Critical Phenomena in a Heterogeneous Excitable System

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1. Aim of the Project

Modelling the changes in the muscular wall of the uterus in the days before labour. In particular, examining the excitation of a tightly coupled system of smooth muscle cells.

2. Background

- The muscular wall of the uterus only acquires the ability to expel a foetus in the final days before labour
- Smooth muscle cells start at a stable equilibrium. Sufficient input current is needed to excite the cell causing a voltage surge (Fig.1)
- Potential returns rapidly to equilibrium after applied current is removed (Fig.2)
- Myometrial cell synchrony is achieved by electrical conduction through connecting microfibrils
- The slow rise and fall of tension results in a contraction that lasts about a minute
- Cells are considered to obey FitzHugh-Nagumo dynamics:
  \[ \frac{dv}{dt} = \frac{1}{\epsilon}(Av(1-v)(v-\alpha)-w-w_c) \]
  \[ \frac{dw}{dt} = v - \gamma w - w_0 \]
- Simulations were run for cells in chains and lattices of an arbitrary size

3. Cell Chains

- Excite a cell with a ‘kick’
- Minimum kick needed to excite highlighted cell increases with coupling (Fig. 3)
- Maximum amplitude is consistent for all coupled cells
- There is a threshold coupling value for exciting coupled cells (Fig.4)

4. Cell Lattices

- Excite middle cell
- Minimum kick needed to excite highlighted cell increases with coupling (Fig.5) faster than for a cell chain
- Maximum amplitude is consistent for all coupled cells and comparable to a cell chain
- There is a threshold for exciting coupled cells (Fig.6) which is lower than for a cell chain

5. Conclusions

- The system has two states:
  - Local excitation which does not spread across the entire tissue
  - Global excitation where all cells are excited
- The phase transition from a locally excited state to a globally excited state has a coupling threshold and is instantaneous at this point
- Excited cells follow the same voltage pathway over time

6. Further Work

- Introduce variable coupling values between cells
- Remove resistor couplings at random and investigate the spread of excitation using percolation theory

7. References


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