What can we learn from B physics?

Sebastian Jäger

Seminar at the University of Warwick,
19/01/2012
Content

- Flavour & CP: what & why
- Observables (selection), some theory issues
- A SUSY GUT model
- Conclusions
Baryogenesis

- There are many photons ... some baryons...

... and essentially no antibaryons in the universe

\[ \eta_B = \frac{n_B}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10} \]

- Can arise dynamically from B=0 if sufficient...
(1) departure from equilibrium and
(2) C and CP violation and
(3) B violation

Sakharov 1967
Thermal leptogenesis

- **CP-violating $v_R$ decay:**
  - Weak CPV phase in $Y_V$
  - CP-conserving phase from loop

- Resulting net lepton numbers $<L_l>$ partially converted to $<B>$ by equilibrium sphalerons
C, P and T

- In local quantum field theory CPT is a symmetry.

```
\begin{align*}
\text{\hspace{1cm}} & e^- & \rightarrow & e^- \\
\text{\hspace{1cm}} & e^+ & \rightarrow & e^+ \\
\text{\hspace{1cm}} \text{``rotation'' by } \pi & \text{ in } tx \text{ plane}
\end{align*}
```

i.e. simultaneously:  
- \( t \rightarrow -t \) (time reversal T)  
- \( x \rightarrow -x \) (parity P - up to a rotation)  
- particles \( \rightarrow \) antiparticles (charge conjugation C)

In particular CPT implies the existence of antiparticles with identical masses and lifetimes, and opposite conserved charges.

(constructive proof at Lagrangian level, or more general proof in axiomatic field theory)
C and P violation

- C, P, T individually need not be symmetries
- Chiral fermions violate C & P maximally [no C,P partners]
- Gauge-fermion theories (renormalisable, only spins 1 and 1/2) preserve CP save for vacuum $\theta$ angle(s)
- Example: SM gauge sector (neglect $\theta_{QCD}$ for now)

$$\mathcal{L}_{gauge} = \sum_{f} \bar{\psi}_f \gamma^\mu D_\mu \psi_f - \sum_{i,a} \frac{1}{4} g_i F^i_{\mu\nu} F^{i\mu\nu}$$

$$f = Q_{Lj}, u_{Rj}, d_{Rj}, L_{Lj}, e_{Rj}\quad j = 1, 2, 3$$

- Conserves CP; large global *flavour* symmetry

$$G_{\text{flavor}} = SU(3)^5 \times U(1)_B \times U(1)_A \times U(1)_L \times U(1)_E$$

$$Q_L \rightarrow e^{i(b/3+a)}V_{QL}Q_L, \quad u_R \rightarrow e^{i(b/3-a)}V_{uR}u_R, \quad d_R \rightarrow e^{i(b/3-a)}V_{dR}d_R$$

$$L_L \rightarrow e^{i(l+a)}V_{LL}L_L, \quad e_R \rightarrow e^{i(l+e-a)}V_{eR}e_R$$

Chivukula, Georgi 1987
CP violation

- Vacuum $\theta$ angle(s) violate CP

$$\mathcal{L} \supset -\theta \frac{g^2}{32 \pi^2} F_{\mu \nu}^a \tilde{F}^{\mu \nu a} \propto \tilde{E}^a \cdot \tilde{B}^a$$

P and CP odd

- CP violation generic if scalars are present

SM Yukawa interactions:

$$\mathcal{L}_Y = -\bar{u}_R Y_U \phi^c Q_L - \bar{d}_R Y_D \phi^d D_L - \bar{e}_R Y_E \phi^e E_L$$

$$Y_U = 1/v \text{ diag}(m_u, m_c, m_t)$$

$$Y_D = 1/v \text{ diag}(m_d, m_s, m_b)$$

$$Y_E = 1/v \text{ diag}(m_e, m_\mu, m_\tau)$$

9 masses

3 mixing angles

1 CP-violating phase

Kobayashi, Maskawa 1972

- CP violation of this type requires 3 generations

- flavour symmetry broken to $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

hadronic electric dipole moments (EDMs)

close connection CP - flavour - EW symmetry breaking (Higgs) sector
Observables

• CP-violating, flavour-conserving
  neutron, electron, atomic EDM’s
  advantage: ultraclean tests of SM and we
  “know” that BSM CP violation exists
  disadvantage: CP violation could be at scales >> TeV
  and possibly out of reach

• CP-violating, flavour-violating
  CPV in K,D, B, B_s mixing and mixing-decay interference
  direct CPV (CPV in decay)
  triple-product asymmetries
  advantage: various clean tests of SM
  disadvantage: TeV scale need not be CPV (see above)

• CP-conserving, flavour-violating
  Rare K, (D,) B, B_s decays: BR’s, kinematic distributions
  lepton flavour violation
  advantage: TeV physics is guaranteed to affect these
  disadvantage: fewer/less clean tests of SM
The CKM picture of flavour & CP violation is consistent with observations.

Within the Standard Model, all parameters (except higgs mass) including CKM have been determined, with good precision.
Flavour of the TeV scale

- Solutions to the hierarchy problem must bring in particles to cut off the top contribution to the weak scale (Higgs mass parameter).

\[ \propto y_t^2 \Lambda_{UV}^2 \]

- The new particles’ couplings tend to break flavour (they do in all the major proposals for TeV physics)

- At least they will have CKM-like flavour violations (minimal flavour violation), so will always affect rare decays
Minimal flavour violation

• in this case, CKM parameters can still be extracted unambiguously beyond the Standard Model

Universal unitarity triangle (UUT)

Buras, Gambino, Gorbahn, SJ, Silvestrini 2000

independent of details of new physics (particle content, masses, couplings)

• however, this is a very restrictive scenario; typically does not apply to dynamical BSM models

• can be generalized (relaxed)

d’Ambrosio et al 2002
Kagan et al 2009
...
SUSY flavour

Supersymmetry associates a scalar with every SM fermion

Squark mass matrices are 6x6 with independent flavour structure:

\[
\mathcal{M}_d^2 = \begin{pmatrix}
\hat{m}_Q^2 + m_d^2 + D_{dLL} & v_1 \hat{T}_D - \mu^* m_d \tan \beta \\
v_1 \hat{T}_D^\dagger - \mu m_d \tan \beta & \hat{m}_d^2 + m_d^2 + D_{dRR}
\end{pmatrix} \equiv \begin{pmatrix}
(M_d^2)_{LL} & (M_d^2)_{LR} \\
(M_d^2)_{RL} & (M_d^2)_{RR}
\end{pmatrix}
\]

3x3 flavour-violating - and supersymmetry-breaking

similar for up squarks, charged sleptons. 3x3 LL for sneutrinos

\[
(\delta_{ij}^{u,d,e,\nu})_{AB} = \frac{(\mathcal{M}_{\tilde{u},\tilde{d},\tilde{e},\tilde{\nu}}^2)_{ij}}{m_f^2}
\]

33 flavour-violating parameters
45 CPV (some flavour-conserving)
SUSY flavour - observables

K-\Bar{K}, B_d-\Bar{B}_d, B_s-\Bar{B}_s mixing

\Delta F=1 decays

lepton flavour violation
\mu \rightarrow e\gamma, \tau \rightarrow e\gamma, \tau \rightarrow \mu\gamma
\tau \rightarrow \mu\mu\mu, ...
\mu \rightarrow e \text{ conversion}
...

B \rightarrow X_s \gamma
B \rightarrow X_s \mu^+\mu^-
B \rightarrow K^*\gamma, B \rightarrow K^*\mu^+\mu^-, B \rightarrow K\pi
B_{s,d} \rightarrow \mu^+\mu^-
K \rightarrow \pi\nu\nu
B \rightarrow K\nu\nu
...

SUSY flavour puzzle

where are their effects?

- elusiveness of deviations from SM in flavour physics seems to make MSSM look unnatural

- pragmatic point of view: flavour physics highly sensitive to MSSM parameters - and SUSY breaking mechanism in particular

\[
\begin{align*}
\left( \delta_{ij}^{u,d,e,\nu} \right)_{AB} & \equiv \frac{\left( \mathcal{M}_{\tilde{u}, \tilde{d}, \tilde{e}, \tilde{\nu}} \right)^{AB}_{ij}}{m_f^2} \\
\end{align*}
\]

\[\begin{array}{c|c}
\text{Quantity} & \text{upper bound} \\
\hline
\sqrt{|\text{Re}(\delta_{ds})_{LL}|} & 4.0 \times 10^{-2} \\
\sqrt{|\text{Re}(\delta_{ds})_{RR}|} & 4.0 \times 10^{-2} \\
\sqrt{|\text{Re}(\delta_{ds})_{LR}|} & 4.4 \times 10^{-3} \\
\sqrt{|\text{Re}(\delta_{ds})_{LL}(\delta_{ds})_{RR}|} & 2.8 \times 10^{-3} \\
\sqrt{|\text{Im}(\delta_{ds})_{LL}|} & 3.2 \times 10^{-3} \\
\sqrt{|\text{Im}(\delta_{ds})_{RR}|} & 3.2 \times 10^{-3} \\
\sqrt{|\text{Im}(\delta_{ds})_{LR}|} & 3.5 \times 10^{-4} \\
\sqrt{|\text{Im}(\delta_{ds})_{LL}(\delta_{ds})_{RR}|} & 2.2 \times 10^{-4} \\
\end{array}\]

\[\begin{array}{c|c}
\text{Quantity} & \text{upper bound} \\
\hline
\sqrt{|\text{Re}(\delta_{dc})_{LL}|} & 9.8 \times 10^{-2} \\
\sqrt{|\text{Re}(\delta_{dc})_{RR}|} & 9.8 \times 10^{-2} \\
\sqrt{|\text{Re}(\delta_{dc})_{LR}|} & 3.3 \times 10^{-2} \\
\sqrt{|\text{Re}(\delta_{dc})_{LL}(\delta_{dc})_{RR}|} & 1.8 \times 10^{-2} \\
\sqrt{|\text{Im}(\delta_{dc})_{LL}|} & 4.8 \times 10^{-1} \\
\sqrt{|\text{Im}(\delta_{dc})_{RR}|} & 4.8 \times 10^{-1} \\
\sqrt{|\text{Im}(\delta_{dc})_{LR}|} & 1.62 \times 10^{-2} \\
\sqrt{|\text{Im}(\delta_{dc})_{LL}(\delta_{dc})_{RR}|} & 8.9 \times 10^{-2} \\
\end{array}\]

\[\text{[Gabbiani et al 96; Misiak et al 97] these numbers from [S], 0808.2044}\]
Flavour - warped extra D

Warped 5D

SM fermions = zero modes (~ ground state WF of a particle in a box) of fields present in the bulk.
also infinitely many massive KK modes (~higher states of particle in box)

Higgs localized on IR brane
light (heavy) fermions localized near UV (IR) brane

Higgs and KK states are localized on the IR.

Higgs and KK states are localized on the IR.

1st KK

bulk

UV

IR

brane

brane

$\phi$

$\frac{\pi}{2}$

$\pi$

$\Pi$

$\Pi$

$\Phi$

$\Phi$

Light fields have highly suppressed coupling to KK modes!

[g Perez, talk at CKM 2010]

couplings (Yukawa and other) given by wave function overlaps

also, dangerous four-fermion operators on the IR brane, but
fermions localized on the UV brane do not “feel” these much

Higgs localized on IR brane

light (heavy) fermions localized near UV (IR) brane

hierarchical SM fermion masses

$u, c, t$

$q, b$

$s, d, h$

$W, W$

$(c, d)$

$(c, d)$

$(e, e)$

$W, W$

$(c, d)$

$(c, d)$

$W, W$

$(c, d)$

$(c, d)$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$

$W, W$
Flavour - warped ED (2)

- dominant contribution to FCNC generically from tree-level KK boson exchange (rather than brane contact terms)

\[ \lambda_{kmn} = \int d\phi \, w(\phi) \, f^{(m)}(\phi) \, f^{(n)}(\phi) \, f^{(k)}_{V}(\phi) \]

KK mode coupling

\[ Y_{mn} \propto f^{(m)}(\pi) \, f^{(n)}(\pi) \]

SM Yukawa coupling

non-minimal flavour violations!

- where are their effects?

- strongest tension generally in Kaon sector, then EW precision tests
Soft-wall ED model

- hard brane replaced by extended, “soft” wall
- Higgs in bulk, localised toward wall
- eases EW precision constraints

![Graph showing the mean values of $\epsilon_{NP}$ and $\Delta m_{NP}$ for the RS model (stars) and the SW model with $c_1 = 1.5$, circles, $c_1 = 1$ (squares), and $c_1 = 0.5$ (diamonds). The CL values are given in (52). For the SW model configuration (A) is plotted in dark blue, (B) in light blue, (C) in cyan, (D) in light green, and (E) in dark green. While for the RS model (A) is plotted in dark red, (B) in light red, (C) in orange, (D) in yellow, and (E) in dark yellow. For both the RS model and the SW model the mass of the first gauge KK mode will be about two times $M_{KK}$. Noted plotted here are the average values. Typically one can always find tuned points, insensitive parameters space, that satisfy the experimental constraints for all configurations and all KK scales considered. $\Omega = 10^{15}$.](Archer, Huber, SJ JHEP 12(2011)101 [arXiv:1108.1433])

flavour still gives strongest constraints on these models

B physics of these models? [Granger, Huber, SJ, w.i.p.]
Other scenarios

- fourth SM generation
  CKM matrix becomes 4x4, giving new sources of flavour and CP violation

- little(st) higgs model with T parity
  (higgs light because a pseudo-goldstone boson)
  finite, calculable 1-loop contributions due to new heavy particles with new flavour violating couplings

- ...

non-minimal flavour violation!
Of all constraints on the unitarity triangle, only the \( \gamma \) and \( |V_{ub}| \) determinations are robust against new physics as they do not involve loops.

It is possible that the TRUE \((\bar{\rho}, \bar{\eta})\) lies here (for example).
“Tree” determinations

Only “robust” measurements of $\gamma$ and $|V_{ub}|$. *Note: the $\gamma(\alpha)$ constraint shown depends on assumptions (absence of BSM $\Delta I=3/2$ contributions in $B\to\pi\pi$); a truly robust $\gamma$ determination should not include $B\to\pi\pi$. Such determinations will be greatly improved by LHCb.*

Certainly there is room for $O(10\%)$ NP in $b\to d$ transitions

Moreover, $b\to s$ transitions are almost unrelated to $(\rho, \eta)$. They are the domain of LHCb
Another view

\[ BR(B \to \tau \nu) = BR(B \to \tau \nu)_{\text{SM}} \times \left| 1 - \frac{M_B^2 \tan^2 \beta}{M_{H^+}^2} \right|^2 \]

could be NP in B_d mixing; leading uncertainty is bag parameter

\[ BR \propto |V_{ub}|^2 \text{ in SM} \]

two-Higgs doublet model (II): 

\[ BR(B \to \tau \nu) = BR(B \to \tau \nu)_{\text{SM}} \times \]
LHCb observables

- **mixing**
  
  theory well understood
  data consistent with SM
  errors still large
  but O(1) mixing phase ruled out

- **hadronic CPV**
  amplitudes
  time-dependent CP violation
  triple products
  $\Delta A_{\text{CP}}$ in D decays

- **semileptonic B decays**
  
  constraints on Wilson coefficients

- *(This is a narrow subset of what I find interesting.)*
Exclusive decays at LHCb

<table>
<thead>
<tr>
<th>final state</th>
<th>strong dynamics</th>
<th>#obs</th>
<th>NP enters through</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptonic</td>
<td>decay constant</td>
<td>O(1)</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow l^+ l^-$</td>
<td>$\langle 0</td>
<td>j_\mu</td>
<td>B\rangle \propto f_B$</td>
</tr>
<tr>
<td>semileptonic, radiative</td>
<td>form factors</td>
<td>O(10)</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow K^<em>l^+ l^-$, $K^</em>\gamma$</td>
<td>$\langle \pi</td>
<td>j_\mu</td>
<td>B\rangle \propto f_{B\pi}(q^2)$</td>
</tr>
<tr>
<td>charmless hadronic</td>
<td>matrix element</td>
<td>O(100)</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow \pi\pi\pi$, $\pi K$, $\phi\phi$, ...</td>
<td>$\langle \pi\pi\pi</td>
<td>Q_i</td>
<td>B\rangle$</td>
</tr>
</tbody>
</table>

Non-radiative modes also NP-sensitive via 4-fermion operators

Decay constants and form factors accessible by QCD sum rules and, increasingly, by lattice QCD.

QCD a big challenge particularly for nonleptonic modes
Hadronic decay amplitudes

- Any SM amplitude can be written

\[ A(\bar{B} \rightarrow M_1 M_2) = e^{-i\gamma} T_{M_1 M_2} + P_{M_1 M_2} \]

\[ T_{M_1 M_2} = V_{uD} |V_{ub}| \left[ C_1 \langle Q^u_1 \rangle + C_2 \langle Q^u_2 \rangle + \sum_{i=3}^{12} C_i \langle Q_i \rangle \right] \]

\[ P_{M_1 M_2} = V_{cD} |V_{cb}| \left[ C_1 \langle Q^c_1 \rangle + C_2 \langle Q^c_2 \rangle + \sum_{i=3}^{12} C_i \langle Q_i \rangle \right] \]

CKM factor (D=d or s)  

\[ \text{tree W exchange} \]

\[ \text{penguins (QCD, magnetic, EW)} \]

\[ \langle Q_i \rangle = \langle M_1 M_2 | Q_i | B \rangle \]: QCD at distances > hc/m_b, strong phases  

required for direct (decay rate) CP asymmetry  

Qi: operators in weak hamiltonian  

Ci: QCD corrections from short distances (< hc/m_b) & new physics
distribution in these three angles is given in terms of the three trans 

two pseudoscalar mesons. This class of decays consists of charmle 

a T-odd) asymmetry is now defined similarly to Eq. (4) as an asymmetr 

Let us consider decays in which each of the two vector mesons in 

vector in the direction of 

where \( \hat{\theta} \)

and \( A \)

where \( A^0 \)

asymmetryi impliesm assuming 

Scalar triple products of three momentum or spin vectors are odd 

of the phenomenology of triple product asymmetries is given in Ref \[s\]o 

asymmetries would be an unambigous signal for New Physics \[sm u\]o 

exercise that does not require either flavour tagging or a time dependent analysiso The 

asymmetries is to measure observable quantities related to triple 

violation in this mode is to measure observable quantities related to triple 

true 

and finalnstate interactionso The former case ha true 

presence of polarization trebles number of amplitudes 

angular analysis allows extraction of all 6 amplitudes 

already relative weak phases imply CP-violating “triple 

products”, ie no strong phase knowledge required 

B→V V 

\[
\frac{d\Gamma}{d\cos \theta_1 d\cos \theta_2 d\phi} = N \left( |A_0|^2 \cos^2 \theta_1 \cos^2 \theta_2 + \frac{|A_\parallel|^2}{2} \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \phi \right. \\
+ \frac{|A_\perp|^2}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi + \frac{\text{Re}(A_0 A_\parallel)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \phi \\
- \frac{\text{Im}(A_\perp A_0^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \phi - \frac{\text{Im}(A_\perp A_\parallel^*)}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi \left. \right)
\]

(for \( B_s \to \phi \phi \) coefficients are time-dependent due to oscillations)

- presence of polarization trebles number of amplitudes
- angular analysis allows extraction of all 6 amplitudes
- already relative weak phases imply CP-violating “triple products”, ie no strong phase knowledge required
Theory approaches I

- **1/N_c**: hierarchies
  
<table>
<thead>
<tr>
<th></th>
<th>T/a_1</th>
<th>C/a_2</th>
<th>P</th>
<th>E/b_1</th>
<th>A/b_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/N</td>
<td>1/N</td>
<td>1/N</td>
<td>1/N</td>
<td>1/N</td>
<td>1</td>
</tr>
<tr>
<td>\Lambda/m_B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>\Lambda/m_B</td>
<td>\Lambda/m_B</td>
</tr>
</tbody>
</table>

  - “naive factorization” for N_c -> infinity
  - strong phases: T, P: O(1/N^2), colour-suppressed tree O(1)
  - main drawback: can’t compute

- QCD light-cone sum rules
  evaluate correlation function off shell;
  OPE & lightcone expansion
  - express hadronic matrix elements
    in terms of simpler objects (form factors etc.) and
    a perturbatively evaluated dispersion integral.
  - works also for form factors themselves (and other objects)
  - main drawback: uncertainty due to “continuum threshold”
    is difficult to quantify

[from Khodjamirian et al, hep-ph/0509049]
Theory approaches II

- heavy-quark expansion in $\Lambda_{\text{QCD}}/m_B$

$T^I, T^I$ computable in perturbation theory in strong coupling

- “naive factorization” for $m_B \rightarrow \infty$
- strong phases [imaginary parts] are $O(\alpha_s)$ or $O(\Lambda_{\text{QCD}}/m_B)$
- annihilation power suppressed altogether
- hierarchies of penguin amplitudes between final states containing pseudoscalars and vectors
- main drawback: $O(\Lambda_{\text{QCD}}/m_B)$ power corrections don’t factorize, in general, and hard to estimate

- flavour SU(3) - relate $b \rightarrow s$ and $b \rightarrow d$; eliminate amplitudes from data. Good if redundant observables ($\gamma$ in SM), less powerful for NP search; SU(3) breaking not controlled

[Beneke, Buchalla, Neubert, Sachrajda (BBNS); Bauer et al]

[QCDF / SCET; pQCD approach]

[Keum, Li, Sanda, ...]

[Zeppenfeld 81; Gronau et al 94; Fleischer, ...]
\[ \langle M_1 M_2 | Q_i | \bar{B} \rangle = \]

\[ f_+^{BM_1}(0) f_{M_2} \int du T_i^I(u) \phi_{M_2}(u) + f_B f_{M_1} f_{M_2} \int du dv d\omega T_i^{II}(u, v, \omega) \phi_{B+}(\omega) \phi_{M_1}(v) \phi_{M_2}(u) + O(\Lambda_{QCD}/m_b) \]

perturbative, includes strong phases

non-perturbative QCD

soft overlap (form factor)

hard spectator scattering

\[ T_i^I \sim 1 + t_i \alpha_s + O(\alpha_s^2) \]

"naive factorization"

BBNS 99-01

Bell 07, 09 (trees), Beneke et al 09 (trees)

\[ T_i^{II} \sim H_i \ast J \]

\[ \sim (1 + h_i \alpha_s + O(\alpha_s^2)) \left( j^{(0)} \alpha_s + j^{(1)} \alpha_s^2 + O(\alpha_s^3) \right) \]

BBNS 99-01

BBNS 99-01


Beneke, SJ 2005 (trees), 2006 (penguins); Kivel 2006; Pilipp 2007 (trees);

Jain, Rothstein, Stewart 2007 (penguins)
Power corrections

- some power-suppressed contributions factorize (later slide); most do not
- varying relevance [size of Wilson/CKM factor multiplying them]
- BBNS proposed & used a (crude) “cut-off-plus-fudge-factor” model to estimate power corrections, including $O(1)$ undetermined soft strong phases on them.

\[ \int_{0}^{1} \frac{dy}{y} \to X_{A}^{M_{1}} \]
\[ X_{A} = (1 + \rho_{A} e^{i\phi_{A}}) \ln \frac{m_{B}}{\Lambda_{h}} \]

- Some authors have attempted to fit power corrections to data [at expense of predictivity]  
  *Feldmann & Hurth; Ciuchini et al*

- In the ‘pQCD’ approach power corrections are (mostly) deemed calculable, but the “perturbative” expressions do not appear [to me] to be dominated by perturbative scales
phenomenological summary

- **Corrections** to naive factorization small for $T$ and $P_{EW}$, stable perturbation series; small uncertainties

- **Corrections** $O(1)$ for $C$ (and $P_{EW}^c$), stable perturbation series large uncertainties (hadronic inputs; large incalculable power correction for final states with pseudoscalars)

- (physical) penguin amplitudes moderately affected by power-suppressed incalculable penguin annihilation (&charm penguin) terms. Spoils precise predictions for direct CP asymmetries

- certain SU(3)-type relations satisfied in good approximation

![Parameter set “G” (fit hadronic parameters to $B \to \pi\pi$ BR’s): $C/T \sim 0.69 + 0.17 i$ large magnitude, small phase](image)
Penguin anatomy: $1/m_b$

$$\alpha^c_4 = a_4 + r^M_\chi a_6$$

However:

$$r^\pi_\chi(\mu) = \frac{2m^2_\pi}{m_b(\mu)(m_u + m_d)(\mu)} \sim \frac{\Lambda_{QCD}}{m_b}$$

but $\sim 1$ numerically

“chiral enhancement”

no chiral enhancement present for vector $M_2$ -> much smaller penguin amplitudes

penguin annihilation [in QCDF terminology]:

$O(1/m_b)$, does not factorize

modeled by naively factorized expression with IR cutoff by BBNS

large and complex in pQCD approach

small in light-cone sum rules

[Keum, Li, Sanda 2000]

[Khodjamirian et al 2005]
Annihilation $\beta_3$

- The colour-leading piece to the annihilation contribution $\beta_3$ to the QCD penguin amplitude has a naively factorizing structure

  (where $Q_6$ has been “Fierzed” to colour singlet x singlet form)

This is proportional to the “scalar form factor”. A QCD sum rule calculation gives a small and approximately real result.

[Khodjamirian Mannel, Melcher, Melic, hep-ph/0509049]

- In contrast, the pQCD approach finds a large and complex value albeit with large uncertainties. [Keum, Li, Sanda 2000]

- This is also the case for the BBNS annihilation model.
Penguins (QCDF) vs data

\[ P_{M_1M_2} / (C_{\pi\pi} + T_{\pi\pi}) \sim \hat{\alpha}_4^c(M_1M_2) / (\alpha_1(\pi\pi) + \alpha_2(\pi\pi)) \]

can be fit to BR, \( A_{CP}(\pi^+K^-) \) and \( BR(\pi^+\pi^-) \) using one SU(3) relation

\[ P_{M_1M_2} \sim \hat{\alpha}_4^c(M_1M_2) = a_4(M_1M_2) \pm r M^2 \alpha_6(M_1M_2) + \beta_3^p(M_1M_2) \]

factorizable power correction

annihilation (modeled a la BBNS)

chirally enhanced for \( M_2 \) pseudoscalar

small for \( M_2 \) vector

pattern (hierarchies & numbers) agree quite well with \( 1/m_b \) expectations (also for \( \rho K, \rho K^* \))

wrong imaginary part for \( \pi K \) unless annihilation is fairly large (well known problem)

[BBNS model of annihilation]
Comparison to data: $S_{CP}$

\[ \sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}}) \]

- Beneke 2005 (NLO QCDF)
- Small corrections (and small errors) to “naive” expectation
- Similar conclusion in BPRS approach [Williamson, Zupan 2006]
- pQCD see Li, Mishima 2006
Theory: $S_{CP}$

$$A_f = \langle f | B \rangle$$

$$B \xrightarrow{\text{mixing}} e^{-2i\beta} \bar{B} \xrightarrow{\text{decay}} f$$

$$\bar{A}_f = \langle f | \bar{B} \rangle$$

$$\frac{BR(B^0(t) \to f) - BR(\bar{B}^0(t) \to f)}{BR(B^0(t) \to f) + BR(\bar{B}^0(t) \to f)} = -S_f \sin(\Delta m_B t) + C_f \cos(\Delta m_B t)$$

$$\sin(2\beta^{\text{eff}})$$

$$-\eta_{CP}(f) \cdot S_f \approx \sin(2\beta) + 2 \cos(2\beta) \sin \gamma \Re \frac{T_f + P_f^u}{P_f^c} + S_f^{\text{N.P.}}$$

need only real part of small amplitude (weak strong-phase dependence)
B$\rightarrow$ππ,πρ,ρρ: P/T, C/T

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value/Range</th>
<th>Value G</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\pi\pi}/T_{\pi\pi}$</td>
<td>$-0.122^{+0.033}<em>{-0.063} + (-0.024^{+0.047}</em>{-0.048})i$</td>
<td>$-0.162 + 0.022i$</td>
</tr>
<tr>
<td>$P_{\rho\rho}/T_{\rho\rho}$</td>
<td>$-0.036^{+0.006}<em>{-0.009} + (-0.009^{+0.007}</em>{-0.007})i$</td>
<td>$-0.037 - 0.009i$</td>
</tr>
<tr>
<td>$P_{\pi\rho}/T_{\pi\rho}$</td>
<td>$-0.034^{+0.015}<em>{-0.028} + (-0.005^{+0.024}</em>{-0.024})i$</td>
<td>$-0.070 + 0.006i$</td>
</tr>
<tr>
<td>$P_{\rho\pi}/T_{\rho\pi}$</td>
<td>$0.042^{+0.039}<em>{-0.023} + (0.004^{+0.030}</em>{-0.030})i$</td>
<td>$0.051 - 0.024i$</td>
</tr>
<tr>
<td>$C_{\pi\pi}/T_{\pi\pi}$</td>
<td>$0.363^{+0.106}<em>{-0.156} + (0.029^{+0.166}</em>{-0.103})i$</td>
<td>$0.691 + 0.165i$</td>
</tr>
<tr>
<td>$C_{\rho\rho}/T_{\rho\rho}$</td>
<td>$0.198^{+0.233}<em>{-0.150} + (-0.009^{+0.145}</em>{-0.097})i$</td>
<td>$0.344 + 0.042i$</td>
</tr>
<tr>
<td>$C_{\pi\rho}/T_{\pi\rho}$</td>
<td>$0.250^{+0.229}<em>{-0.143} + (-0.012^{+0.127}</em>{-0.096})i$</td>
<td>$0.467 + 0.071i$</td>
</tr>
<tr>
<td>$C_{\rho\pi}/T_{\rho\pi}$</td>
<td>$0.134^{+0.199}<em>{-0.156} + (-0.024^{+0.152}</em>{-0.117})i$</td>
<td>$0.283 + 0.138i$</td>
</tr>
<tr>
<td>$T_{\rho\pi}/T_{\pi\rho}$</td>
<td>$0.869^{+0.275}<em>{-0.207} + (0.014^{+0.058}</em>{-0.055})i$</td>
<td>$0.945 - 0.004i$</td>
</tr>
</tbody>
</table>

S parameter gives good γ determination, small corrections to naive factorisation
C parameter - direct CPV zero in naive factorisation

$\gamma = 80^\circ$  $\gamma = 70^\circ$  $\gamma = 60^\circ$

$[|V_{ub}/V_{cb} = 0.09|$ in P/T extraction]
Comparison to data: annihilation

- Annihilation power suppressed, small branching fractions predicted (but with large uncertainties)

- LHCb has published data on $B_s \rightarrow \pi \pi$ and $B^0 \rightarrow K K$

$$BR(B_s^0 \rightarrow \pi^+ \pi^-) = (0.98^{+0.23}_{-0.19} \pm 0.11) \times 10^{-6}$$

$$BR(B^0 \rightarrow K^+ K^-) = (0.13^{+0.06}_{-0.05} \pm 0.07) \times 10^{-6}$$

consistent with CDF

The $B_s$ BF is in excess of estimates, whereas the $B^0$ decay fits nicely. Both decays are SU(3)-related.

However, BF is quadratic in annihilation (other processes are affected at linear order), need (only) about factor 2-3 enhancement of an annihilation contribution

- more an issue for SU(3) than for factorisation (which implies SU(3) relations) per se. Could this be NP?
Polarisation & NP

• Triple-product asymmetries in $B \to \phi K^*$

$$A_T^{(1)\text{chg-avg}} \equiv \frac{[\Gamma(S > 0) + \bar{\Gamma}(\bar{S} > 0)] - [\Gamma(S < 0) + \bar{\Gamma}(\bar{S} < 0)]}{[\Gamma(S > 0) + \bar{\Gamma}(\bar{S} > 0)] + [\Gamma(S < 0) + \bar{\Gamma}(\bar{S} < 0)]}$$
$$= \frac{2\sqrt{2}}{\pi} \frac{\text{Im}(A_\perp A_0^* - \bar{A}_\perp \bar{A}_0^*)}{(|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2) + (|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2)}$$

$$A_T^{(2)\text{chg-avg}} \equiv \frac{[\Gamma(\sin 2\phi > 0) + \bar{\Gamma}(\sin 2\bar{\phi} > 0)] - [\Gamma(\sin 2\phi < 0) + \bar{\Gamma}(\sin 2\bar{\phi} < 0)]}{[\Gamma(\sin 2\phi > 0) + \bar{\Gamma}(\sin 2\bar{\phi} > 0)] + [\Gamma(\sin 2\phi < 0) + \bar{\Gamma}(\sin 2\bar{\phi} < 0)]}$$
$$= \frac{4}{\pi} \frac{\text{Im}(A_\perp A_0^* - \bar{A}_\perp \bar{A}_0^*)}{(|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2) + (|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2)}.$$

[Valencia 1989, ...]

[Datta, Duraisamy, London; Gronau, Rosner 2011]

• HFAG data for the entire set of polarization amplitudes exists; Triple products at most 5-10% in either case

[Grónau, Rosner 2011]

• A SM calculation in QCD factorization (based on the heavy-quark expansion) is consistent with the HFAG data

[Beneke, Rohrer, Yang 2006]

• Also “fake” triple-product asymmetries which require strong phases - small in QCDF, small in obs.
Polarisation observables

- “Factorization predicts $f_L \approx 1$, in disagreement with data.” Really?
- comprehensive phenomenological analysis of polarisation observables in (QCD) factorization exists.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Theory</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_L/%$</td>
<td>$\phi K^{*-}$</td>
<td>$45^{+0+58}_{-0-36}$</td>
</tr>
<tr>
<td></td>
<td>$\phi \bar{K}^{*0}$</td>
<td>$44^{+0+59}_{-0-36}$</td>
</tr>
<tr>
<td>$\phi_{</td>
<td></td>
<td>}/^\circ$</td>
</tr>
<tr>
<td></td>
<td>$\phi \bar{K}^{*0}$</td>
<td>$-42^{+0+87}_{-0-54}$</td>
</tr>
<tr>
<td></td>
<td>$\phi \phi$</td>
<td>$-39^{+0+86}_{-0-57}$</td>
</tr>
<tr>
<td>$\Delta \phi_{</td>
<td></td>
<td>}/^\circ$</td>
</tr>
<tr>
<td></td>
<td>$\phi \bar{K}^{*0}$</td>
<td>$0^{+0+0}_{-0-0}$</td>
</tr>
<tr>
<td></td>
<td>$0^{+0+0}_{-0-1}$</td>
<td>$0^{+0+1}_{-0-1}$</td>
</tr>
</tbody>
</table>

- transverse polarisation fractions can be large, naive factorisation is not reliable; $f_\perp$ & $f_{||}$ depend on incalculable power corrections so $1-f_L$ not a good probe of new physics.
- QCDF does give negligible relative weak phases in the SM (this is because it preserves dominance of penguin amplitudes)

[CP-averaged phase difference (mostly strong phase difference)]
[CP-asymmetric phase difference (mostly weak phase difference)]
Polarisation & NP

• Triple-product asymmetries in $B_s \rightarrow \phi \phi$
  - similar pair of TP asymmetries
  - time-dependence -> mixing-decay interference
  - one can define two combinations $A_U$, $A_V$ sensitive to

$$\text{Im}[A_{\perp}(t)A_i^*(t) + \bar{A}_{\perp}(t)\bar{A}_i^*(t)]$$

$i = 0, ||$

[Gronau, Rosner 2011]

• CDF
  $$A_U = -0.007 \pm 0.064 \text{(stat)} \pm 0.018 \text{(syst)}$$
  $$A_V = -0.120 \pm 0.064 \text{(stat)} \pm 0.016 \text{(syst)}.$$

[arXiv:1107.4999]

• LHCb
  $$A_U = -0.064 \pm 0.057 \text{(stat.)} \pm 0.014 \text{(syst.)}$$
  $$A_V = -0.070 \pm 0.057 \text{(stat.)} \pm 0.014 \text{(syst.)}.$$

[LHCb-CONF-2011-052]

• No quantitative theoretical calculation exists at the moment but qualitatively it is clear that the SM predicts both TP asymmetries to be small (strong penguin dominance)
• 1/m_b expansion predicts a hierarchy \( \bar{A}_0 : \bar{A}_- : \bar{A}_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^2 \) in \( \bar{B} \) decay (+/- interchanged in B decays); however, the suppression of the negative-helicity amplitude is numerically spoiled by annihilation contributions [Korner, Goldstein 1979]

• A nonvanishing \textit{positive}-helicity amplitude could be a sign of NP and could even be turned into quantitative information on “right-handed currents” [Kagan 2004]

• The (presumable) smallness of the \textit{negative}-helicity amplitude suppresses one of the two triple-product asymmetries, making it a probe of right-handed currents
EWP effect in $B \rightarrow VV$

- If NP involves a right-handed dipole operator $Q_7^r$ this can give a sizable $A_+$
- would be present in $B_s \rightarrow \phi \phi$
- full polarisation analysis would be interesting

low-virtuality photon, makes $A_-$ formally leading (but $\alpha_{EM}$ suppressed), important contribution in the SM

[Bekeke, Rohrer,Yang 2005]
LHCb has measured [essentially] the difference
\[ \Delta A_{CP} = A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) \]
\[ \Delta A_{CP} = [-0.82 \pm 0.21\text{(stat.)} \pm 0.11\text{(sys.)}] \% \]  

• SU(3) symmetry predicts equal and opposite relative sign between the two asymmetries, i.e. no cancellation expected

• but GIM cancellations suggest, in the SM, strong suppression of the penguin amplitude (\(|P/T| \sim 10^{-3}\))

\[ \propto V_{us} V_{cs}^* = \mathcal{O}(\lambda) \]
\[ \propto V_{ub} V_{cb}^* = \mathcal{O}(\lambda^5) \]

• to explain in SM would need about an order of magnitude enhancement of the penguin amplitude. Current theoretical control much worse than for B decays; recent discussion in

[Brood, Kagan, Zupan 1111.5000]
Semileptonic decay

\[ A(B \rightarrow K^* \ell^+ \ell^-) = \]

- kinematics described by dilepton invariant mass \( q^2 \) and three angles

- Systematic theoretical description based on heavy-quark expansion \( (\Lambda/m_b) \) for \( q^2 \ll m^2(J/\psi) \) (SCET)
  also for \( q^2 \gg m^2(J/\psi) \) (OPE)

Theoretical uncertainties on form factors, power corrections

Grinstein et al; Beylich et al 2011
B_d \rightarrow K^* \mu^+ \mu^-

- **Most well-known observable: forward-backward asymmetry**

\[ \frac{dA_{FB}}{dq^2}[\text{GeV}^{-2}] \]

Zero crossing to 0.3 GeV^2 in SM

\[ q_0^2[K^{*0}] = 4.36^{+0.33}_{-0.31} \text{ GeV}^2, \quad q_0^2[K^{*+}] = 4.15^{+0.27}_{-0.27} \text{ GeV}^2 \]


- **Many more observables to consider**

See also Bobeth et al 2008, 10, 11; Egede et al 2009, 2010; Alok et al 2010, Altmannshofer et al 2011 for recent analyses
Constraints on NP

Bobeth et al 1111.2558
see also Descotes-Genon et al 2011,
Altmannshofer, Paradisi, Straub 2011
SUSY (again)

- SUSY virtues
  - solves naturalness problem
  - gauge coupling unification
  - dark matter, strings, ...

- many ‘soft’ parameters in absence of a theory of SUSY
  breaking violate flavour: flavour puzzle

\[
(\delta_{ij}^{u,d,e,\nu})_{AB} \equiv \frac{(M_{\tilde{u},\tilde{d},\tilde{e},\tilde{\nu}}^2)_{ij}}{m_f^2}
\]

33 flavour-violating parameters
45 CPV (some flavour-conserving)

- flavour probes the SUSY breaking; GUT relations
CMSSM / mSUGRA

- standard approach: “CMSSM” (“mSUGRA”)
  - universal scalar mass, gaugino mass, A-terms (A_{ij}=a \, Y_{ij}) at the GUT scale, sign(\mu)
  - 3 parameters & 1 sign, RG evolution down to TeV scale
- flavour puzzle absent [CMSSM still needs to be justified]
- Straightforward interpretation of experimental constraints

![Graph of the CMSSM/mSUGRA limits](ATLAS-CONF-2011-064)
Grand unification

| (\(u_L\), \(d_L\)) | \(u_R\) | \(d_R\) | \(c_L\) | \(c_R\) | \(t_L\) | \(t_R\) | \(Q = +2/3\) |
| \(\nu_{eL}\) | \(-\) | \(\nu_{\mu L}\) | \(-\) | \(\nu_{\tau L}\) | \(-\) | \(Q = 0\) |
| \(e_L\) | \(e_R\) | \(\mu_L\) | \(\mu_R\) | \(\tau_L\) | \(\tau_R\) | \(Q = -1\) |

- SM in highly reducible representations of the gauge group
  SM gen = \((3,2)_{1/6} + (\bar{3},1)_{-2/3} + (\bar{3},1)_{1/3} + (1,2)_{-1/2} + (1,1)_1\)

- however,
  SM gen = \([10 + \bar{5}]_{\text{SU}(5)}\)
  SM gen + \(\nu_R^c\) = \(16_{\text{SO}(10)}\)

- if either group is gauged, no gauge invariant distinction of baryons and leptons - baryon & lepton number violation

what about flavour?
Flavour of SUSY GUTs

- small, hierarchical mixing in the quark sector

\[ K = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i\eta) & -A \lambda^2 & 1
\end{pmatrix} \]

- large mixings in the lepton sector

\[ U = \begin{pmatrix}
c e^{i\alpha_1/2} & s e^{i\alpha_2/2} & s_{13} e^{-i\delta} \\
-s e^{i\alpha_1/2} & c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \\
s e^{i\alpha_1/2} & -c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix} \quad s = O(1) \]

SUSY radiative corrections can “transfer” leptonic mixing angles to the hadronic sector
Barbieri&Hall 1994, Barbieri,Hall,Strumia 1995
CMM Model

- SO(10) gauge theory with superpotential

\[ W_Y = \frac{1}{2} 16_i Y_{i}^{ij} 16_j 10_H + 16_i Y_{i}^{ij} 16_j \frac{45_H}{2 M_{Pl}} 10'_H + 16_i Y_{i}^{ij} 16_j \frac{16_H}{2 M_{Pl}} 16_H \]

\( i = 1, 2, 3 \)

- SO(10) spinor \( 16_i = (Q, u^c, d^c, L, e^c, \nu^c)_i \)

- assumptions:
  - \( Y_1 \) and \( Y_N \) simultaneously diagonisable
  - breaking via SU(5)

\[ \text{SO}(10) \xrightarrow{\langle 16_H \rangle, \langle 16_H \rangle, \langle 45_H \rangle} \text{SU}(5) \xrightarrow{\langle 45_H \rangle} G_{\text{SM}} \equiv \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \]

\[ \xrightarrow{\langle 10_H \rangle, \langle 10'_H \rangle} \text{SU}(3)_C \times \text{U}(1)_{\text{em}} \]

- MSSM Higgs doublets in different copies of 10 of SO(10)

\[ 10_H = (\ast, 5_H) = (\ast, (3_H, H_u)) \]

\[ 10'_H = (\bar{5}_H, \ast) = ((\bar{3}_H, H_d), \ast) \]

Nonrenormalizable \( Y_2 \) term gives naturally small \( \tan(\beta) \)

- keep universal ("CMSSM-like") SUSY breaking, at \( M_{\text{Planck}} \)
Flavour structure

\[ W_Y = \frac{1}{2} 16_i Y_1^{ij} 16_j 10_H + 16_i Y_2^{ij} 16_j \frac{45_H 10'_H}{2 M_{Pl}} + 16_i Y_N^{ij} 16_j \frac{16_H 16_H}{2 M_{Pl}} \]

\[ Y_1 = L_1 D_1 L_1^\top, \]
\[ Y_2 = L_2 D_2 R_2^\dagger, \]
\[ Y_N = R_N D_N P_N R_N^\top \]

\[ L_1^\dagger R_N = 1 \] (\( Y_1 \) and \( Y_N \) simultaneously diagonalisable)

\[ V_q = L_1^\top L_2^* \] CKM quark mixing matrix

\[ U_D = P_N^* R_2^\dagger L_1^* \] PMNS lepton mixing matrix

- Now fix a U-basis where \( Y_1 \) and \( Y_N \) are diagonal. Then

\[ Y_2 = V_q^* D_2 U_D \]

contains all flavour violation
In the SM, \( U_D \) is unphysical in hadronic physics.
Flavour structure (2)

- work in the (U) basis

\[ Y_2 = V^*_q D_2 U_D \]

\[ M_D = v_d Y_2 \]

rotating to mass eigenstates eliminates \( U_D \)

\[ M_L = v_d Y_2^T \]

rotating to mass eigenstates eliminates \( V_q \)

\( M_U \) brought out of diagonal form, but only by CKM \( (V_q) \) angles

no physical effect of \( U_D \) in the SM, or unbroken SUSY theory

However, the large top Yukawa coupling in \( Y_1 \) fixes the U-basis as the universal mass eigenbasis for the sfermions
Observables

• There is now a mismatch of the sfermion and fermion mass bases for the right-handed down-type particles and the left-handed leptons

\[ m_D^2 = U_D m_D^2 U_D^\dagger = \begin{pmatrix} m_d^2 & 0 & 0 \\ 0 & m_d^2 - \frac{1}{2} \Delta_d & -\frac{1}{2} \Delta_d e^{i\xi} \\ 0 & -\frac{1}{2} \Delta_d e^{-i\xi} & m_d^2 - \frac{1}{2} \Delta_d \end{pmatrix} \]

• Diagonalizing the matrix introduces flavour violation into neutral current vertices
Soft flavour violation

\[
\begin{align*}
l_i^- & \rightarrow \tilde{\nu}_{Lj} \quad \tilde{\chi}_k^- \quad -i \left( \frac{e}{\sin \theta_W} Z_{1i}^l P_L + y_{l_i}^- Z_{2i}^{2*} P_R \right) U_{ij} \\
l_i^- & \rightarrow \tilde{l}_{Lj} \quad \tilde{\chi}_k^0 \quad i \left[ \frac{e}{\sqrt{2}} \cos \theta_W Z_{1k}^l N + \frac{e}{\sqrt{2}} \sin \theta_W Z_{2k}^l N \right] P_L + y_{l_i}^- Z_{3k}^{2*} P_R \right) U_{ij} \\
d_{ib} & \rightarrow \tilde{d}_{Rjc} \quad \tilde{g}_3^A \quad i \sqrt{2} g_3 T^A_{cl} P_R U_{ij}
\end{align*}
\]

large effects in $b \rightarrow s$ transitions, CP violation correlations of hadronic & leptonic observables

2 $\rightarrow$ 1 and 3 $\rightarrow$ 1 transitions less clearly correlated

but see Trine et al 2009, Girrbach et al 2010
Phenomenology: RG evolution

- 2-loop RGE for gauge couplings and $y_t$, analytic formulas for soft terms, matched at SUSY, SU(5) and SO(10) thresholds

- relate Planck-scale inputs to set of low-energy inputs:

  at $M_Z$  
  $m_{u_1}^2 (M_Z)$,  
  $m_{d_1}^2 (M_Z)$,  
  $a_{1d}^d (M_Z) \equiv \left[ a^d (M_Z) \right]_{11}$

  evolve to $M_{\text{GUT}}$  
  $m_{\tilde{\psi}_1}^2 (t_{\text{GUT}}) = m_{\tilde{u}_1}^2 (t_{\text{GUT}})$,  
  $m_{\tilde{\Phi}_1}^2 (t_{\text{GUT}}) = m_{\tilde{d}_1}^2 (t_{\text{GUT}})$

  evolve to $M_{10}$  
  $m_{16_1}^2 (t_{\text{SO}(10)}) = \frac{1}{4} \left[ 3m_{\tilde{\psi}_1}^2 (t_{\text{SO}(10)}) + m_{\tilde{\Phi}_1}^2 (t_{\text{SO}(10)}) \right]$

  evolve to $M_{\text{Pl}}$  
  $m_0^2 = m_{16_1}^2 (t_{\text{Pl}})$  
  similarly for $a_{1d}^d$

  evolve all soft terms down to $M_Z$, calculate spectrum & observables
Example

\[ M_{\tilde{q}} = 1500 \text{ GeV}, \ m_{\tilde{g}_3} = 500 \text{ GeV}, \ \alpha_1^d(M_Z)/M_{\tilde{q}} = 1.5, \ \arg(\mu) = 0 \text{ and } \tan \beta = 6 \]

\[ a_0 = 1273 \text{ GeV}, \quad m_0 = 1430 \text{ GeV}, \quad m_{\tilde{g}} = 184 \text{ GeV} \]

\[ \hat{A}_u(M_{\text{GUT}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 46 \end{pmatrix} \text{ GeV}, \quad \hat{A}_d(M_{\text{GUT}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.3 & -3.5 \end{pmatrix} \text{ GeV} \]

\[ \hat{A}_\nu(M_{\text{GUT}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0013 & 0.0023 & 43.4 \end{pmatrix} \text{ GeV}, \]

\[ m_{\tilde{\phi}}(M_{\text{GUT}}) = \text{diag}(1426, 1426, 1074) \text{ GeV}, \]

\[ m_{\tilde{\nu}}(M_{\text{GUT}}) = \text{diag}(1444, 1444, 1077) \text{ GeV}, \]

\[ m_{\tilde{N}}(M_{\text{GUT}}) = \text{diag}(1459, 1459, 1078) \text{ GeV}, \]

\[ m_{H_u}(M_{\text{GUT}}) = 1126 \text{ GeV}, \quad m_{H_d}(M_{\text{GUT}}) = 1446 \text{ GeV}, \]

\[ m_{\tilde{g}}(M_{\text{GUT}}) = 211 \text{ GeV}. \]

\[ m_{\tilde{g}_1} = 83 \text{ GeV}, \quad m_{\tilde{g}_2} = 165 \text{ GeV}, \]

\[ m_{\tilde{\chi}_i^0} = (640, 632, 159, 81) \text{ GeV} \]

\[ m_{\tilde{\chi}_i^\pm} = (640, 159) \text{ GeV} \]

\[ M_{\tilde{l}_i} = (1427, 1427, 1074, 1462, 1462, 1095) \text{ GeV} \]

\[ M_{\tilde{u}_i} = (1519, 1519, 934, 1501, 1501, 485) \text{ GeV} \]

\[ M_{\tilde{d}_i} = (1519, 1519, 908, 1498, 1498, 1164) \text{ GeV}. \]
Figure 3: Relative mass splitting $\Delta_{d}^{\text{rel}} = 1 - m_{d3}^2/m_{d2}^2$ among the bilinear soft terms for the right-handed squarks of the second and third generations with $\tan\beta = 3$ (left) and 6 (right) in the $M_{\tilde{q}}(M_Z) - a_{1d}(M_Z)/M_{\tilde{q}}(M_Z)$ plane for $m_{\tilde{g}_3} = 500$ GeV and $\text{sgn}(\mu) = +1$. 

Exemplarily, we present the output for one CMM model parameter point. We choose the same inputs as in Sec. 3.7 where the parameters at the GUT scale have been discussed:

$m_{\tilde{g}_3} = 1500$ GeV, $m_{\tilde{g}_3} = 500$ GeV, $\frac{a_{1d}}{M_{\tilde{q}}} = 1$, $\text{sgn}(\mu) = +1$, $\tan\beta = 3$, $\text{arg}(\mu) = 0$, $\tan\beta = 6$. (130)

The sparticle spectrum at the electroweak scale is given as (mass eigenvalues):

$m_{\tilde{g}_1} = 83$ GeV, $m_{\tilde{g}_2} = 165$ GeV, $m_{\tilde{\chi}^0_1} = (640, 632, 159, 81)$ GeV (131)

$m_{\tilde{\chi}^\pm_1} = (640, 159)$ GeV (132)

$M_{\tilde{l}_i} = (1427, 1427, 1074, 1462, 1462, 1095)$ GeV (133)

$M_{\tilde{u}_i} = (1519, 1519, 934, 1501, 1501, 485)$ GeV (134)

$M_{\tilde{d}_i} = (1519, 1519, 908, 1498, 1498, 1164)$ GeV (135)

The lightest neutralino is identified as the LSP (underlined number). The first three entries in $M_{\tilde{f}_i}, \tilde{f}_i = \tilde{l}, \tilde{u}, \tilde{d}$ correspond to sfermions with a larger left-handed component and the last three with a larger right-handed component, where the third generation masses are printed in bold face. The typical mass splitting is quite evident. The mixing angle between the two stop eigenstates with 485 GeV and 934 GeV is $\theta_{\tilde{t}} = 11^\circ$ and left-right mixing in the down sector is negligible, owing to the small value of $\tan\beta$. While $M_{2\tilde{d}_4} = M_{2\tilde{d}_5} = m_{2\tilde{d}_1} = m_{2\tilde{d}_2}$, the flavor composition of the two
Figure 4: Correlation of FCNC processes as a function of $M_{\tilde{q}}(M_Z)$ and $a_1^d(M_Z)/M_{\tilde{q}}(M_Z)$ for $m_{\tilde{g}_3}(M_Z) = 500$ GeV and sgn $\mu = +1$ with tan $\beta = 3$ (left) and tan $\beta = 6$ (right). $\mathcal{B}(b \rightarrow s\gamma)[10^{-4}]$ solid lines with white labels; $\mathcal{B}(\tau \rightarrow \mu\gamma)[10^{-8}]$ dashed lines with gray labels. Black region: $m_{\tilde{f}}^2 < 0$ or unstable $|0\rangle$; dark blue region: excluded due to $B_s - \overline{B}_s$; medium blue region: consistent with $B_s - \overline{B}_s$ but excluded due to $b \rightarrow s\gamma$; light blue region: consistent with $B_s - \overline{B}_s$ and $b \rightarrow s\gamma$ but inconsistent with $\tau \rightarrow \mu\gamma$; green region: compatible with all three FCNC constraints.
Higgs mass & CPV in $B_s$ mixing

$m_{\tilde{g}_3} = 500$ GeV, $\text{sgn}(\mu) = +1$, $\tan \beta = 3$

$m_{\tilde{g}_3} = 500$ GeV, $\text{sgn}(\mu) = +1$, $\tan \beta = 6$

- Excludes whole green region at $\tan \beta = 3$
- Max possible $B_s$ mixing phase (degrees)

- Higgs mass bound can be satisfied for $\tan \beta = 6$ (or greater)
A very brief history of flavour

1934    Fermi proposes Hamiltonian for beta decay

\[ H_W = -G_F (\bar{p}\gamma^\mu n)(\bar{e}\gamma_\mu \nu) \]

1956-57  Lee&Yang propose parity violation to explain “θ-τ paradox”.
Wu et al show parity is violated in β decay
Goldhaber et al show that the neutrinos produced in \(^{152}\text{Eu} K\)-capture always have negative helicity

1957    Gell-Mann & Feynman, Marshak & Sudarshan

\[ H_W = -G_F (\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma_\mu P_L \nu_e) - G(\bar{p}\gamma^\mu P_L n)(\bar{e}\gamma_\mu P_L \nu_e) + \ldots \]

**V-A** current-current structure of weak interactions. Conservation of vector current proposed
Experiments give \( G = 0.96 \ G_F \) (for the vector parts)
1960-63 To achieve a universal coupling, Gell-Mann & Levy and Cabibbo propose that a certain superposition of neutron and Λ particle enters the weak current. Flavour physics begins!

1964 Gell-Mann gives hadronic weak current in the quark model

\[ H_W = -G_F J^\mu J^\dagger_\mu \]

\[ J^\mu = \bar{u} \gamma^\mu P_L (\cos \theta_c d + \sin \theta_c s) + \bar{\nu}_e \gamma^\mu P_L e + \bar{\nu}_\mu \gamma^\mu P_L \mu \]

1964 CP violation discovered in Kaon decays (Cronin & Fitch)

1960-1968 \( J_\mu \) part of triplet of weak gauge currents. Neutral current interactions predicted and, later, observed at CERN.

However, the predicted flavour-changing neutral current (FCNC) processes such as \( K_L \rightarrow \mu^+ \mu^- \) are not observed!
1970  To explain the absence of $K_L \rightarrow \mu^+\mu^-$, Glashow, Iliopoulos & Maiani (GIM) couple a “charmed quark” to the formerly “sterile” linear combination

$$-\sin \theta_c d_L + \cos \theta_c s_L$$

The doublet structure eliminates the Zsd coupling!

1971  Weak interactions are renormalizable (‘t Hooft)

1972  Kobayashi & Maskawa show that CP violation requires extra particles, for example a third doublet. CKM matrix

1974  Gaillard & Lee estimate loop contributions to the $K_L$-$K_S$ mass difference

Bound $m_c < 5$ GeV

1974  Charm quark discovered
1977  τ lepton and bottom quark discovered
1983  W and Z bosons produced
1987  ARGUS measures B_d - B_d mass difference
      First indication of a heavy top
      The diagram depends quadratically on m_t
1995 top quark discovered at CDF & D0

<table>
<thead>
<tr>
<th></th>
<th>u_L</th>
<th>u_R</th>
<th>c_L</th>
<th>c_R</th>
<th>t_L</th>
<th>t_R</th>
<th>b_L</th>
<th>b_R</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_L</td>
<td></td>
<td></td>
<td>s_L</td>
<td>s_R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Q</td>
</tr>
<tr>
<td>e_L</td>
<td>ν_e</td>
<td></td>
<td>ν_μ</td>
<td></td>
<td>ν_τ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ Q = +2/3 \]
\[ Q = -1/3 \]
\[ Q = 0 \]
\[ Q = -1 \]

2012-

SUSY, new strong interactions, extra dimensions, ...
Summary: what can we learn?

- The case for flavour is strong (if there is anything at TeV or not too far above).

- For hadronic decays at LHCb, strong QCD dynamics is the main theory obstacle, but less so in some observables than in others

- Observables not depending on strong phases preferred [calculable phases $O(as)$ ~ incalculable ones $O(L/mb)$]

- Feedback from experiment important (to fit/constrain some amplitudes, develop theory). Look at sine coefficients, TP’s, and of course CP-conserving data - specifically “wrong polarisations” can probe RH currents

- Illustrated the power to probe fundamental scales within a SUSY GUT model
BACKUP
“msugra GUTs”

Assume that SUSY breaking is flavour blind and universal (like msugra) at or near the Planck scale

\[
\mathcal{L}_{\text{soft}} = -\tilde{1}_6^i m_{16}^{2ij} \tilde{1}_6^j - m_{10H}^2 10^*_H 10_H - m_{10}_H^2 10^*_H 10_H' \\
- m_{16H}^2 \tilde{1}_6^i \tilde{1}_6^j - m_{16}^2 16^*_H 16_H - m_{45H}^2 45^*_H 45_H \\
- \left( \frac{1}{2} \tilde{1}_6^i A^{jj}_1 \tilde{1}_6^j 10_H + \tilde{1}_6^i A^{jj}_2 \tilde{1}_6^j 10_H' \right) + \frac{1}{2} \tilde{1}_6^i A^{jj}_N \tilde{1}_6^j \tilde{1}_6^i \tilde{1}_6^j 16_H \tilde{1}_6^j + \text{h.c.} \right)
\]

\[
m_{16i}^2 = m_0^2 \mathbb{1}, \quad m_{10H}^2 = m_{10_H}^2 = m_{16}^2 = m_{16_H}^2 = m_{45_H}^2 = m_0^2
\]

\[
A_1 = a_0 Y_1, \quad A_2 = a_0 Y_2, \quad A_N = a_0 Y_N,
\]

radiative corrections lead to a \textit{nonuniversal} sfermion mass matrix at the GUT scale, \textit{diagonal in the U-basis}

[Hall, Kostelecky, Raby 86; Barbieri, Hall, Strumia 95]

\[
m_{16_3}^2 = m_0^2 - \Delta
\]

\[
m_{16_1}^2 \approx m_{16_2}^2 = m_0^2 + \delta
\]
Higgs mass constraint

- like in mSUGRA, the weak scale gives one relation between \( \mu \) and the soft SUSY breaking parameters
- like always in the MSSM, the Higgs ‘likes’ to be light tree level
  - (very) small values of \( \tan \beta \) disfavoured
- one & two loops
  \[
  \begin{align*}
  m_H &= \frac{3G_F\sqrt{2m_t}}{\pi^2} \left\{ -\ln \left( \frac{m_t^2}{M_S^2} \right) + \frac{|X_t|^2}{M_S^2} \left( 1 - \frac{|X_t|^2}{12M_S^2} \right) \right\} \\
  &\quad - \frac{3G_F\sqrt{2\alpha_s m_t}}{\pi^3} \left\{ \ln^2 \left( \frac{m_t^2}{M_S^2} \right) + \left[ \frac{2}{3} - 2\frac{|X_t|^2}{M_S^2} \left( 1 - \frac{|X_t|^2}{12M_S^2} \right) \right] \ln \left( \frac{m_t^2}{M_S^2} \right) \right\}
  \end{align*}
  \]
  - larger \( \tan \beta \) reduces \( y_t \) and size of flavour effects
- could be relaxed by allowing the Higgs multiplets to have different Planck-scale masses from the sfermions (similarly to the ‘non-universal Higgs model’ (NUHM))
Theoretical description

\[ A(B \to K^{*}ll^-) = \]

![Diagram showing the process of the decay of a B meson to a K* meson and a muon-antimuon pair, with various decay channels and Wilson coefficients.]

"naively" factorize into form factors and "effective" Wilson coefficients \( C_{9\text{eff}}, C_{10} \)

partly short distance

Form factor
(lattice, QCD sum rules)

\[ T_{1,2,3} \times C_7 \]

partly long distance

Wilson coefficient (may receive NP corrections)

\( q = \text{charm} / u / d / s \)
not calculable in terms of form factors

[Fig C Bobeth]
Long-distance effects

no known way to treat charm resonance region to the necessary precision (would need << 1% to see short-distance contribution) “solution”: cut out 6 GeV$^2 < q^2 < 14$ GeV$^2$

above (high-$q^2$) charm loops calculable in OPE

Grinstein et al; Beylich et al 2011

at low $q^2$, long-distance charm effects also suppressed, but photon can now be emitted from spectator without power suppression


small Wilson coefficients

more significant for $b \to s$ transitions

$$\frac{\pi^2}{N_c} \frac{f_B f_{K^*,a}}{M_B} \Xi_a \sum_{\pm} \int d\omega \frac{\Phi_{B,\pm}(\omega)}{\omega} \int_0^1 du \Phi_{K^*,a}(u) T_{a,\pm}(u,\omega)$$

light-cone wave functions calculable

long-distance “resonance” effects as in top figure (q=u,d,s) CKM and power suppressed