Number Systems I

CIS008-2 Logic and Foundations of Mathematics

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Outline

R. Dedekind

"Numbers are free creations of the human mind that serve as a medium for the easier and clearer understanding of the diversity of thought."

Systems of numbers

- natural numbers,
  $1, 2, 3, \ldots$
- integers,
  $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
- rational numbers
- real numbers
- *complex numbers* (not covered in this course)

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Natural Numbers - $\mathbb{N}$

- The set of Natural numbers are symbolised by $\mathbb{N}$
- Sometimes called ‘counting numbers’
- All positive integers belong to the set of natural numbers.
- Zero is **not** a natural number
- $1, 2, 3, \ldots$

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Integers - $\mathbb{Z}$

- The set of integers are symbolised by $\mathbb{Z}$
- The set of natural numbers belong to the set of integers
- Integers are whole numbers, including zero
- The set of natural numbers supplemented with zero and negative whole numbers is the set of integers
- $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Rational numbers - $\mathbb{Q}$

- The set of Rational numbers are symbolised by $\mathbb{Q}$
- Ratios of integers are rational numbers
- Rational numbers produce other rational numbers when added, multiplied, subtracted, or divided.
- Integers belong to the set of rational numbers

Real numbers - $\mathbb{R}$

- The set of Real numbers are symbolised by $\mathbb{R}$
- Not all numbers are included in the set of integers and rational numbers
- $\pi = 3.1415, \ldots$ cannot be represented as any ratio of integers
- the solution to $x^2 - 2 = 0$ cannot be represented by any rational number
- numbers that cannot be represented by ratios of integers are known as irrational numbers
- The set of rational numbers, together with the set of irrational numbers is the set of real numbers

Decimal Number System

- 10 symbols are used (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
- Read from right to left.
- 1^{st} symbol represents 1's (10^0), 2^{nd} represents 10's (10^1), 3^{rd} represents 100's (10^2), ... etc.
- We call the value on which the number system is based the **base** of the system (base 10 in the decimal system).

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Binary Number System

- 2 symbols are used (0, 1)
- Read from right to left.
- 1^{st} symbol represents 1's (2^0), 2^{nd} represents 2's (2^1), 3^{rd} represents 4's (2^2), . . . etc.
- The **base** of the binary system is 2.

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Binary Number System

Example

Binary to decimal

$$
\begin{aligned}
101101_2 &= (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) \\
&\quad +(0 \times 2^1) + (1 \times 2^0) \\
101101_2 &= (1 \times 32) + (0 \times 16) + (1 \times 8) + (1 \times 4) \\
&\quad +(0 \times 2) + (1 \times 1) \\
101101_2 &= 32 + 8 + 4 + 1 \\
101101_2 &= 45_{10}
\end{aligned}
$$

Hexadecimal Number System

- 16 symbols are used (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)
- Read from right to left.
- 1^{st} symbol represents 1's (16^0), 2^{nd} represents 16's (16^1), 3^{rd} represents 256's (16^2), ... etc.
- The **base** of the Hexadecimal system is 16.

Hexadecimal Number System

Example

Hexadecimal to decimal

$$
\begin{aligned}
B4F_{16} &= (11 \times 16^2) + (4 \times 16^1) + (15 \times 16^0) \\
B4F_{16} &= (11 \times 256) + (4 \times 16) + (15 \times 1) \\
B4F_{16} &= 2816 + 64 + 15 \\
B4F_{16} &= 2895_{10}
\end{aligned}
$$

Converting a decimal integer into Base b

Example

Convert the decimal number 3941 to an Octal number (Base 8):
Successive division by 8, recording the remainder

$3941 \div 8$	remainder 5	1's place
$492 \div 8$	remainder 4	8's place
$61 \div 8$	remainder 5	8^2's place
$7 \div 8$	remainder 7	8^3's place

gives:

$$3941_{10} = 7545_8$$

Converting an integer from Base b to Decimal

Write and algorithm in pseudocode that returns the decimal value of the base b integer $c_n c_{n-1} \dots c_1 c_0$. The variable n is used as an index in the sequence c.

Converting a Decimal integer into Base b

Write ans algorithm in pseudocode that converts the positive integer m into the base b integer $c_n c_{n-1} \dots c_1 c_0$. The variable n is used as an index in the sequence c. The value of $m \bmod b$ is the remainder when m is divided by b. The value of $[m/b]$ is the quotient when m is divided by b.

Adding Binary Numbers

Write and algorithm in pseudocode that adds the binary numbers $b_n b_{n-1} \dots b_1 b_0$ and $b'_n b'_{n-1} \dots b'_1 b'_0$ and returns the sum $s_{n+1} s_n s_{n-1} \dots s_1 s_0$.

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