Parker’s solar wind model

The corona cannot remain in static equilibrium but is continually expanding. The continual expansion is called the solar wind.

Assume that the expanding plasma of the solar wind is **isothermal** and **steady**.

The governing equations can be obtained from the MHD equations setting $\partial/\partial t = 0$:

\begin{align}
\nabla \cdot (\rho \mathbf{V}) &= 0, \quad (1) \\
\rho (\mathbf{V} \cdot \nabla) \mathbf{V} &= -\nabla p + \rho \mathbf{g}, \quad (2) \\
p &= \rho RT, \quad (3)
\end{align}

and

\[ T = T_0. \quad (4) \]

Also, we restrict our attention to the spherically symmetric solution. The velocity $\mathbf{V}$ is taken as purely radial, $\mathbf{V} = ve_r$ and the gravitational acceleration $\mathbf{g} = ge_r$ obeys the inverse square law,

\[ g = -\frac{GM_\odot}{r^2}. \quad (5) \]

The temperature and, consequently, the sound speed

\[ C_s^2 = p/\rho, \quad (6) \]

are constant.

We are interested, for simplicity, in the dependence on the $r$ coordinate only. Thus, the expressions for the differential operators in the spherical coordinates are

\[ \nabla a = \frac{da}{dr}, \quad \nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{d}{dr} \left( r^2 A_r \right). \]

In the spherical geometry, the governing equations describing and the radially symmetric values are

\begin{align}
\rho v \frac{dv}{dr} &= -\frac{dp}{dr} - \frac{GM_\odot \rho}{r^2}, \quad (7) \\
\frac{d}{dr} (r^2 \rho v) &= 0 \quad \Rightarrow \quad r^2 \rho v = \text{const.} \quad (8)
\end{align}

Substituting (6) into (7) we exclude the pressure from the equations,

\begin{align}
\rho v \frac{dv}{dr} &= -C_s^2 \frac{d \rho}{dr} - \frac{GM_\odot \rho}{r^2}, \quad (9) \\
or \quad v \frac{dv}{dr} &= -C_s^2 \frac{1}{\rho} \frac{d \rho}{dr} - \frac{GM_\odot}{r^2}. \quad (10)
\end{align}
To exclude $\rho$, we use (8),
\[
\frac{d}{dr}(r^2 \rho v) = \rho \frac{d}{dr}(r^2 v) + r^2 v \frac{d\rho}{dr} = 0,
\]
and obtain
\[
\frac{1}{\rho} \frac{d\rho}{dr} = -\frac{1}{r^2 v} \frac{d}{dr}(r^2 v).
\]
Now, Eq. (10) becomes
\[
v \frac{dv}{dr} = \frac{C_s^2}{r^2} \frac{d}{dr}(r^2 v) - \frac{GM_\odot}{r^2}.
\]
Rewriting this equation, we obtain
\[
\left(v - \frac{C_s^2}{v}\right) \frac{dv}{dr} = \frac{2C_s^2}{r} - \frac{GM_\odot}{r^2},
\]
and, then
\[
\left(v - \frac{C_s^2}{v}\right) \frac{dv}{dr} = 2 \frac{C_s^2}{r^2} (r - r_c),
\]
where $r_c = GM_\odot/(2C_s^2)$ is the critical radius showing the position where the wind speed reaches the sound speed, $v = C_s$.

This is a separable ODE, which can readily be integrated,
\[
\int \left(v - \frac{C_s^2}{v}\right) dv = \int \frac{2C_s^2}{r^2} (r - r_c) dr,
\]
giving the solution
\[
\left(\frac{v}{C_s}\right)^2 - \log \left(\frac{v}{C_s}\right)^2 = 4 \log \left(\frac{r}{r_c}\right) + 4 \frac{r_c}{r} + C.
\]
The constant of integration $C$ can be determined from boundary conditions, and it determines the specific solution. Several types of solution are present in the figure:
Types I and II are double valued (two values of the velocity at the same distance), and are non-physical.

Types III has supersonic speeds at the Sun which are not observed.

Types IV seem also be physically possible. (The “solar breeze” solutions).

The unique solution of type V passes through the critical point \((r = r_c, v = C_s)\) and is given by \(C = -3\). It can be obtained from the general solution (17) by putting the coordinates of the critical point. This is the “solar wind” solution (Parker, 1958). It was discovered by Soviet Luna-2, Luna-3 and Venera-1 probes in 1959.

Let us estimate the critical radius \(r_c\). For a typical coronal sound speed of about \(10^5\) m/s, and the critical radius is

\[
r_c = \frac{GM_\odot}{2C_s^2} \approx 6 \times 10^9 \text{ m} \approx 9 - 10R_\odot.
\]

At the Earth’s orbit, the solar wind speed can be obtained by substituting \(r = 214R_\odot\) to Eq. (17), which gives \(v = 310\) km/s.

For the radial flow, the rotation of the Sun makes the solar magnetic field twist up into a spiral.

Suppose the magnetic field is inclined at an angle \(\phi\) to the radial solar wind velocity:

The component of the vector \(\mathbf{V}\) perpendicular to the vector \(\mathbf{B}\), \(v \sin \phi\), must equal the speed of the field line in that direction, because the field is frozen in the plasma. But,
the field is dragged by the solar rotation with the angular frequency $\Omega$. The normal component of the speed of the field line is $\Omega(r - R_{\odot})$. Consequently,

$$v \sin \phi = \Omega(r - R_{\odot}) \cos \phi,$$

which gives us

$$\tan \phi = \frac{\Omega(r - R_{\odot})}{v}$$

Taking $v \approx 310$ km/s and calculating the frequency of the equatorial rotation period, which is about 26 days, $\Omega = \frac{2\pi}{(26 \times 24 \times 60 \times 60)} \approx 2.8 \times 10^{-6}$ rad/s, we obtain that near the Earth’s orbit, $r \approx 214 R_{\odot}$, the angle is about 45°.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{solar_wind_diagram.png}
\end{figure}

\textit{In-situ} observations have established that there are actually two component in the solar wind,

\begin{itemize}
  \item relatively low-speed streams ($v < 350$ km/s) - the “slow solar wind” and
  \item high-speed streams ($v$ up to 800 km/s) - the “fast wind”.
\end{itemize}

The slow wind is denser and carries greater flux of particles. The presence of the fast wind has been observed at higher solar latitudes:
Realtime monitoring of the solar wind near the Earth’s orbit:
Forecasting of the arrival time of coronal mass ejections (CMEs):