Determining the type of neutrino mass hierarchy

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IPPP – Durham and CERN
• **Present knowledge of neutrino masses**

• **Absolute mass scale** (direct mass searches, cosmology)

• **Type of hierarchy**
  1) Neutrinoless double beta decay
  2) LBL $\nu$-oscillation experiments:
     - matter effects
     - solving degeneracies

• **Super-NO$\nu$A**: finding the type of hierarchy with no degeneracies.

• **Conclusions**
2 – Present status

- Solar neutrinos and KamLAND

\[ \Delta m_{\odot}^2 |_{BF} = 8.0 \times 10^{-5} \text{ eV}^2, \]

[SK, KamLAND 2004, SNO 2005]

- Atmospheric and accelerator neutrinos

\[ \Delta m_{\text{atm}}^2 |_{BF} = 2.1 \times 10^{-3} \text{ eV}^2, \]
\[ \Delta m_{\text{atm}}^2 |_{BF} = 2.8 \times 10^{-3} \text{ eV}^2. \]

[SK 2005, K2K 2005]

Stay tuned for MINOS results soon to come!!

\[ \Delta m_{\odot}^2 \ll \Delta m_{\text{atm}}^2 \]
The existence of \textit{neutrino oscillations} is crucial in our understanding of neutrino physics as it implies that

\textbf{NEUTRINOS ARE MASSIVE AND THEY MIX.}

Future questions:

- **nature of neutrinos**: Dirac vs Majorana particles.
- absolute values of \textit{neutrino masses}.
- establish CP-violation in the lepton sector.
- Number of neutrinos (sterile $\nu$).
$\Delta m_\odot^2 \ll \Delta m_{\text{atm}}^2$ implies at least 3 neutrinos.

**Normal ordering**

\[
\begin{align*}
&\Delta m^2 \\
&\Delta m_\odot^2 \\
&\Delta m_\odot^2 \\
\end{align*}
\]

**Inverted ordering**

\[
\begin{align*}
&\Delta m^2 \\
&\Delta m_\odot^2 \\
&\Delta m_\odot^2 \\
\end{align*}
\]

\[
m_1 = m_{\text{MIN}}
\]

\[
m_2 = \sqrt{m_{\text{MIN}}^2 + \Delta m_\odot^2}
\]

\[
m_3 = \sqrt{m_{\text{MIN}}^2 + \Delta m_{\text{atm}}^2}
\]

Measuring neutrino masses requires to know $m_{\text{MIN}}$ and $\text{sign}(\Delta m_{31}^2)$.

We can identify 3 types of spectra:

**NH:** $m_1 \ll m_2 \ll m_3$

**IH:** $m_3 \ll m_1 \simeq m_2$

**QD:** $m_1 \sim m_2 \sim m_3$. 
Neutrino masses

Absolute mass scale

$n$ oscillations are sensitive only to $\Delta m^2$.

- Direct mass searches in tritium beta decay experiments.

The present limit is $m_0 < 2.2$ eV (Troiztk and Mainz).

KATRIN can reach a sensitivity to $m_0 \sim 0.2$ eV, covering all the QD spectrum!!!
Cosmological observations.

Exploiting the effects of massive neutrinos on structure formation in the Early Universe it is possible to constrain the sum of neutrino masses.

Present: $\Sigma \sim 0.5$–1.5 eV  
Prospects: $\Sigma \sim 0.1$ eV.
3 – Neutrino masses

**Type of hierarchy**

- **$(\beta\beta)_{0\nu}$-decay exp**: it can distinguish between different types of spectra (NH, IH, QD). The role of $\sin^2 \theta_{13}$ is subdominant.

  [e.g. S.P., Petcov, Rodejohann, Schwetz]

- **Neutrino oscillations by exploiting matter effects** (requires sizable $\sin^2 \theta_{13}$):
  a) LBL neutrino oscillation experiments. Degeneracies arise with the CP-violating phase $\delta$.
  [e.g. Aoki, Barger, Burguet-Castell, de Gouvea, Donini, Fogli, Freund, Hagiwara, Huber, Kajita, Kayser, Lindner, Lisi, Marfatia, Meloni, Mena, Minakata, Nunokawa, Okamura, Palomares-Ruiz, Parker, S. P., Petcov, Rigolin, Yasuda, Whisnant, Winter, Jenkins]

  b) atmospheric neutrinos. e.g. [Bernabeu, Gandhi, Ghosal, Goswami, Hüber, Indumathi, Maltoni, Mehta, Murthy, Palomares-Ruiz, Petcov, Schwetz]

- **Vacuum neutrino oscillations**: require very high precision measurement of $\Delta m^2_{\text{atm}}$.
  [e.g. de Gouvea, Jenkins, Kayser, Nunokawa, Parke, Petcov, Piai, Zukanovich Funchal]
neutrinoless double beta decay: \((A, Z) \rightarrow (A, Z + 2) + 2e^-,\) is the most sensitive of processes \((\Delta L = 2)\) which can probe the nature of neutrinos (Dirac vs Majorana).

\((\beta\beta)_{0\nu}\)-decay has a special role in the study of neutrino properties, as it probes the violation of global lepton number, and it might provide information on the neutrino mass spectrum, absolute neutrino mass scale and CP-V.
The half-life time, $T_{0\nu}^{1/2}$, of the ($\beta\beta$)$_{0\nu}$-decay can be factorized, for light Majorana neutrinos, as:

$$\left( T_{0\nu}^{1/2}(0^+ \rightarrow 0^+) \right)^{-1} \propto |M_F - g_A^2 M_{GT}|^2 |< m >|^2$$

$|< m >|$ is the effective Majorana mass parameter:

$$|< m >| \equiv |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}}|$$

The present bound on $|< m >|$ reads $|< m >| < (0.35 - 1.05) \text{ eV}$ (Heidelberg-Moscow), $|< m >| < (0.20 - 1.05) \text{ eV}$ (Cuoricino) and $|< m >| < (0.65 - 2.8) \text{ eV}$ (NEMO3).

In 2002 and with a new more detailed analysis, evidence of ($\beta\beta$)$_{0\nu}$-decay was reported [Klapdor-Kleingrothaus et al., PLB 586 198 (2004)]:

$|< m >|_{BF} = 440 \text{ meV} \hspace{1cm} 120 \text{ meV} \leq |< m >| \leq 700 \text{ meV}.$

The next generation of ($\beta\beta$)$_{0\nu}$-decay exp (SuperNEMO, EXO, Majorana, CUORE, Cobra ...) aim to $|< m >| \sim 10^{−} - 30 \text{ meV}$. 
The predictions for $|<m>|$ depend strongly on the type of spectrum.

- **NH:**
  $$|<m>| \simeq \sqrt{\Delta m^2_{\odot}} \cos^2 \theta_1 \sin^2 \theta_{\odot} + \sqrt{\Delta m^2_{\text{atm}}} \sin^2 \theta_1 e^{i \alpha_{32}}$$

- **IH:**
  $$\sqrt{\Delta m^2_{\text{atm}}} \cos 2\theta_{\odot} \leq |<m>| \simeq \sqrt{\left(1 - \sin^2(2\theta_{\odot}) \sin^2 \alpha_{21} \right)} \Delta m^2_{\text{atm}} \leq \sqrt{\Delta m^2_{\text{atm}}}$$

$|<m>|$ has a significant lower bound

$$0.01 \text{ eV} \lesssim |<m>| \lesssim 0.06 \text{ eV}$$

$|<m>|$ is in the range of sensitivity of the upcoming $(\beta\beta)_{0\nu}$-decay experiments.

- **QD:**
  $$|<m>| \simeq m_{\bar{\nu}_e} \left( \cos^2 \theta_{\odot} + \sin^2 \theta_{\odot} e^{i \alpha_{21}} \right) \cos^2 \theta_1 + \sin^2 \theta_1 e^{i \alpha_{31}}$$

All the allowed range for $|<m>|$ is in the range of sensitivity of present and upcoming $(\beta\beta)_{0\nu}$-decay experiments.
• For the IH and QD spectra, present and/or future exp will probe the allowed values of $|<m>|$.

• As $\cos 2\theta_\odot > 0.25$, $|<m>|_{NH} < |<m>|_{IH}$.

$(\beta\beta)_{0\nu}$-decay exp can distinguish the NH vs IH and QD cases. This is independent from the value of $\theta_{13}$.

[S.P., Petcov, PLB2003; S.P., Petcov, Rodejohann, 2003]
A careful analysis yields:

\[
\sin^2 \theta_{13} = 0.03 \pm 0.006, \quad \sin^2 \theta_{12} = 0.31 \pm 3\%, \quad \Delta m_{21}^2 = 8 \times 10^{-5} \pm 2\%, \quad |\Delta m_{31}^2| = 2.2 \times 10^{-3} \pm 3\%
\]

The nuclear matrix uncertainties have a mild impact.

[S.P., Petcov and Schwetz, Accepted in NPB]
Long base-line neutrino oscillation experiments are sensitive to matter effects: neutrinos travelling through the Earth are affected by matter and a potential $V$ appears in the Hamiltonian ($V = \sqrt{2}G_F(N_e - N_n/2)$).

The mixing angle changes with respect to the vacuum case:

$$\sin 2\theta_m = \frac{(\Delta m^2/2E) \sin 2\theta}{\sqrt{\left(\frac{\Delta m^2}{2E} \sin 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E} \cos 2\theta - V\right)^2}}$$

For $\Delta m^2 > 0$, mixing gets enhanced for neutrinos and suppressed for antineutrinos.

For $\Delta m^2 < 0$, the opposite happens.
Matter effects imply that

\[ P(\nu_l \rightarrow \nu_l') \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_l') \]

If \( U \) is complex we have CP-violation:

\[ P(\nu_l \rightarrow \nu_l') \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_l') \]

There are degenerate solutions:

\[ \Delta m^2_{31}, \theta_{13}, \delta, \theta_{23} \]

\[ \Delta m^2_{31}', \theta_{13}', \delta', \theta_{23}' \]

P, \bar{P}

It is necessary to disentangle true CP-V effects due to the \( \delta \) phase from the ones induced by matter: degeneracies.
In the range of energies \((E \sim 0.5 \div 4 \text{ GeV})\) and length \((L \sim 200 \div 1000 \text{ Km})\), of interest, the oscillation probability for \(\nu_\mu \rightarrow \nu_e\), in the 3-neutrino mixing case, is given by:

\[
P(\bar{P}) \simeq \frac{s_{23}^2}{2} \sin^2 2\theta_{13} \left( \frac{\Delta_{13}}{A + \Delta_{13}} \right)^2 \sin^2 \frac{(A + \Delta_{13})L}{2} \\
+ \tilde{J} \frac{\Delta_{12}}{A} \frac{\Delta_{13}}{A + \Delta_{13}} \sin \frac{AL}{2} \sin \frac{(A + \Delta_{13})L}{2} \cos \left( \mp \delta + \frac{\Delta_{13}L}{2} \right) \\
+ c_{23}^2 \sin^2 2\theta_{12} \left( \frac{\Delta_{12}}{A} \right)^2 \sin^2 \frac{AL}{2}
\]

with \(\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12}\) and \(\Delta_{13} \equiv \Delta m_{31}^2 / (2E)\). \(A \equiv \sqrt{2} G_F \bar{n}_e\).
In the vacuum case, for simplicity, we identify 2-, 4- and 8- fold degeneracies [Barger, Marfatia, Whisnant]:

- \((\theta_{13}, \delta)\) degeneracy [Koike, Ota, Sato; Burguet-Castell et al.]:
  \[
  \delta' = \pi - \delta \\
  \theta'_{13} = \theta_{13} + \cos \delta \sin 2\theta_{12} \frac{\Delta m_{12}^2 L}{4E} \cot \theta_{23} \cot \frac{\Delta m_{13}^2 L}{4E}
  \]

- \((\text{sign}(\Delta m_{13}^2), \delta)\) degeneracy [Minakata, Nunokawa]:
  \[
  \delta' = \pi - \delta \\
  \text{sign}'(\Delta m_{13}^2) = -\text{sign}(\Delta m_{13}^2)
  \]

- \(\theta_{23}, \pi/2 - \theta_{23}\) degeneracy [Fogli, Lisi].
LBL oscillation experiments

\[ \sin^2 2 \theta_{13} = 0.058 \]
\[ \delta = 300 \]
\[ L = 810 \text{ km} \]

[from O. Mena, S. Palomares-Ruiz, S.P.]
Degeneracies worsen the sensitivity of future experiments to $\theta_{13}$, the type of hierarchy and CP-violation.
5 – LBL oscillation experiments

Sensitivity to the sign of $\Delta m^2_{31}$

- Systematic
- Correlation
- Degeneracy

- JHF–SK
- NuMI
- JHF–HK
- NuFact–I
- NuFact–II

$10^{-6}$  $10^{-5}$  $10^{-4}$  $10^{-3}$  $10^{-2}$  $10^{-1}$

$\sin^2 2\theta_{13}$

[Refs: Lindner et al.]
It is crucial to resolve such degeneracies. For example:

- combination of different channels: gold ($\nu_\mu \rightarrow \nu_e$) + silver channels ($\nu_e \rightarrow \nu_\tau$). [Burguet-Castell et al.]

- use different energies and/or baselines: NUMI + T2K. [Minakata, Nunokawa, Parke; Mena, Parke].

- use energy spectrum. [Rubbia]

- reactors neutrinos + LBL exp. [Minakata, Sugiyama, Yasuda]

- atmospheric neutrinos + LBL. [Huber, Schwetz, Winter]
1. Superbeams.

2. Neutrino factories.

Superbeams (NO$\nu$A, T2K, BNL):
a very intense $\nu_\mu$ beam produced in $\pi^\pm$, $K^\pm$ decays to search for $\nu_\mu$
disappearance, $\nu_e$ appearance and $\nu_\tau$ appearance.
Off-axis detector to achieve a narrow energy beam.
Energy of hundred of MeV to few GeV.
Distance 700–1000 Km.
Due to the sufficiently long baseline $\sim 800$ Km and beam intensity, NO$\nu$A would be the first LBL experiment sensitive to the type of hierarchy.

However, the degeneracy with $\delta$ limits the capability of NO$\nu$A to determine the hierarchy.
I propose a new approach for lifting the degeneracy and determine **the type of hierarchy free of degeneracies** using just **one experiment** (the NUMI off-axis beam and 2 detectors):

**Super-NO\(\nu\)A**


The improvement with respect to NO\(\nu\)A is remarkable!
Super-NO\(\nu\)A requires:

- NUMI beam
- 2 detectors off-axis: NO\(\nu\)A detector + WC or LiAr with the same \(L/E\).
- sequencing of the experiment:
  - phase I as in NO\(\nu\)A
  - phase II, construction and data-taking of the second detector.
In order to achieve a narrow energy spectrum, the proposed experiments (NOvA, T2K) plan to put the far detector off-axis by a small angle (few mrad).

Due to the kinematics of the $\pi$ decay, the spectrum peaks at a lower energy:

Choosing the off-axis angle it is possible to control the peak energy.
For the near-off-axis detector, I exploit the fact that the Earth is curved.

Putting a detector on the surface at closer distance (few hundred Km), the angle of the neutrinos which reach the detector with respect to the on-axis, is given by

$$\alpha \sim \frac{L_{\text{FAR}} - L_{\text{NEAR}}}{2R}$$
The normalised difference of the oscillation $P$ at baselines $L_N$ and $L_F$, for the same $E/L$, is:

$$D_1 \equiv \frac{P_N - P_F}{P_N + P_F} \sim \frac{A_N L_N - A_F L_F}{2} \left( \frac{1}{\Delta_{13} L/2} - \frac{1}{\tan(\Delta_{13} L/2)} \right)$$

There is no degeneracy with the CPV phase as the dominant CPV term cancels out.
The NO and IO cases are well distinguishable.
Choosing a type of second off-axis detector and placing it at the optimal 200 km distance (the larger the baseline difference the stronger the effect):
The degeneracies weaken and the sensitivity increases for larger second off-axis detector.
For the determining the type of hierarchy, no antineutrino run would be required. However NO$\nu$A plans to run in antineutrinos. This open up the possibility to search for CP-violation.

Phase I: 3+3 years. Phase 2: 3+1 with a proton driver years.
The NO$\nu$A proposal itself considers the possibility of a second off-axis detector but at second oscillation maximum. The cancellation of CP-violating effects does not take place in this case and both neutrino and antineutrino running need to be considered.
Further in the future other possibilities can be considered.

**Atmospheric neutrinos**

By exploiting matter effects in large detectors it is possible to obtain information on the type of hierarchy.

Either a Mton WC detector and $\geq 10$ years of data taking is required or a 100kton iron magnetized detector (charge identification!!!).

**INO** proposal for an iron detector in India.

The typical sensitivities for type of hierarchy for $\sin^2 2\theta_{13} > 0.02 - 0.03$. 
Due to the short distance T2K alone has no sensitivity to matter effects.

- Add a second detector:

[M. Ishitsuka et al.;
See also K. Hagiwara, N. Okamura, K. Senda]
• Combine with atmospheric neutrino data:

In either case a (or 2) Mton detector is required and the time scale is \( \gg 2010 \).
Neutrino factories:

$\nu_\mu$ and $\nu_e$ are produced in very relativistic muon decays.

Energies of 20–50 GeV.

Distance 3000–7000 Km.

At present a new study ISS is being performed.
Beta-beams:

$\nu_e$ are produced by beta decay of accelerated ions.

Energies 200-2000 MeV.

Distance 100-1000 Km.

For short baselines (CERN-Frejus) no sensitivity to matter effects. At least 600–700 Km are required, which implies high gamma factors ($> 300 - 400$).
Ultimate sensitivity in neutrino factories or beta-beams:

[Huber et al. 2005]
(\beta\beta)_{0\nu}-decay and LBL exp are complementary in the quest for \nu-masses.

- They exploit completely different physics effects.

1) If \sin^2 2\theta_{13} \ll 0.01 - 0.001, future generation LBL experiments will not be able to resolve the type of hierarchy. (\beta\beta)_{0\nu}-decay is a viable alternative.

2) If \nu's are Dirac particles, (\beta\beta)_{0\nu}-decay will not find any signal. We have to rely on LBL exp.
• Strong synergy: important information can be obtained from **combining** the results.

Probing Dirac particles and lepton number conservation:
If LBL establishes that is IH and \((\beta\beta)_{0\nu}\)-decay reach a sensitivity of \(\vert< m >\vert \sim 10\) meV, without finding a signal, this implies that neutrinos are **Dirac particles**.

Solving one degeneracy in LBL experiments.

\[\vert< m >\vert < 10\) meV with positive signal \(\Rightarrow\) NH, this solves the \(\Delta m_{31}^2 - \delta\) LBL degeneracy and improves on the sensitivity to CP-violation.
• Establishing the **absolute values of neutrino masses** is one of the crucial questions in neutrino physics.

• The type of hierarchy can be probed in:
  
  1) \((\beta\beta)_{0\nu}\)-decay: for Majorana neutrinos, no dependence on \(\theta_{13}\).
  
  2) LBL exp exploiting matter effects.

In the next future only NO\(\nu\)A has sensitivity to the hierarchy. Problem of degeneracies.

**Super-NO\(\nu\)A!**: a setup with two detectors and only a neutrino beam can resolve the type of hierarchy for \(\sin^2 2\theta_{13} \gtrsim 0.02\), with no degeneracies.