



Angular
Momentum is
DWD

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Angular Momentum and Stability: Double White Dwarfs as Progenitors of AM CVn

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Northwestern University & CIERA

Third International Workshop on AM CVn stars
April 16–20, 2012



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Motivation

Exploring the possibility of Double White Dwarfs as a formation channel for AM CVn

Problems

- Double White Dwarfs (DWDs) are small enough and H-poor enough, but the mass transfer (MT) process does not appear to be dynamically stable
- A fact which largely amounts to angular momentum losses during MT



Background

An (incomplete) history of the DWD formation channel

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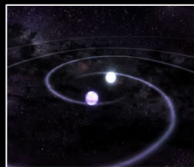
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Paczynski (1967)

Suggests semi-detached dwarfs can explain the newly discovered AM CVn

Pringle & Webbink (1975), Tutukov & Yungleson (1979)

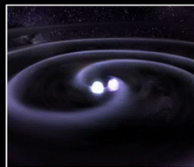
Examine gravitational radiation as a method to induce MT in close binaries



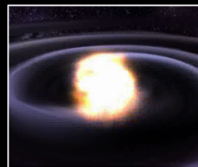
TODAY



40 million years from now



60 million years from now



61 million years from now

Image credit: NASA/GSFC/D.Berry



Eventually MT Occurs

Gravitational waves decrease J_{orb}

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J_{orb} directly affects the separation, a

$$J_{\text{orb}} = \left(\frac{Ga}{M_D + M_A} \right)^{1/2} M_D M_A$$

$$\frac{\dot{J}_{\text{orb}}}{J_{\text{orb}}} = (1 - q) \frac{\dot{M}_D}{M_D} + \frac{1}{2} \frac{\dot{a}}{a}$$

Since $R_L \propto a$ and $R_D \propto M_D$, mass transfer is stable only if

Stability Criterion*

$$q < \frac{5}{6} + \frac{\zeta(M_D)}{2}$$

Where $\zeta(M_D) = d \log R_L / d \log M_D \approx -1/3$

* Assumes all J from stream returned to the orbit



Background, cont'd

Skipping a few years...

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Nelemans et al. (2001)

- Noted that for DWD, the accretor is large compared to the binary separation
- Ballistic trajectories of Lubow & Shu (1975) would cause direct impact of the stream onto the accretor
- No disk means its harder to return angular momentum (J) back to the orbit.

$$J_{\text{stream}} \rightarrow J_{\text{Spin,A}}$$

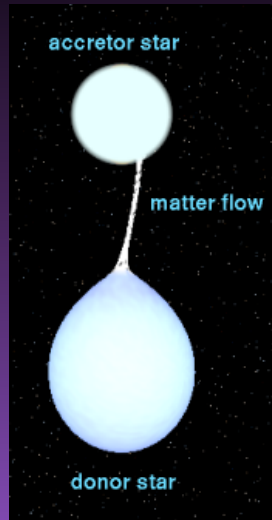


Image credit Andrzej Krolak via einstein-online.info



This is a problem because...

see, e.g., Nelemans et al. (2001); Marsh, Nelemans, and Steeghs (2004)

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Since $R_L \propto a$ and $R_D \propto M_D$, mass transfer is stable only if

Stability Criterion*

$$q < \frac{5}{6} + \frac{\zeta(M_D)}{2} + f(J_{\text{stream}})$$

* Assumes the average J of a ballistic particle is removed from the orbit.



This is a problem because...

see, e.g., Nelemans et al. (2001); Marsh, Nelemans, and Steeghs (2004)

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$$\frac{\dot{J}_{\text{orb}}}{J_{\text{orb}}} = (1 - q) \frac{\dot{M}_D}{M_D} + \frac{1}{2} \frac{\dot{a}}{a}$$

Since $R_L \propto a$ and $R_D \propto M_D$, mass transfer is stable only if

Stability Criterion*

$$q < \frac{5}{6} + \frac{\zeta(M_D)}{2} - \sqrt{(1 + q)r_h}$$

* Assumes the average J of a ballistic particle is removed from the orbit. (r_h from Lubow & Shu (1975) ; Verbunt & Rappaport (1988))



But now we have tides

Marsh, Nelemans, and Steeghs (2004)

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Tides return J to the orbit from the accretor on a timescale τ_A

$$\dot{J}_{\text{tides,A}} = \frac{kM_A R_A^2}{\tau_A} (\Omega_A - \Omega_{\text{orb}})$$

So our orbital angular momentum changes like

Angular Momentum Balance

$$\dot{J}_{\text{orb}} = \dot{J}_{\text{GW}} + \dot{J}_{\text{MT}} + \dot{J}_{\text{tides}}$$



But now we have tides

Marsh, Nelemans, and Steeghs (2004)

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Angular Momentum Balance

$$\dot{J}_{\text{orb}} = \dot{J}_{\text{GW}} + \sqrt{GM_A r_h a} \dot{M}_D + \frac{kM_A R_A^2}{\tau_A} (\Omega_A - \Omega_{\text{orb}})$$



White Dwarf Stability

Marsh, Nelemans, and Steeghs (2004)

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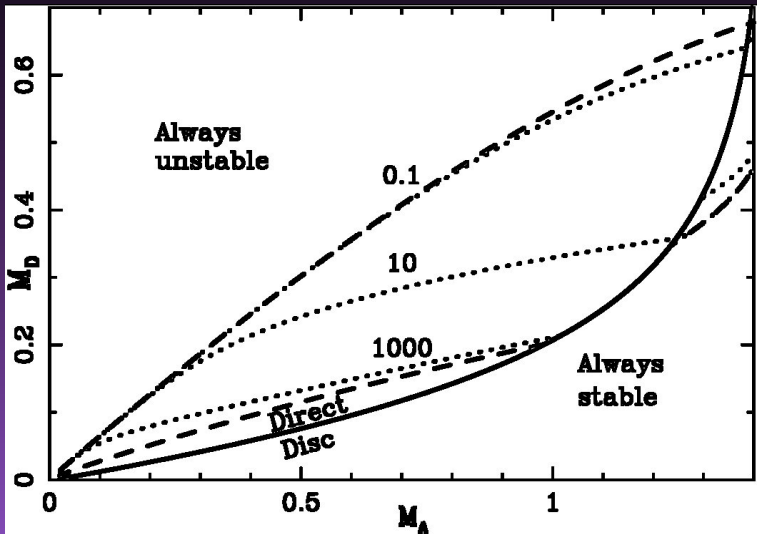
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Angular Momentum and Mass Transfer

Lubow & Shu (1975), Verbunt & Rappaport (1988), Nelemans et al. (2001)

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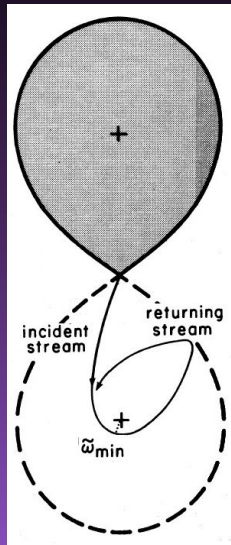
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ΔJ due to Mass Transfer

- Mass, M_P , ejected from L_1 with some velocity \vec{v} , removing J from orbit
- Which changes over the path of the mass
- $\langle J_P \rangle \rightarrow \Delta J_{MT} \rightarrow \sqrt{GM_A R_h M_P}$
- Upon accretion:
 $\Delta J_{\text{spin},A} = J_P$





Angular Momentum and Mass Transfer

Lubow & Shu (1975), Verbunt & Rappaport (1988), Nelemans et al. (2001)

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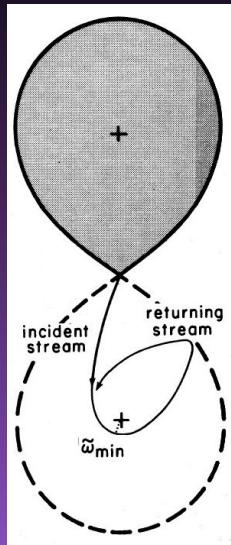
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But...

- J_P is not constant
 J_P at impact is not equal to $\langle J_P \rangle$
- Angle of impact matters





Angular Momentum and Mass Transfer

Lubow & Shu (1975), Verbunt & Rappaport (1988), Nelemans et al. (2001)

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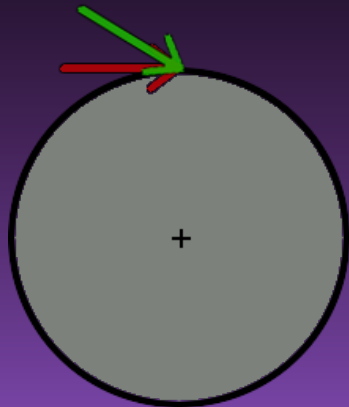
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But...

- J_P is not constant
 J_P at impact is not equal to $\langle J_P \rangle$
- Angle of impact matters

Impact Angle

- Oblique impacts will change both the Spin *and* Orbital Angular Momentum of the accretor





New (Basic?) Model

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Our Method

Rigid Spheres Conserving Both Linear and Angular Momentum

Sepinsky et al. (2010)

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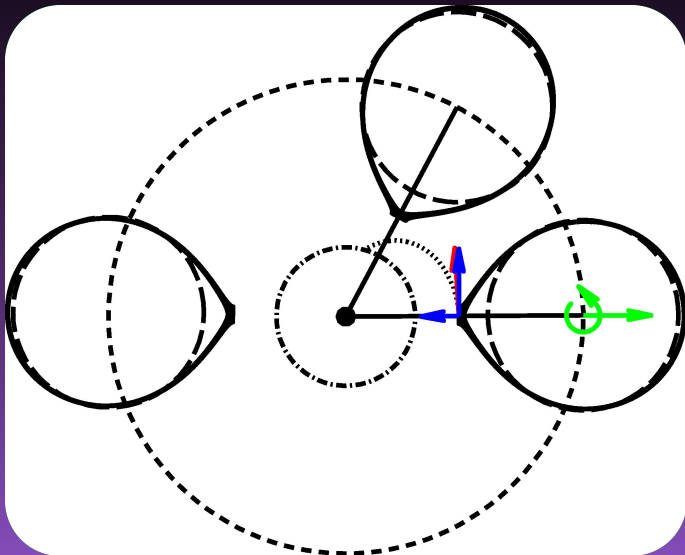
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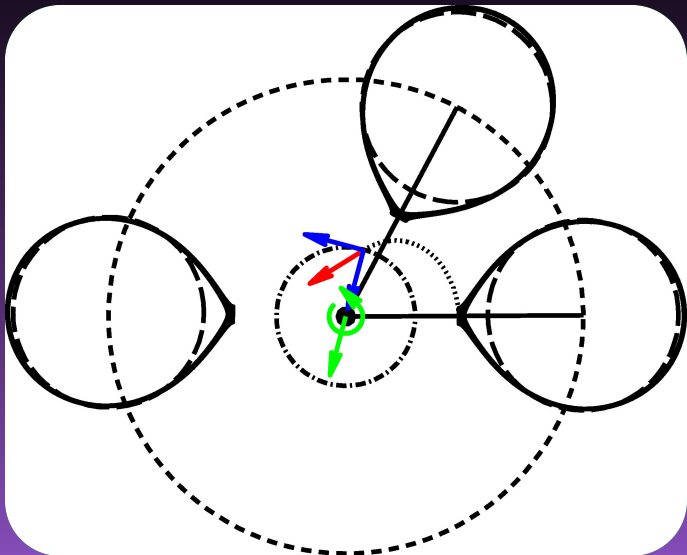
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What Changes After One Orbit?

Highlighted momenta changes are *not* included in the standard model

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Angular Momenta

- **Donor Orbital Angular Momentum** → Changes in size/shape of orbit
- **Accretor Orbital Angular Momentum** → Changes in size/shape of orbit
- **Accretor Spin Angular Momentum** → Sink of Angular Momentum; Source of Tidal Angular Momentum
- **Donor Spin Angular Momentum** → Source of Angular Momentum; Sink of Tidal Angular Momentum



What Changes After One Orbit?

Highlighted momenta changes are *not* included in the standard model

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- **Accretor Spin Angular Momentum** → Sink of Angular Momentum; Source of Tidal Angular Momentum
- **Donor Spin Angular Momentum** → Source of Angular Momentum; Sink of Tidal Angular Momentum

Angular Momentum Balance

$$\dot{J}_{\text{orb}} = \dot{J}_{\text{GW}} + \dot{J}_{\text{MT}} + \dot{J}_{\text{tides,A}} + \dot{J}_{\text{tides,D}}$$



Ballistic Numerical Integrations

Consistent with Lubow & Shu (1975)

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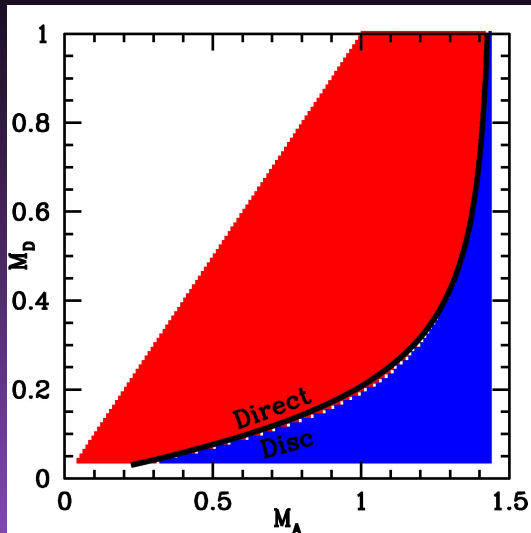
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Solid Line: where r_{min} from Lubow & Shu (1975) equals the Accretor Radius



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Angular Momentum Changes

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Calculations

- Eject a ballistic particle from the L_1 of the Donor WD, conserving Linear and Angular Momentum
- Numerically integrate the trajectory, conserving total momentum and energy
- For direct impact: Accrete the ballistic particle, conserving angular and linear momentum of the donor
- Calculate the total changes in the orbital and spin angular momentum of each component



Angular Momentum Changes

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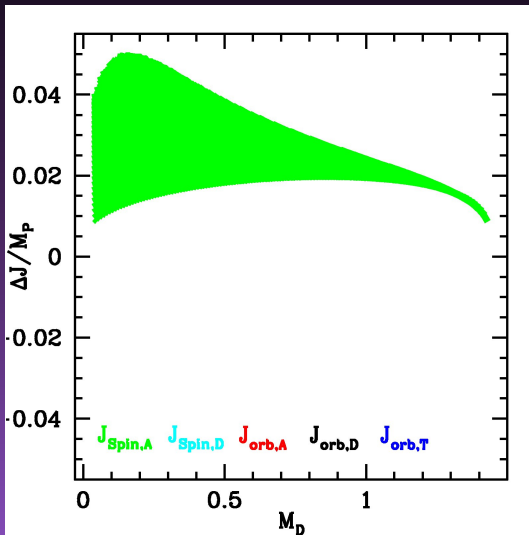
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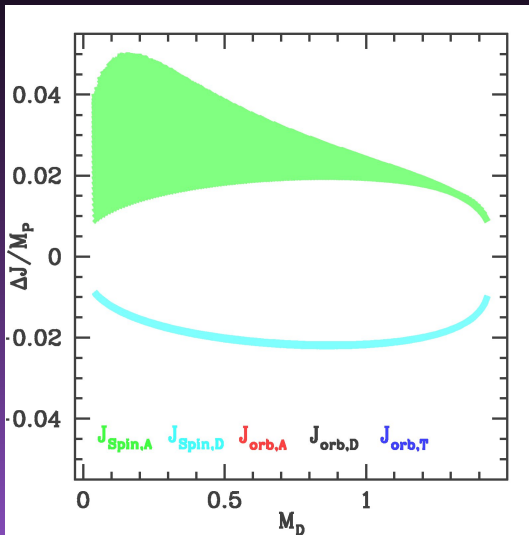
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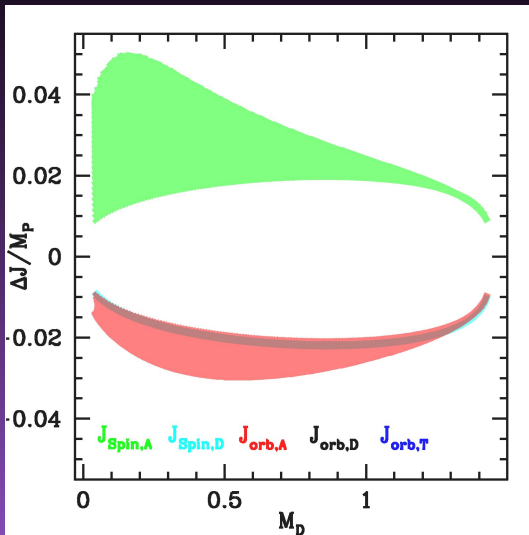
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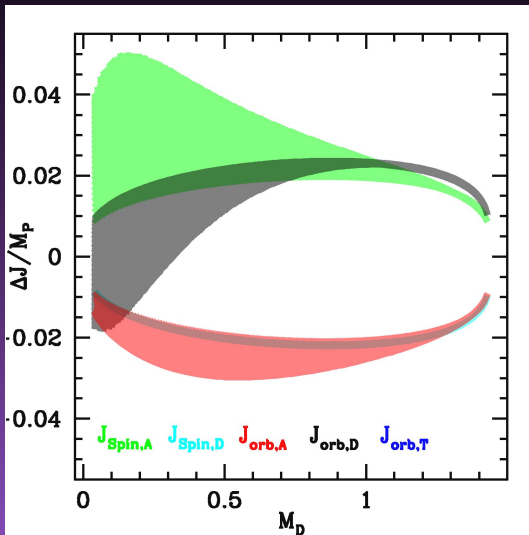
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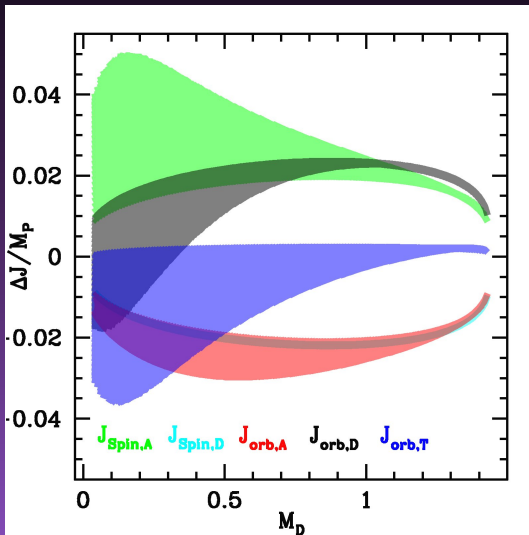
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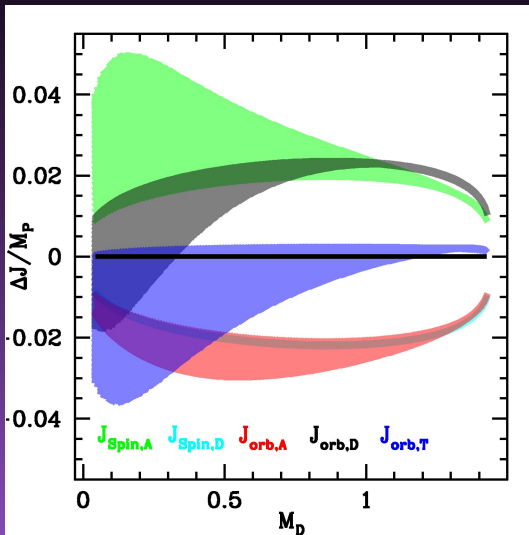
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Angular Momentum Changes

The Orbital Angular Momentum can Increase!

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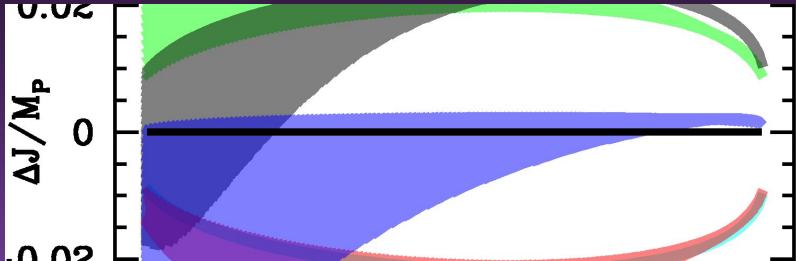
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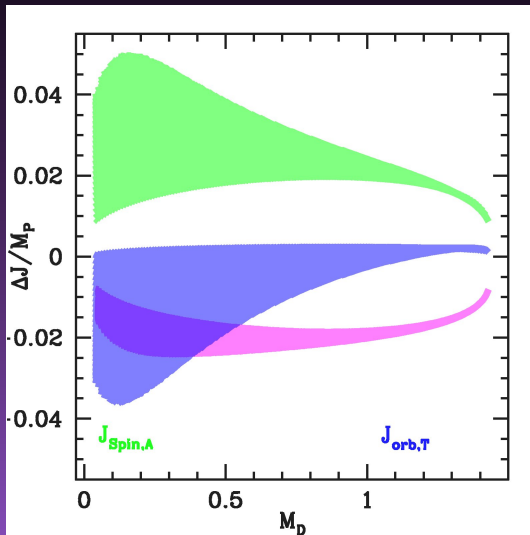
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Comparison with Previous Work

Magenta: Average particle orbital angular momentum from Verbunt & Rappaort (1988)





Angular Momentum and Stability

So what does all this mean?

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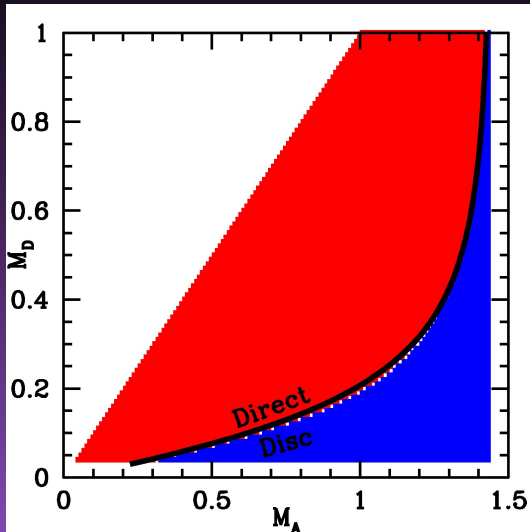
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Solid Line: where r_{min} from Lubow & Shu (1975) equals the Accretor Radius



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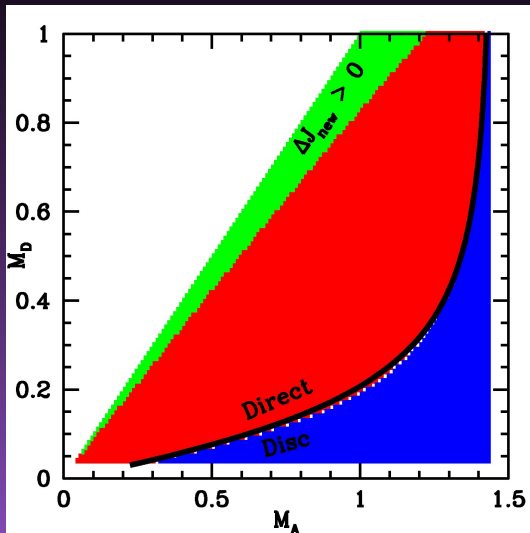
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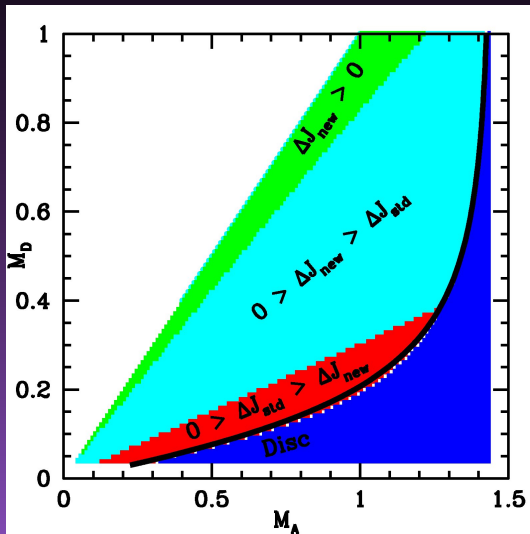
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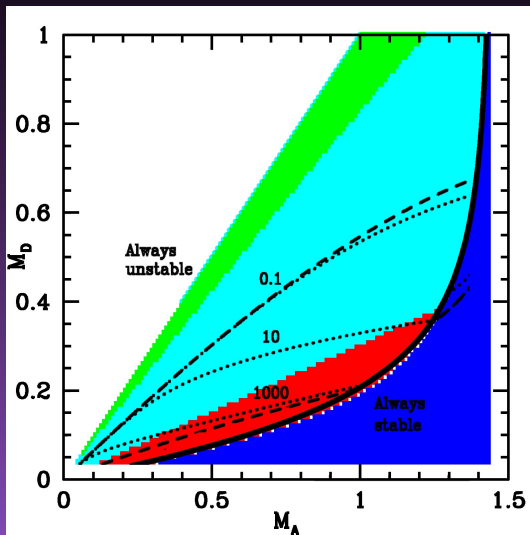
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Assuming Donor Remains Synchronous

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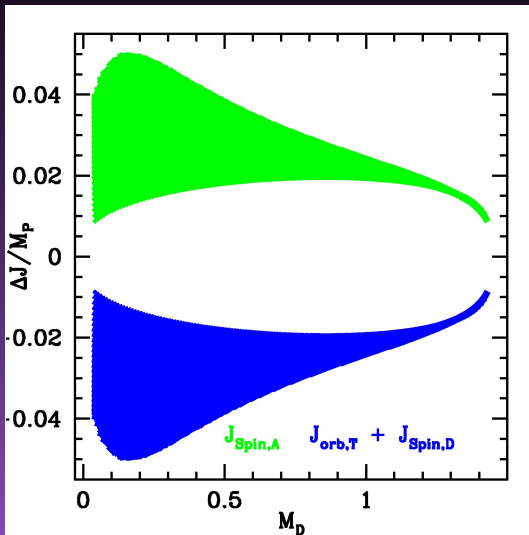
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Assuming Donor Remains Synchronous

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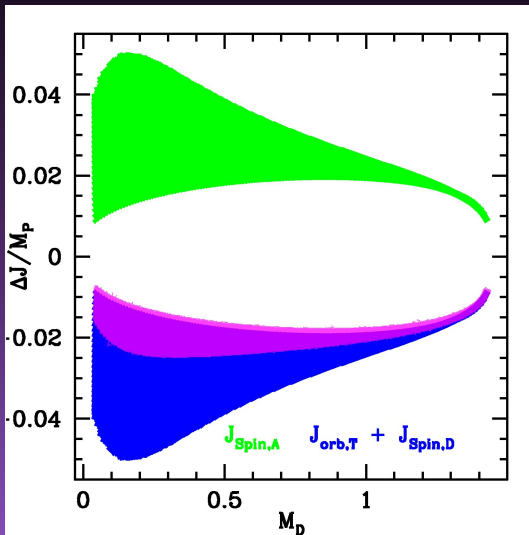
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Conclusions

- Conservation of angular momentum during ejection and accretion change the spin of each star, as well as their orbital angular momentum
- The resultant change in the total angular momentum can be significantly different than the standard model and may *increase* the orbital angular momentum of the system
- This may affect the stability of mass transfer and may significantly change the survivability of DWD into AM CVn



Conclusions & Future Work

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Future Work

The new rates of change of orbital angular momentum bring up three important areas of future work

- How do the new momenta changes affect the instantaneous stability?
- What does the long term, steady-state system look like?
- What role does an asynchronous donor play in the evolution of the system?



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Thank you! Any Questions?



The Equations

On Ejection (For more details see Sepinsky et al. (2010))

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Shift in the Center of mass:

$$\Delta \vec{R}_D = -\frac{M_P}{M_D - M_P} (\vec{R}_P - \vec{R}_D) \quad (1)$$

Shift in the Linear Momentum:

$$\Delta \vec{V}_D = -\frac{M_P}{M_D - M_P} (\vec{V}_P - \vec{V}_D) \quad (2)$$

Change in Spin Angular Momentum:

$$\Delta \mathbf{J}_{\text{Spin},D} = -\frac{M_D M_P}{M_D - M_P} [(\vec{R}_P - \vec{R}_D) \times (\vec{V}_P - \vec{V}_D)] \quad (3)$$



The Equations

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Then A Numerical Integration Occurs...



The Equations

On Accretion (For more details see Sepinsky et al. (2010))

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Shift in the Center of mass:

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Shift in the Linear Momentum:

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Change in Spin Angular Momentum:

$$\Delta J_{\text{Spin},A} = \frac{M_A M_P}{M_A + M_P} [(\vec{R}_P - \vec{R}_A) \times (\vec{V}_P - \vec{V}_A)] \quad (3)$$