloured by opinion.

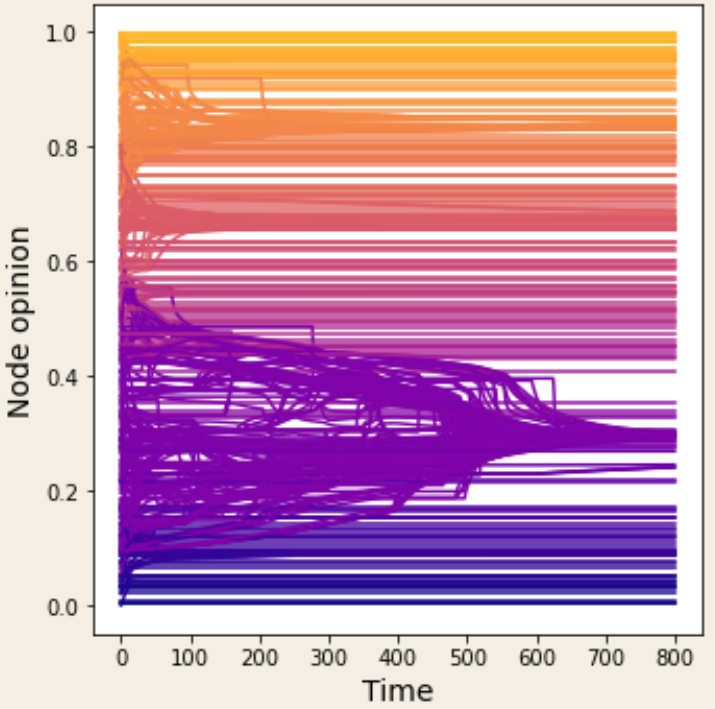
Social Network Model



$R = 0.2$



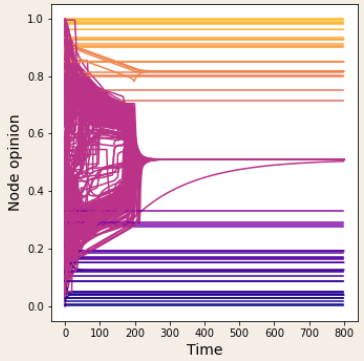
$R = 0.15$



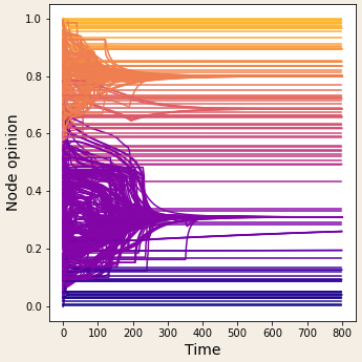
$R = 0.1$

Example network model dynamics.

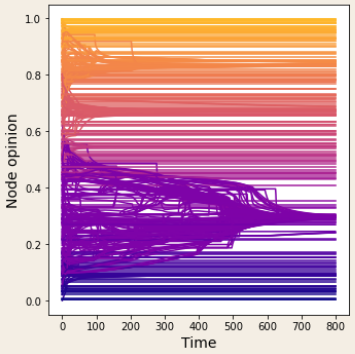
Social Network model



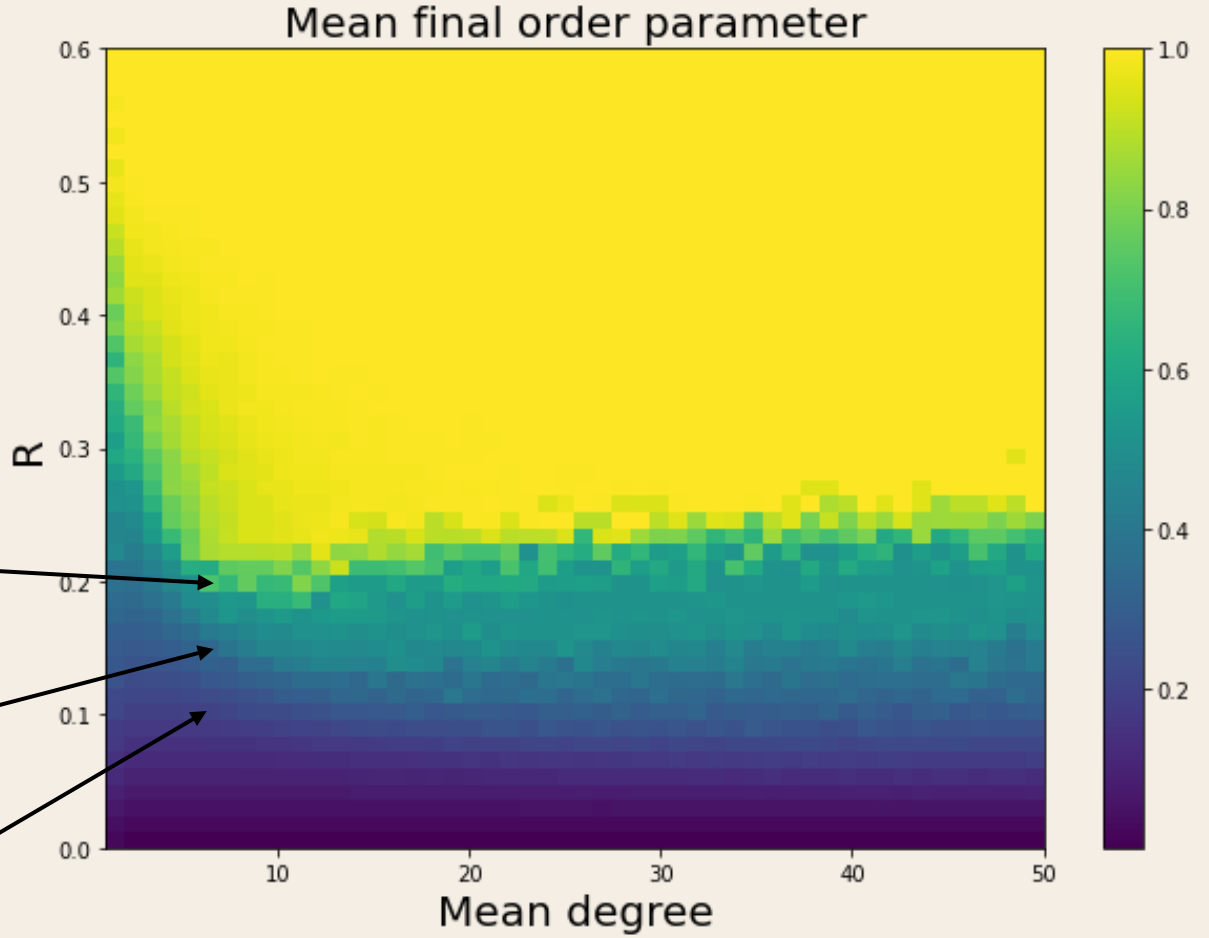
$R = 0.2$



$R = 0.15$



$R = 0.1$



Order parameter heatmap for various values of R and network mean degree.

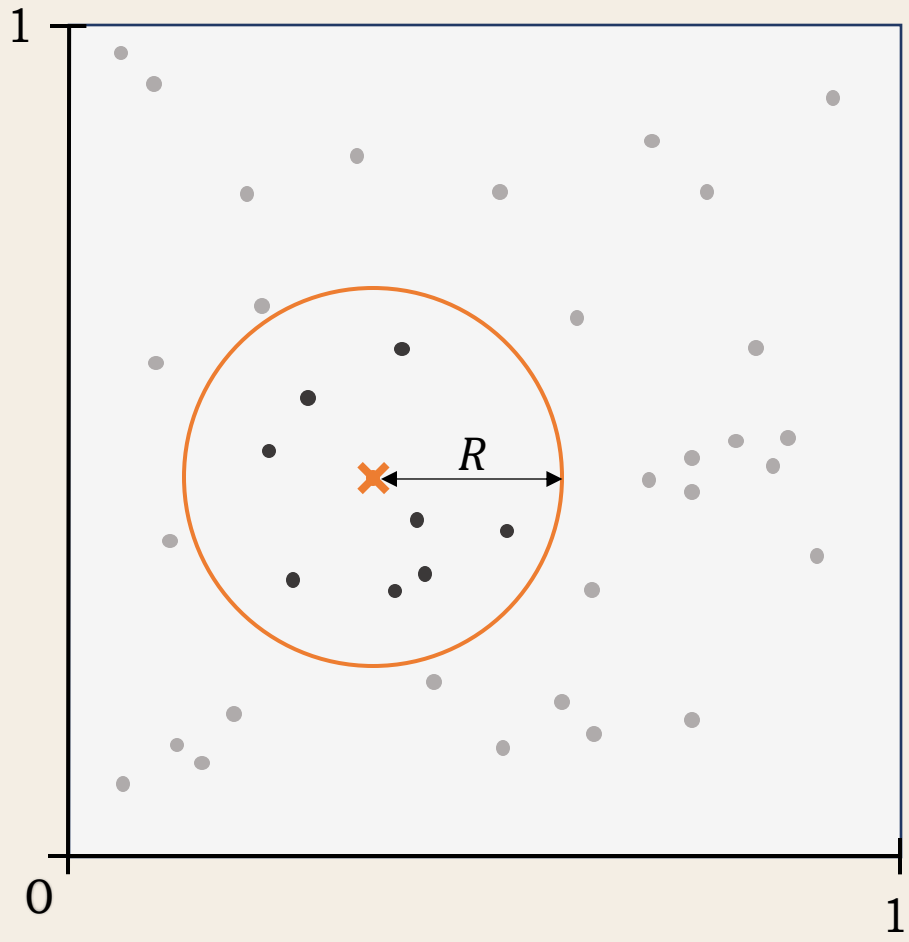
Including multiple topics

Opinion dynamics:

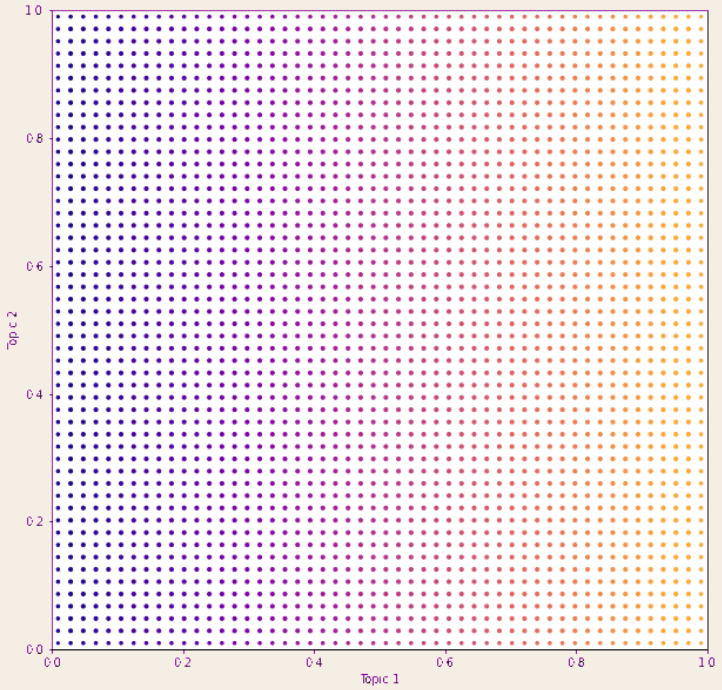
$$\frac{dx_i}{dt} = \frac{1}{N} \sum_{j \in I(i,x)} (x_j - x_i) \ , t \in [0, T]$$

Can define the confidence set using the Euclidean norm:

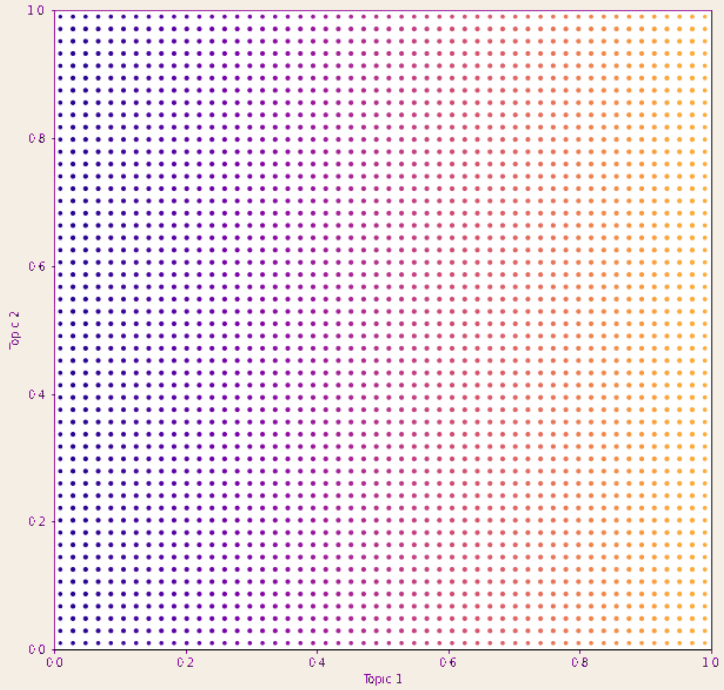
$$I(i,x) = \{ j : \|x_i - x_j\| < R \}$$



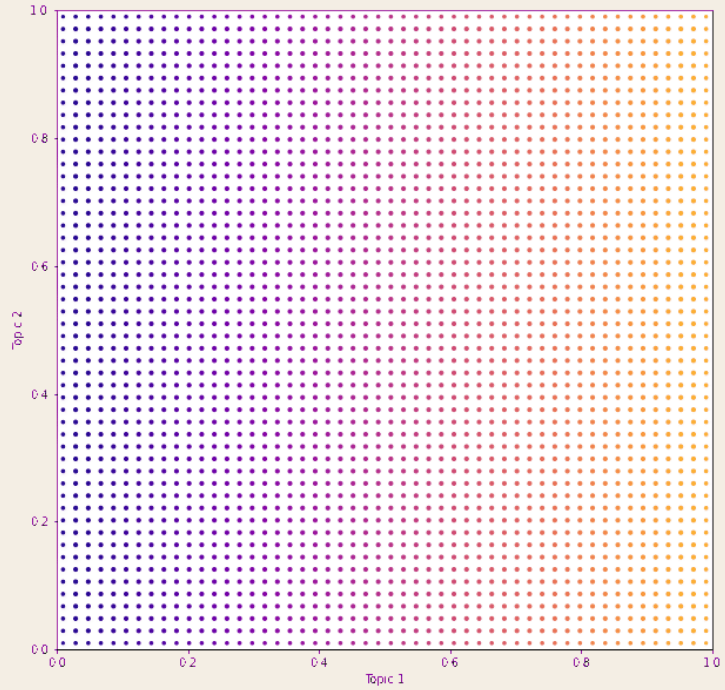
Including multiple topics



$R = 0.05$

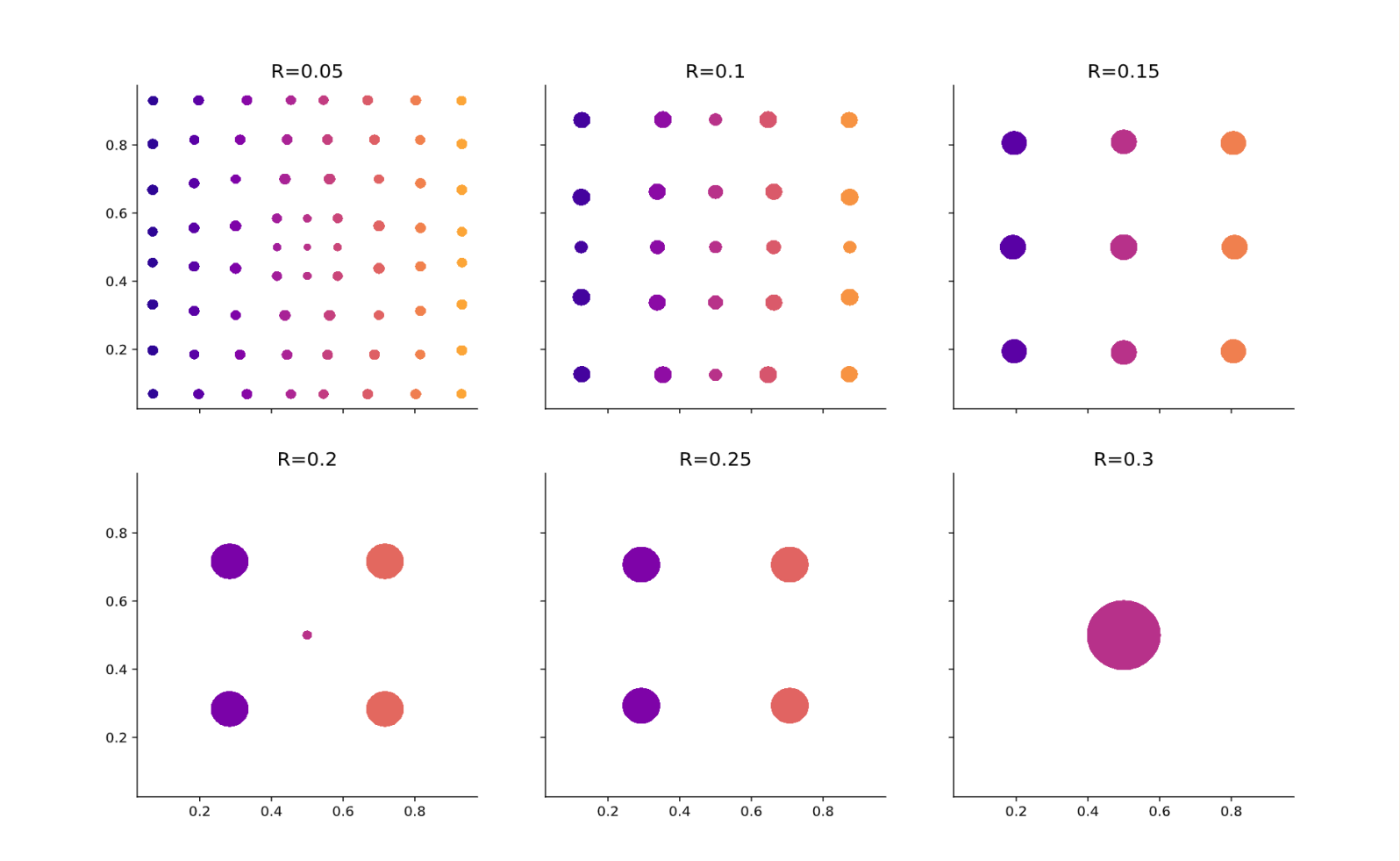


$R = 0.15$



$R = 0.30$

Including multiple topics



Example two-dimensional dynamics with the Euclidean norm.

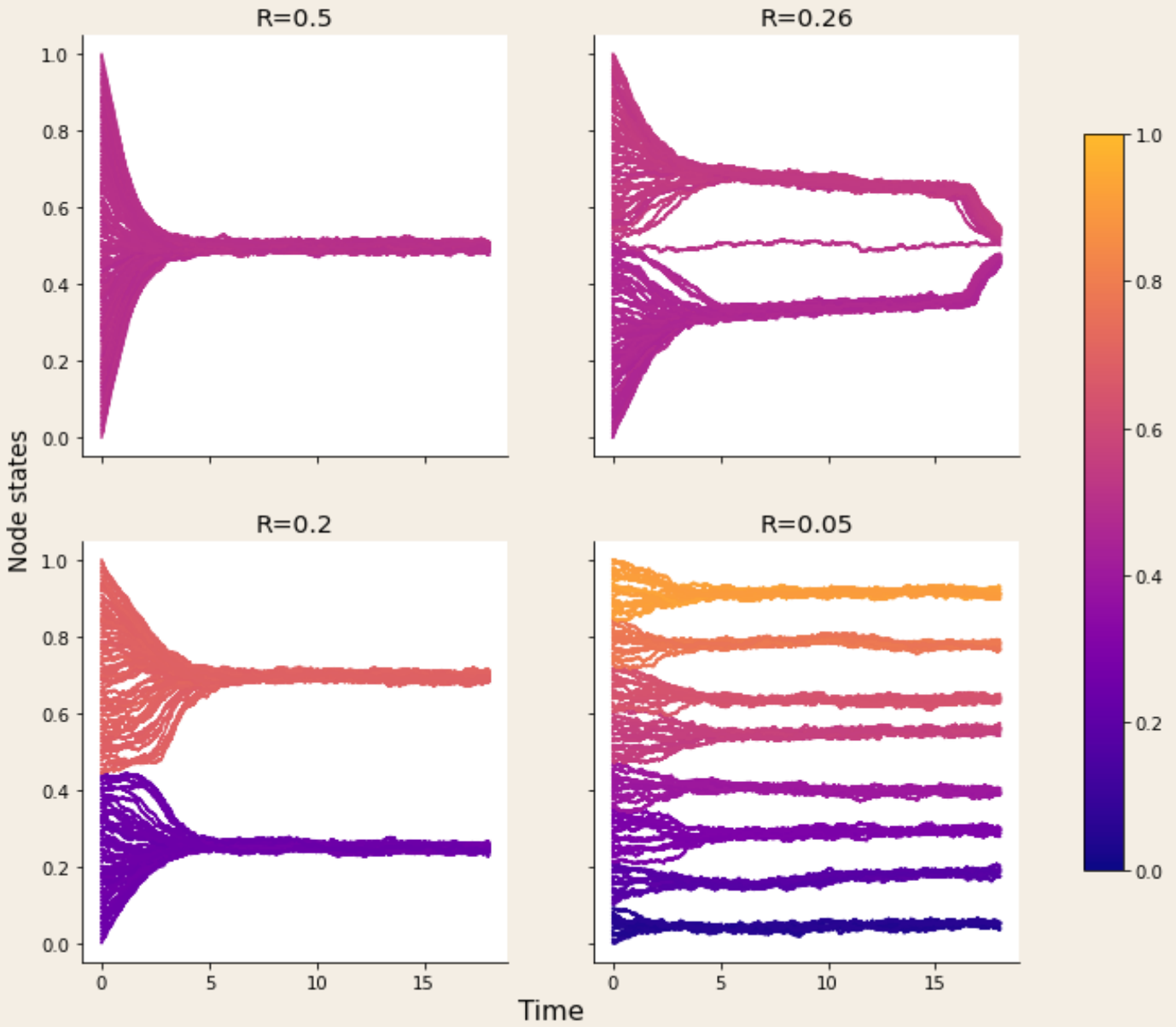
Noisy opinion model

Opinion dynamics:

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_{j \in I(i,x)} (x_j - x_i) \ , t \in [0, T]$$

Noisy opinion dynamics:

$$dx_i = \underbrace{\left(\frac{1}{N} \sum_{j \in I(i,x)} (x_j - x_i) \right) dt}_{\text{drift term}} + \underbrace{\sigma \, d\beta_t^{(i)}}_{\text{diffusion term}}$$



Example of noisy Hegselmann-Krause model dynamics.

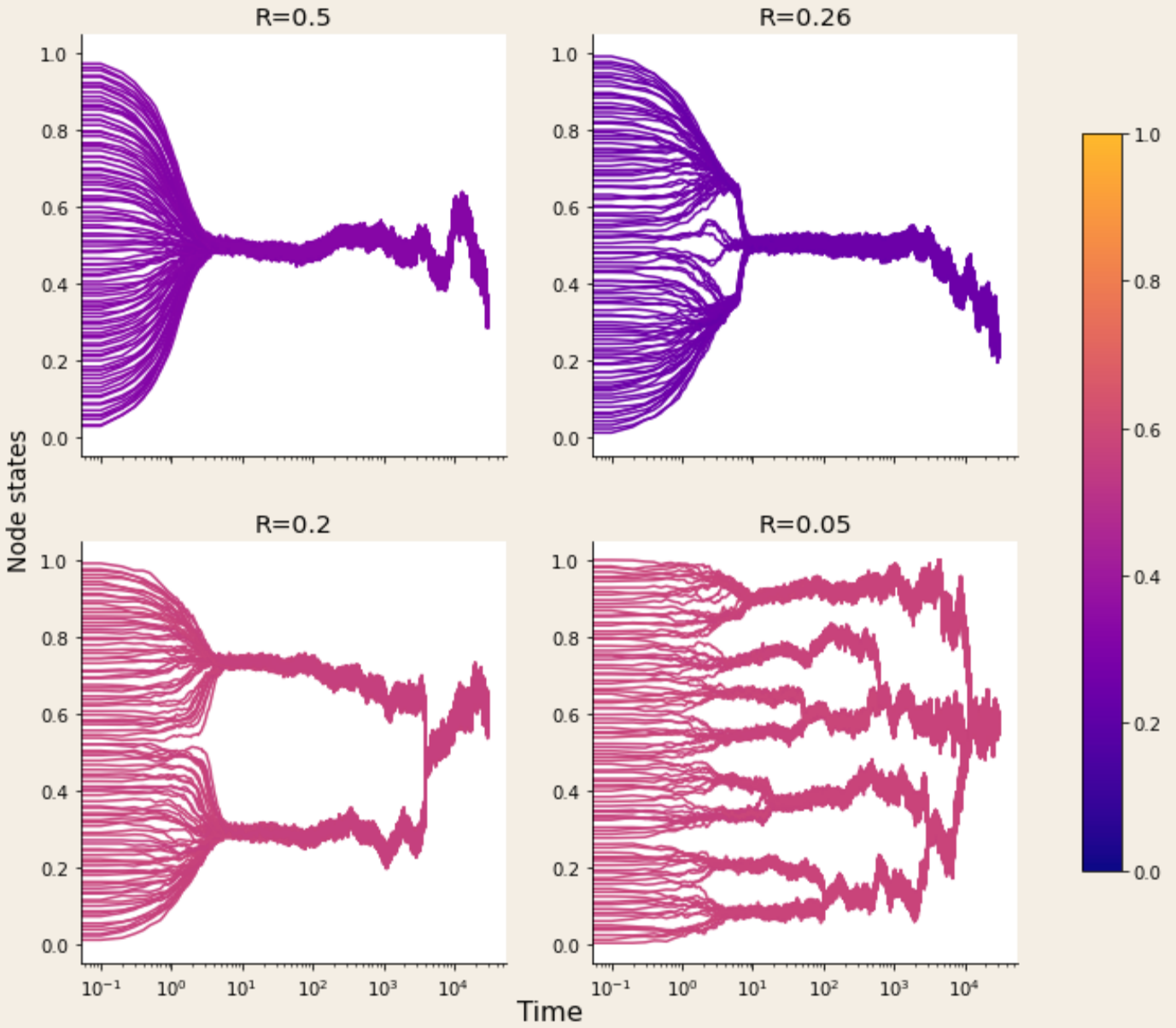
Noisy opinion model

Centre of mass of a cluster:

$$c = \frac{1}{M}\sum_{m} x_m$$

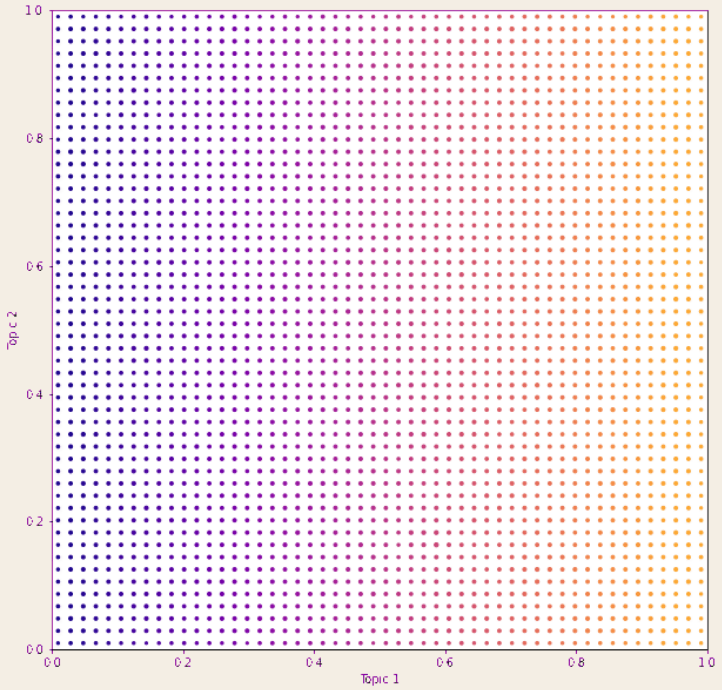
$$dc = \left(\frac{1}{M}\frac{1}{N}\sum_{m}\sum_{j}(x_j - x_m)\right)dt + \frac{\sigma}{M}\sum_{m} d\beta_t^{(m)}$$

See *Noisy Hegselmann-Krause Systems: Phase Transition and the 2r-Conjecture* by Wang et al.

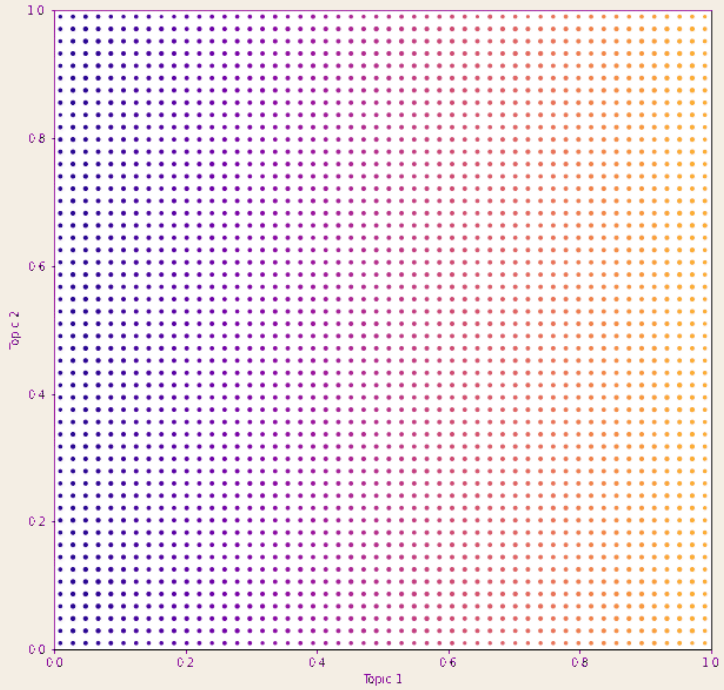


Long-term noisy Hegselmann-Krause model dynamics.

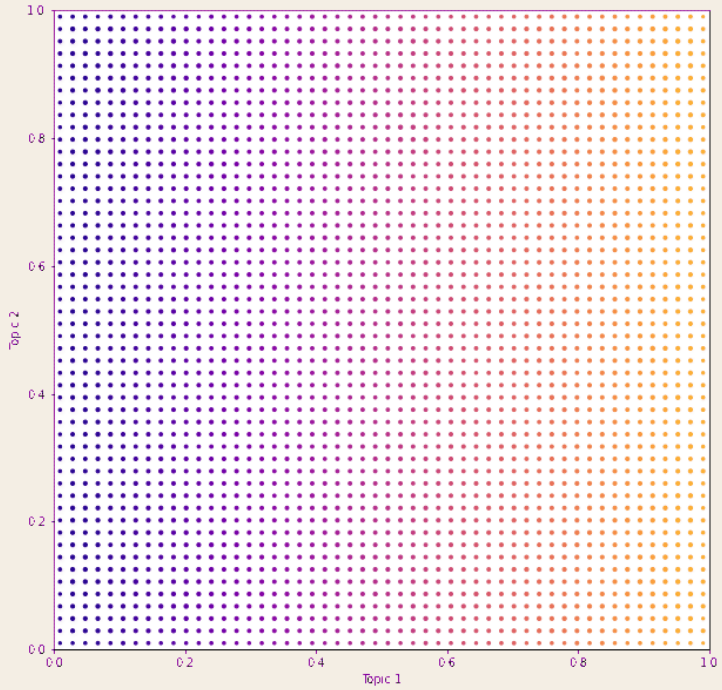
Including multiple topics & noisy opinions



$R = 0.05$



$R = 0.15$



$R = 0.26$

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