Magnetohydrodynamic waves in coronal polar plumes

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Polar plumes are cool, dense, linear, magnetically open structures that arise from predominantly unipolar magnetic footpoints in the solar polar coronal holes. As the Alfvén speed is decreased in plumes in comparison with the surrounding medium, these structures are natural waveguides for fast and slow magnetoacoustic waves. The simplicity of the geometry of polar plumes makes them an ideal test ground for the study of magnetohydrodynamic (MHD) wave interaction with solar coronal structures. The review covers recent observational findings of compressible and incompressible waves in polar plumes with imaging and spectral instruments, and interpretation of the waves in terms of MHD theory.

Keywords: solar corona; EUV emission; magnetohydrodynamic waves

1. Introduction

Solar polar plumes are observed as bright quasi-radial rays in coronal holes at heights up to 10 or more solar radii by various coronal instruments in the EUV and VL channels (figure 1). They are remarkably quiescent structures and sometimes stay unchanged for days. Plumes originate in photospheric unipolar magnetic flux concentrations (DeForest et al. 1997), and highlight open magnetic structures in the corona. Over the origin, plumes expand super-radially (with a half-cone angle of 45°) in their lowest 20–30 Mm, and more slowly above that, with a linear super-radial expansion factor of about 3 at 4–5 solar radii and about 6 at 15 solar radii. There is some indication that pressure balance structures which are a common feature in high-latitude, fast solar wind near solar minimum are remnants of coronal polar plumes (Yamauchi et al. 2002). Plumes are believed to be filled with a low-β plasma and are regions of plasma density enhancements. Across plumes, the total pressure balance should be kept and consequently they are regions of Alfvén speed minima. Lower than 10 solar radii, outward plasma flows in plumes are slower than in inter-plume regions, the speed difference can be up to several 100 km s⁻¹ (Suess 1998).

For coronal wave studies, polar plumes are interesting for several reasons. First of all, coronal holes are the regions of the solar wind acceleration which is observed to happen at heights below five solar radii. The acceleration can be connected with the deposition of mechanical momentum carried by low-frequency waves (e.g. Ofman & Davila 1998). Plumes are well observed exactly...
at the heights of the wind acceleration. The enhanced emission coming from polar plumes—they are brighter than the inter-plume medium—makes the detection of the waves more probable. Also, the relatively high number of photons coming from bright plumes allows for the resolution of waves with shorter periods. Moreover, the relatively simple geometry of polar plumes permits the use of simpler theoretical models. In addition, as plumes are the regions of the Alfvén speed minima, they are natural fast magnetoacoustic wave guides. As the propagation of other magnetohydrodynamic (MHD) modes, the Alfvén wave and the slow magnetoacoustic wave, is confined to the magnetic field, polar plumes are also Alfvén and slow wave guides. Thus, the study of MHD wave activity in polar plumes is an important and interesting branch of solar coronal physics.

2. Compressible waves

(a) Observations

The first observational indication of the presence of compressible perturbations in polar plumes was found by Withbroe (1983) in the data obtained with the Harvard Skylab experiment. Statistically significant short period variations of Mg II emission intensity were detected, with the amplitudes of about 10% and possibly moving at speeds lower than 100–200 km s\(^{-1}\).

Soon after the launch of the Solar and Heliospheric Observatory (SOHO), Ofman et al. (1997) using the white light channel of SOHO/UVCS discovered in coronal holes at the height of about 1.9 \( R_\odot \) polarized brightness (density) fluctuations with periods of about 9 min. The signal was integrated over a 14\(\times\)14\(''\) area with an exposure time of 60–180 s and a cadence time of 90–210 s. Developing this study and analysing longer UVSC data sequences with a 75–125 s cadence time, Ofman et al. (2000b) determined the fluctuation periods...
as 7–10 min. The estimation of the propagation speed of the fluctuations gave values in the range of 160–260 km s\(^{-1}\) at 2 \(R_\odot\). This range is consistent with the phase speed of slow magnetoacoustic waves (possibly, slightly accelerated by nonlinearity).

A similar phenomenon was observed by Morgan et al. (2004) as a quasi-periodic variation of the Ly\(\alpha\) intensity observed with SOHO/UVCS as the light scattered at heights between 1.5 and 2.2 \(R_\odot\), with the period 7–8 min. Statistically significant correlations were found between oscillation patterns at neighbouring pixels along a vertical slit, suggesting that the variations was positioned along the polar plumes and were possibly associated with compressible waves. Perhaps, the same waves are also observed in the transition region with SOHO/CDS (Banerjee et al. 2000) as periodic (10–25 min) intensity fluctuations of the O \(\text{V} \ 629\ \text{Å}\) line, corresponding to a formation temperature of 0.25 MK.

Deforest & Gurman (1998) analysed SOHO/EIT 171 Å datasets and constructed time–distance maps for vertical strips over several plumes at the heights 1.01–1.2 \(R_\odot\). These maps contained diagonal stripes. Such a diagonal stripe exhibits an EUV brightness disturbance which changes its position in time and, consequently, propagates along the path. The variation of the intensity in the disturbances suggests that they are perturbations of density and, consequently, the disturbances are compressive. The observed amplitude of the intensity variations was estimated as 10–20%. Assuming that the emission intensity is proportional to the density squared, we obtain that the amplitude of the density perturbations was about 5–10%. Outwardly propagating waves were observed only. The wave periods were 10–15 min, gathered in envelopes of 3–10 periods. The duty cycle of the waves was roughly symmetric. The projected speeds were about 75–150 km s\(^{-1}\), which was lower than the estimated sound speed for this bandpass, of about 152 km s\(^{-1}\). Further analysis revealed that the wave amplitude grows with height, which was consistent with the interpretation of the waves as slow magnetoacoustic waves.

It is not yet clear whether the 10 min period fluctuations discovered with UVCS and the waves observed with EIT are the same waves. Correlation studies between plume images taken simultaneously with EIT and UVCS have, as yet, been inconclusive.

(b) Theoretical modelling

For typical plume parameters: the electron temperature of about 1.6 MK and the concentration of \(10^8\ \text{cm}^{-3}\), the electron collisional frequency is about 2.3 Hz (Ballai & Marcu 2004). For the proton collisions the frequency is about 40 times smaller. Waves with frequencies much lower than this value, including the waves mentioned above, can be satisfactorily described by MHD theory.

A theory of slow magnetoacoustic waves in polar plumes has been created by Ofman et al. (1999, 2000\(^b\)). The model incorporates several physical mechanisms of the wave evolution, namely dissipation due to thermal conduction and viscosity, effects of stratification and magnetic field divergence, and weak nonlinearity. The latter mechanism is important because slow magnetoacoustic waves propagating upwards in a stratified atmosphere are amplified and consequently become more and more nonlinear, provided the effect of dissipation does not prevent it.
In the model discussed, the magnetic field is taken to be radially divergent

$$B(r) = B_0(r) \frac{R_\odot^2}{r^2},$$

(2.1)

where $B_0$ is the magnetic field strength at the base of the corona ($r = R_\odot$) and $r$ is the radius vector (see figure 2). Such a configuration corresponds to a magnetic monopole and is obviously not correct for modelling a global structure of the solar magnetic field. However, this configuration models the local magnetic structure of a plume over the lowest 20–30 Mm very well. The temperature $T$ and consequently, the sound speed $C_s$ are taken to be constant, however, it is not difficult to generalize the model to the case of radially stratified temperature. The hydrostatic equilibrium profile of the gravitationally stratified density is

$$\rho_0(r) = \rho_{00} \exp \left( \frac{-R_\odot}{H} \left( 1 - \frac{R_\odot}{r} \right) \right),$$

(2.2)

where $H$ is the scale height, $H$/Mm $\approx 50T$/MK, and $\rho_{00}$ is density at $r = R_\odot$. The vertical dependence of the gravitational acceleration is $g = GM_\odot/r^2$, where $M_\odot$ is the mass of the Sun and $G$ the gravitational constant.

Spherically symmetric slow magnetoacoustic waves, which perturb the density $\rho$ and the vertical component $V_r$ of the plasma velocity are described by the wave equation

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} - \frac{C_s^2}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \tilde{\rho}}{\partial r} \right) - g \frac{\partial \tilde{\rho}}{\partial r} = \text{RHS},$$

(2.3)

where nonlinear and dissipative terms are gathered on the right-hand side and represented by RHS. Here $\tilde{\rho}$ is the absolute value of the density perturbation.

When the considered wavelengths $\lambda$ are much less than both the scale height $H$ and $R_\odot$, equation (2.3) can be solved in the WKB (or the single wave) approximation, introducing the small parameter $\epsilon = \lambda/H \ll 1$. The nonlinear, quadratic in the weakly nonlinear limit, and dissipative terms can be considered to be of the same order in $\epsilon$. Following an upwardly propagating wave and
passing to the moving frame of reference \( \xi = \mathbf{r} - C_s t \), \( R = \epsilon \mathbf{r} \), we obtain

\[
\frac{\partial \tilde{\rho}}{\partial R} + \left( \frac{1}{R} + \frac{g(R)}{2C_s^2} \right) \tilde{\rho} + \frac{1}{\rho_0(R)} \tilde{\rho} \frac{\partial \tilde{\rho}}{\partial \xi} - \frac{2\eta_0}{3C_s\rho_0(R)} \frac{\partial^2 \tilde{\rho}}{\partial \xi^2} = 0, \tag{2.4}
\]

where \( \eta_0 \) is the dissipation coefficient connected with bulk viscosity. Accounting for effects of thermal conduction will change the qualitative value of the dissipative coefficient but not the structure of equation (2.4).

Equation (2.4) is the spherical Burgers equation. Neglecting the nonlinear and dissipative terms, we obtain the ideal linear solution

\[
\tilde{\rho} = \rho(R_\odot) \frac{1}{R} \exp \left[ -\frac{R_\odot}{2H} \left( 1 - \frac{R_\odot}{R} \right) \right]. \tag{2.5}
\]

This expression shows that the absolute amplitude of the density perturbation decreases with height, while the ratio of the density perturbation amplitude and the local equilibrium density value grows,

\[
\frac{\tilde{\rho}}{\rho_0} \propto \frac{1}{R} \exp \left[ \frac{R_\odot}{2H} \left( 1 - \frac{R_\odot}{R} \right) \right]. \tag{2.6}
\]

Such behaviour of slow magnetoacoustic waves has been confirmed observationally, see figure 3.

The altitude growth of slow waves is limited by nonlinear effects connected with the finite amplitude of the waves. According to Ofman et al. (1999, 2000a) the waves observed with UVCS and EIT can develop into shocks and become subject to nonlinear dissipation. For the detected parameters of the waves
(relative amplitude about several percent, periods of about 10 min) the shock formation heights are from 1.1 to 1.4 solar radii. Similar heights were obtained by Cuntz & Suess (2001) in the model incorporating plume geometrical spreading, thermal conduction and radiative damping. Waves with shorter periods are subject to nonlinear dissipation at lower heights.

The physical mechanisms driving slow waves in polar plumes and responsible for their periodicity have not yet been understood. Theoretically, there may be several possibilities: the waves can result from certain overstability (e.g. thermal instability), they can be driven by p-modes (e.g. by the mechanism suggested by De Pontieu et al. (2004), through the parametric resonance (Zaqarashvili et al. 2005), etc.

3. Alfvén waves

(a) Theoretical modelling

In the model introduced in §2b (see figure 2), from equations (2.1) and (2.2), the vertical profile of the Alfvén speed $C_A$ is given by

$$C_A(r) = \frac{B_0(R_\odot)R_\odot^2}{r^2[4\pi\rho_0(R_\odot)]^{1/2}} \exp\left(\frac{R_\odot(r - R_\odot)}{2Hr}\right). \quad (3.1)$$

In this geometry, linear Alfvén waves perturb the $\phi$-components of the magnetic field and velocity and are described by the wave equation,

$$\frac{\partial^2 V_\phi}{\partial t^2} - \frac{B_0(r)}{4\pi\rho_0(r)} \frac{\partial^2}{\partial r^2} [r B_0(r) V_\phi] = \text{RHS}, \quad (3.2)$$

where the RHS are nonlinear and dissipative terms.

Following the upwardly propagating waves and applying the WKB approximation discussed in §2b, Nakariakov et al. (2000) derived the evolutionary equation

$$\frac{\partial V_\phi}{\partial R} - \frac{R_\odot^2}{4H} \frac{1}{R^2} V_\phi - \frac{1}{4C_A(C_A^2 - C_s^2)} \frac{\partial V_\phi^3}{\partial r} - \frac{\nu}{2C_A^3} \frac{\partial^2 V_\phi}{\partial r^2} = 0, \quad (3.3)$$

which is a spherical analogue of the scalar Cohen–Kulsrud–Burgers equation. The main difference between this equation and the spherical Burgers equation (2.4) is the order of the nonlinearity, which affects the characteristic wave signature. In particular, if an initially harmonic acoustic wave develops into a saw-tooth wave, the Alfvén wave develops into a square wave (‘meander’).

In the linear and dissipationless regime, the third and the fourth terms of equation (3.3) can be neglected and the height evolution of the wave amplitude is given by the linear solution

$$V_\phi = V_\phi(R_\odot)\exp\left(\frac{R_\odot(R - R_\odot)}{4HR}\right). \quad (3.4)$$

This coincides with the result $V_\phi \propto \rho_0^{-1/4}(r)$ following from the Poynting flux conservation.
(b) Indirect evidence

The growth of the Alfvén wave amplitude with height, theoretically predicted by equation (3.4), can be observed through the non-thermal broadening of spectral lines. The width of an emission line can be expressed as

$$\Delta \lambda / \lambda \propto \left( kT/m_i + \phi + \xi^2 \right)^{1/2}, \quad (3.5)$$

where the first term on the right-hand side represents the width of the line due to the temperature (with the mass of the emitting ion in the denominator), the second term corresponds to the instrumental width and the third term is due to unresolved motions along the line-of-sight. Those motions can be connected with plasma turbulence and/or with waves. If this broadening is caused by linear Alfvén waves and is described by equation (3.4), it should increase with height.

O’Shea et al. (2003) detected the growth of non-thermal broadening of the Mg×625 Å line in both plume and inter-plume plasmas with SOHO/CDS, qualitatively consistent with the altitude growth given by equation (3.4). It was found that the line width at the same altitude is usually wider in the inter-plume regions than in the plume. At certain heights the broadening was seen to level off. A similar behaviour was earlier found by Banerjee et al. (1998) in inter-plume regions. Consequently, there is indirect evidence for the presence of unresolved Alfvén and possibly fast magnetoacoustic waves in polar plumes.

4. Effects of transverse structuring

More advanced models for wave propagation in polar plumes require taking into account effects of transverse structuring. Usually, spatial scales of transverse and longitudinal inhomogeneities are very different, making it possible to consider them separately.

(a) A polar plume as a magnetoacoustic waveguide

In the lower corona both plume and inter-plume regions are filled with a low-β plasma. Hence, as plumes are regions with plasma density enhancements, the Alfvén speed there is decreased. The regions of the decreased Alfvén speed are stretched along the radial magnetic field. Thus, according to the theory of MHD modes of plasma cylinders see, (e.g. Roberts et al. 1984), polar plumes are waveguides for fast magnetoacoustic waves. The guiding mechanism is either reflection or refraction of the waves into the regions with lower speed. An important class of magnetoacoustic modes of a plasma cylinder are trapped modes, which are confined to the cylinder and are evanescent in the external medium. There can be magnetoacoustic guided modes of various kinds, with properties determined by the symmetry of the cylinder perturbations (e.g. sausage, kink and ballooning). Phase speeds of trapped modes are higher than the Alfvén speed inside the cylinder but slower than this speed outside. When the wavelength of the mode is comparable to cylinder radius, the mode is highly dispersive. Phase speeds of all trapped fast magnetoacoustic modes decrease with the growth of their wave numbers.
Recently, Ballai & Marcu (2004) developed this theory further, accounting for gas pressure anisotropy (see also Nakariakov & Oraevsky 1995). The anisotropy can be connected with the difference in the transport processes operating along and across the background magnetic field. No significant changes of dispersive properties of fast magnetoacoustic modes were found in the low-$\beta$ case. However, the anisotropy effect can be significant at higher altitudes, where $\beta$ is closer to unity. Slow magnetoacoustic waves can be affected by the anisotropy at lower heights.

No direct observational evidence for the presence of fast magnetoacoustic modes guided by polar plumes has been found yet.

(b) Phase mixing

In a 1D transverse inhomogeneity, Alfvén waves are subject to phase mixing: the waves propagate on different magnetic surfaces with different speeds and, in some time, perturbations of different magnetic surfaces become uncorrelated with each other. This effect generates very high transverse gradients, leading to enhanced dissipation of the waves (Heyvaerts & Priest 1983) because of viscosity or resistivity. In the developed stage of phase mixing, the Alfvén wave amplitude decays faster than exponentially,

$$V_\phi (r) \propto \exp \left\{ -\frac{\nu \omega^2}{6 C^0_A(x)} \left[ \frac{d C_A(x)}{dx} \right]^2 r^3 \right\}.$$ (4.1)

(Here, $r$ is the vertical, field-aligned coordinate, and $x$ is the coordinate across the plume boundary.) This effect is accompanied by enhanced nonlinear excitation of oblique fast magnetoacoustic waves (Nakariakov et al. 1997), which may be responsible for the generation of compressive perturbations in plumes.

If instead of a monochromatic wave in the longitudinal direction a localized Alfvén pulse is considered, Heyvaerts and Priest’s expression for the exponential decay equation (4.1) should be replaced by the power law, $V_\phi \propto r^{-3/2}$, see Hood et al. (2002) and Tsiklauri et al. (2003) for two methods of derivation and comparison with results of numerical modeling.

Two-dimensional inhomogeneity modifies the dissipation law (4.1) too. Taking into account that longitudinal structuring is much weaker than transversal, Ruderman et al. (1998) investigated the effect of longitudinal inhomogeneity connected with stratification, non-radial field expansion and spherical geometry, on the efficiency of phase mixing. It was concluded that the rate of wave damping due to phase mixing in two-dimensional magnetic configurations depends strongly on the particular geometry of the configuration and can be either weaker or stronger than that given by equation (4.1).

Numerical 2D modelling of Alfvén wave interaction with a polar plume has been carried out by Ofman & Davila (1998). It was found that the nonlinear excitation of oblique fast waves is not efficient in plumes, while the rapid dissipation because of structuring certainly takes place and competes with nonlinear dissipation (§3a).

(c) Wave-flow interaction

Coronal holes are regions of the enhanced acceleration of solar wind. There is some observational evidence that the efficiency of the acceleration is different in...
plumes and in inter-plume regions—plumes flow more slowly than inter-plume plasma inside 10 solar radii. This would lead to structuring of the radial flow $U_0(x)$ across the plume. Sufficiently sharp flow gradients would modify the phase mixing altitude dependence given by equation (4.1) (Nakariakov et al. 1998),

$$V_\phi(r) \propto \exp\left\{-\frac{v\omega^2}{6C_\Lambda^2(x)} \left[ \frac{d}{dx}(U_0(x) + C_\Lambda(x)) \right]^2 r^3 \right\}. \quad (4.2)$$

When the flow shear exceeds some certain threshold, the plasma configuration is subject to various instabilities, e.g. the Kelvin–Helmholtz instability. This effect can be responsible for plume—inter-plume mixing and consequently for the plume destruction. According to Suess (1998), polar plumes can become unstable to this instability at the heights over 10 solar radii. However, the lowest threshold instabilities are the family of the negative energy wave instabilities: dissipative, radiative (Joarder et al. 1997) and resonant flow (Hollweg et al. 1990). In the presence of the flow shear, certain wave modes of the plume can have negative energy, in other words their amplitude is growing while its energy reduces because of some dissipative process or wave leakage outside the plume or to another mode. In particular, this can invert the rate of resonant absorption, amplifying certain wave modes. In this case, the energy comes to the wave from the steady flow.

Andries et al. (2000) considered a plume as a straight jet of a dense zero-\(\beta\) plasma and demonstrated that when the flow shear is strong enough for the frequency of the originally (without the flow) forward-propagating non-leaky mode to be in the frequency range of the backward propagating waves, the mode becomes overstable. In particular, it was found out that the threshold of the instability is sensitive to the density contrast between the plume and the inter-plume medium. The threshold value of the velocity shear decreases with the growth of the density contrast. That study was continued in (Andries & Goossens 2001) accounting for finite-\(\beta\) effects. It was shown that the instability which most probably occurs in plumes is due to an Alfvén resonance of slow body modes. However, the studies mentioned above were restricted to simple 1D transverse inhomogeneities and consequently were local. Incorporation of radial stratification and radial variation of the flow shear in this model would be very interesting.

In the vicinity of the resonant layer, the wave amplitude can become sufficiently high for nonlinear effects to come into play. Modification of the resonant flow instability by the nonlinearity has not been well understood yet (see, however, Erdélyi et al. (2001) for recent progress in the study of the weakly nonlinear regime of the instability).

5. Conclusions

The main conclusions may be summarized as following:

(i) There is direct evidence of the presence of slow magnetoacoustic waves in solar coronal polar plumes obtained with EUV imagers. The wave amplitudes are several percent and periods are several minutes.
for Alfvén and fast magnetoacoustic waves is indirect yet abundant in both imaging and spectral data. The source of these waves and the physical mechanism responsible for the observed periodicity has not yet been understood.

(ii) Simple 1D analytical models allow us to take into account effects of stratification, spherical geometry, weak nonlinearity and dissipation. MHD wave evolution predicted by these models is in satisfactory qualitative agreement with the observational findings.

(iii) According to theoretical findings, the effects of transverse structuring in plumes can be crucial for wave dynamics. Polar plumes provide us with an ideal test ground for the study of such effects as phase mixing, guided wave propagation, weak nonlinearity and negative energy waves.

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References


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