Lattice QCD meets experiment

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HPQCD collaboration

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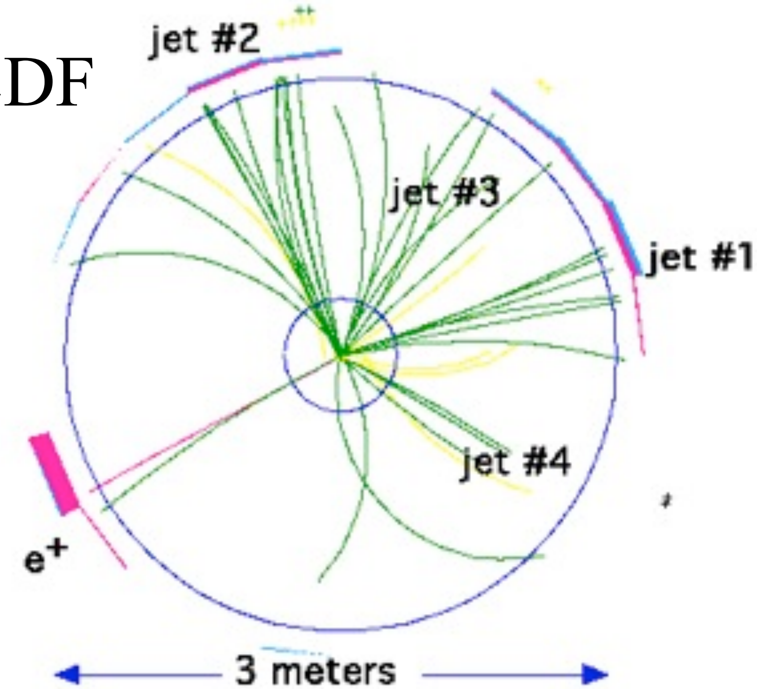
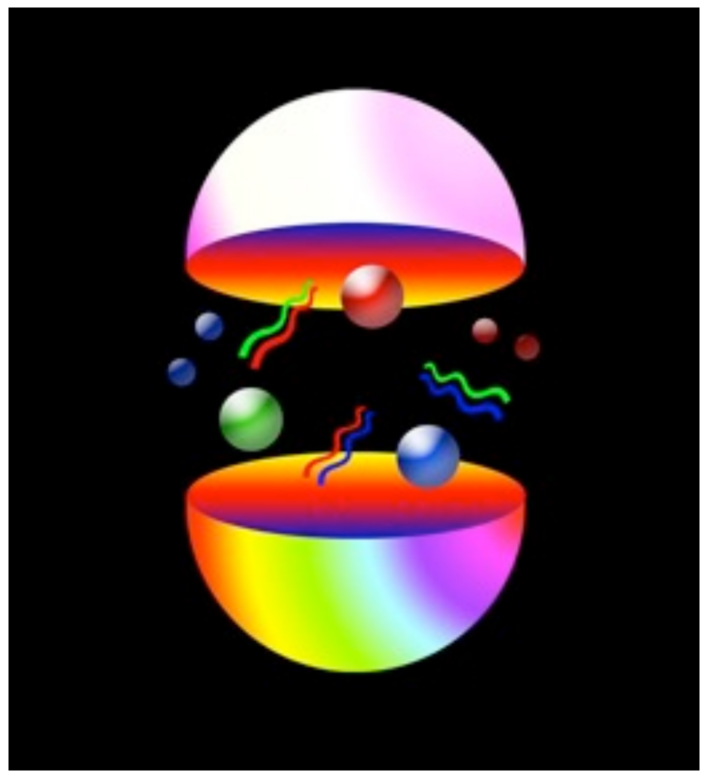
QCD is a key part of the Standard Model but quark confinement complicates things.

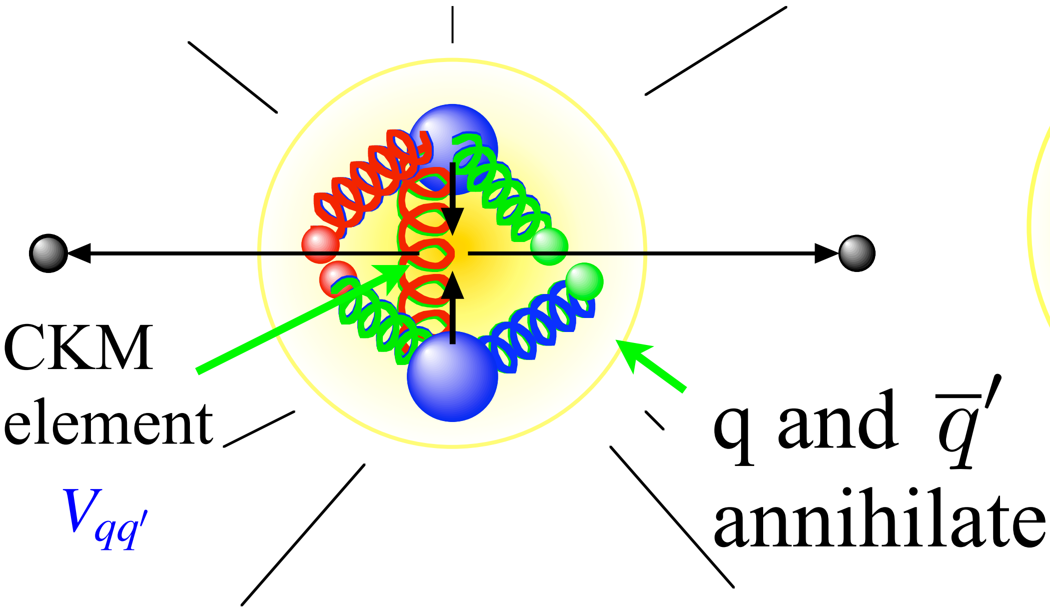
CDF

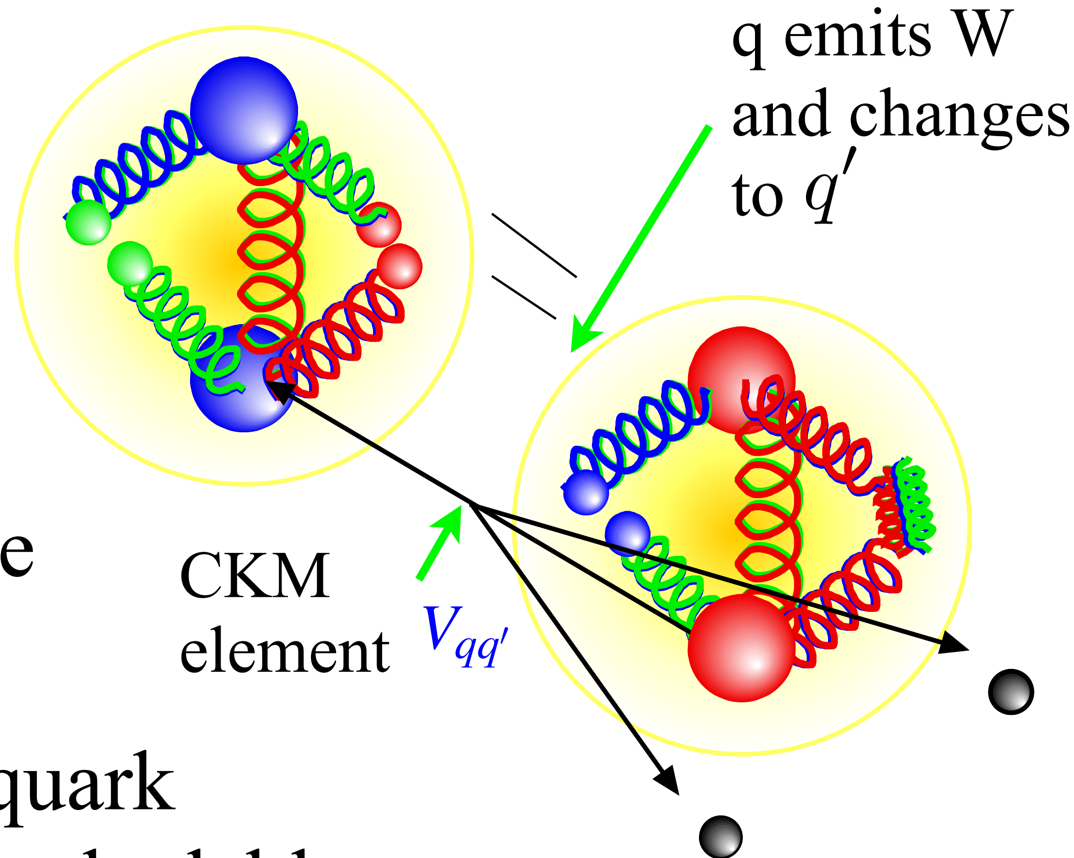


QCD only tested to 5-10% level at high energies from comparison of e.g. jet phenomena to pert.th.

But properties of hadrons calculable from QCD if fully nonperturbative calc. is done - can test QCD and determine parameters very accurately (1%).





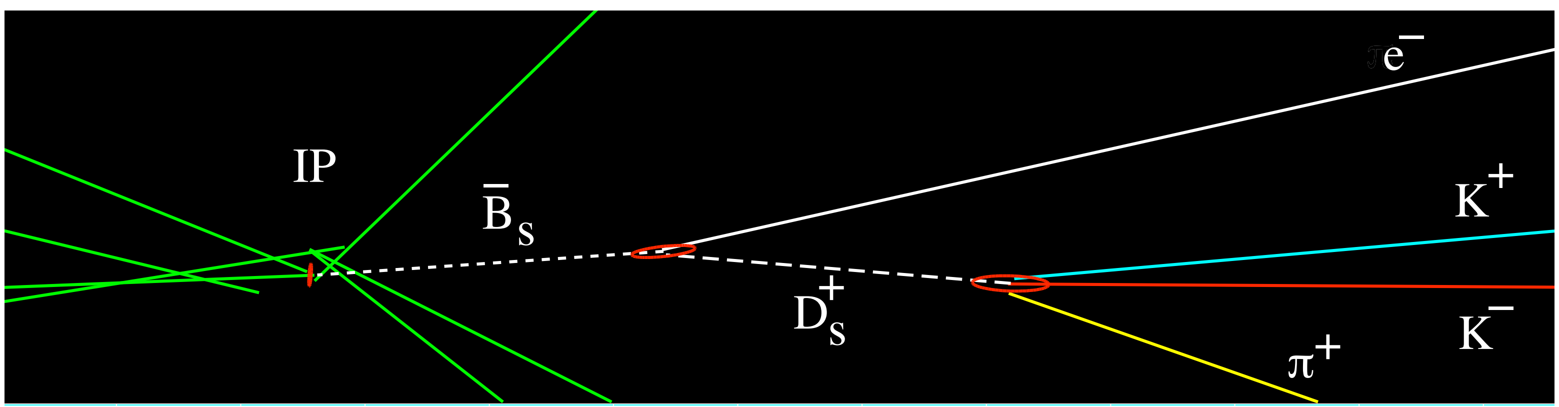


Rates for simple weak or em quark processes inside hadrons also calculable, but *not* multi-hadron final states.

$\overline{B}_s \longrightarrow D_s e^- \nu$

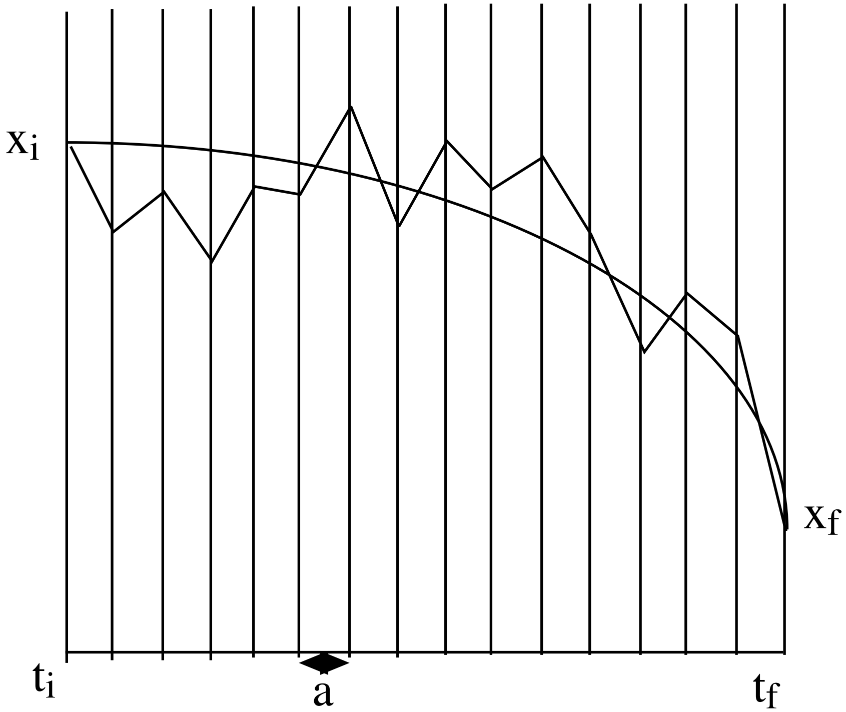
ALEPH $(D_S \longrightarrow K^+ K^- \pi^+)$

Compare to exptl rate gives $V_{qq'}$ accurately



Solving a path integral: quantum mechanical case

Solve Schrödinger's eq. for eigenvalues/fns of H or:



discretise time and integrate over all paths possible weighted by e^{iS}

$$S = \int dt \mathcal{L}; \quad \mathcal{L} = \frac{1}{2} m \dot{x}^2 - V(x)$$

classical path is $m\ddot{x} = V'$
qm path fluctuates about this.

In Euclidean time solve numerically, by making sets of $x(t_i)$

$$< x(t_2)x(t_1) >= \frac{\int \mathcal{D}x \, x(t_2)x(t_1)e^{-S}}{\int \mathcal{D}x e^{-S}} = \sum_n A_n e^{-(E_n - E_0)(t_2 - t_1)}$$

average over 'ensemble' of paths - paths chosen with prob. e^{-S}

fit as fn of time can extract excitation energies - amps are transition matrix elements

`further reading: G.P.Lepage, hep-lat/0506036`

Solving a path integral: QCD

Now path integral over gluon and quark fields on a 4-d space-time lattice - quarks anticommute so do by hand.

$$\mathcal{L}_{QCD} = \frac{1}{2} Tr F_{\mu\nu}^2 + \overline{\psi}(\gamma \cdot D + m)\psi$$

= a huge matrix, M

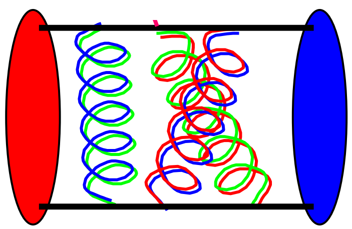
$$\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\overline{\psi} O(\psi, \overline{\psi}) e^{-S_{QCD}} \rightarrow$$

Integral over gluon fields only

$$\int \mathcal{D}U O(M^{-1}) e^{-(S_g - \ln(\det M))}$$

valence quarks inc. in operator

complicated prob, distn for gluons - inc. effects of sea quarks

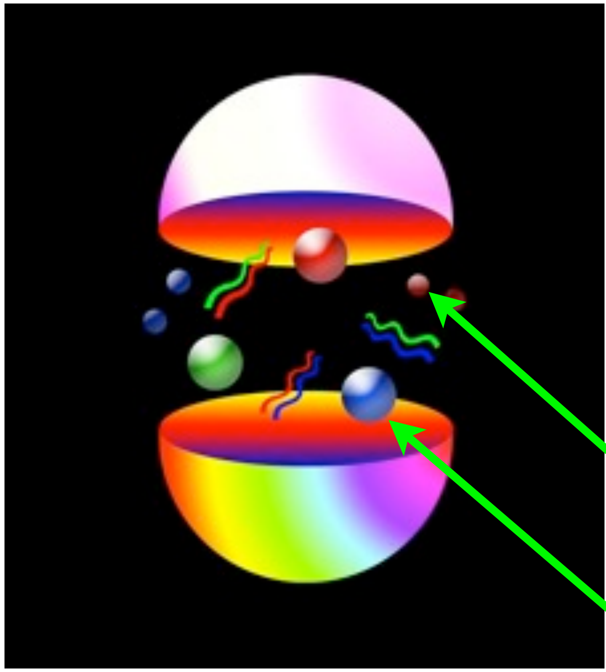


$$< O > = < H(t) H^{\dagger}(0) > = \sum_n A_n e^{-E_n t}$$

Fit as fn of t to get hadron mass and transn amp.

ensemble average

Lattice QCD = fully nonperturbative QCD calculation



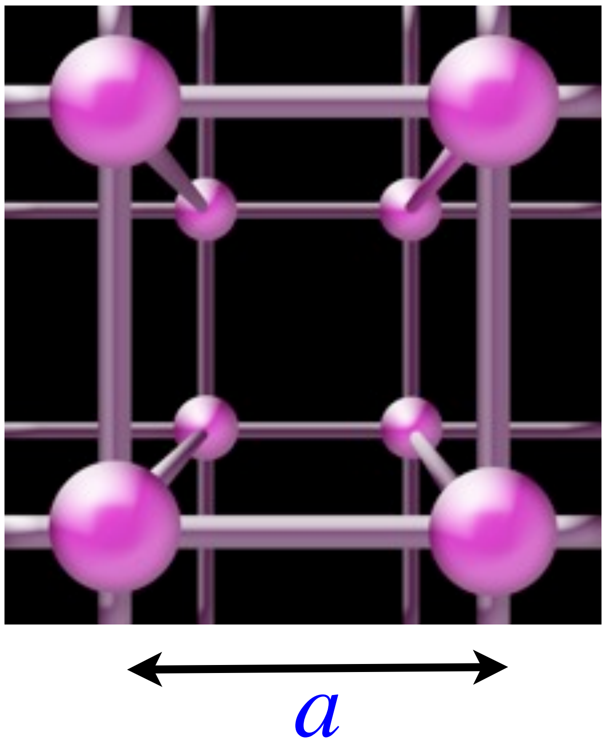
RECIPE

- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d and s sea quarks)

- Calculate averaged “hadron correlators” from valence q props.

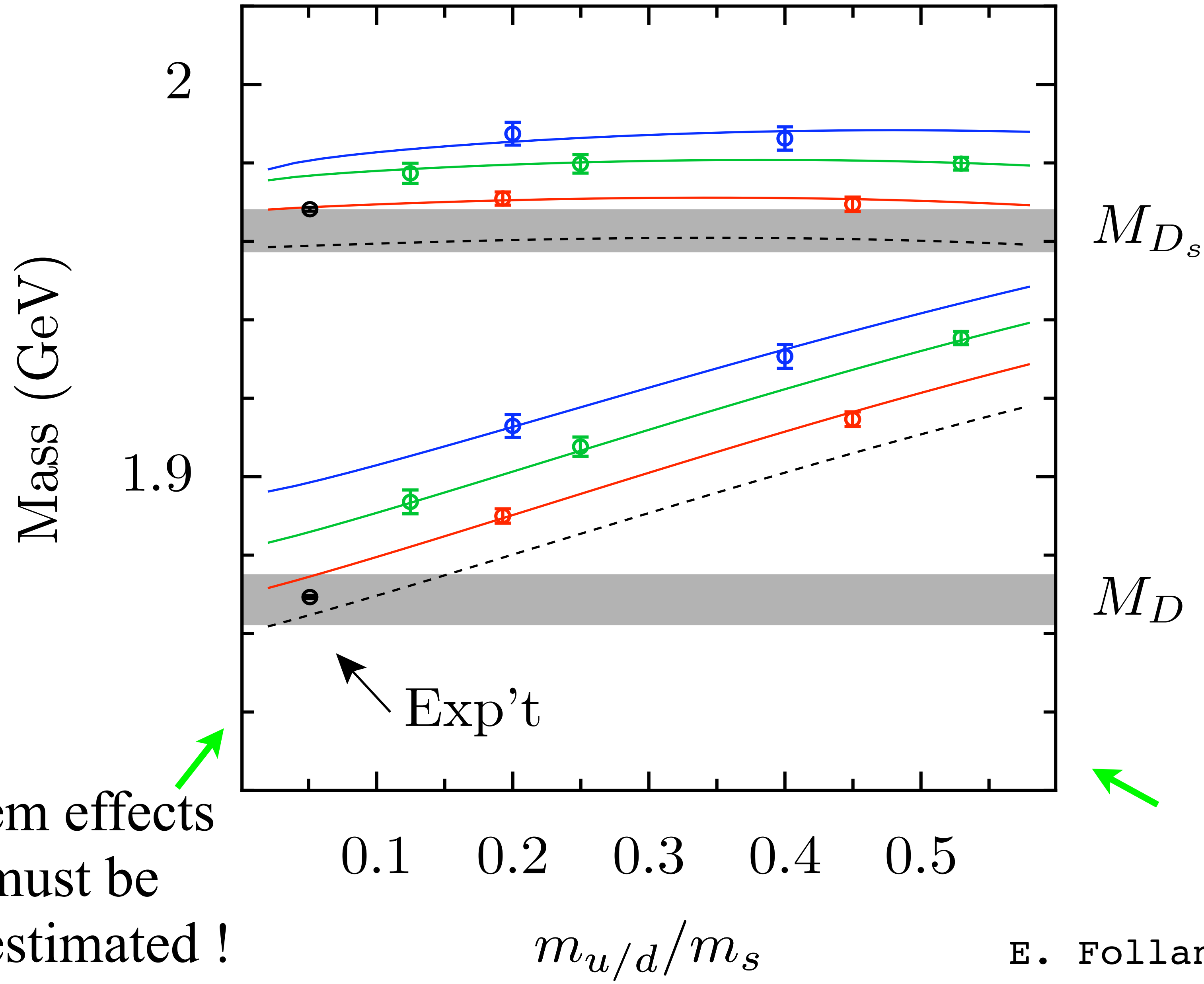
- Fit for masses and simple matrix elements

- Determine a and fix m_q to get results in physical units.



- extrapolate to $a = 0, m_{u,d} = phys$ for real world

Lattice results need to be extrapolated to the real world where a=0 and $m_{u/d}$ = small.



To do this well needs:

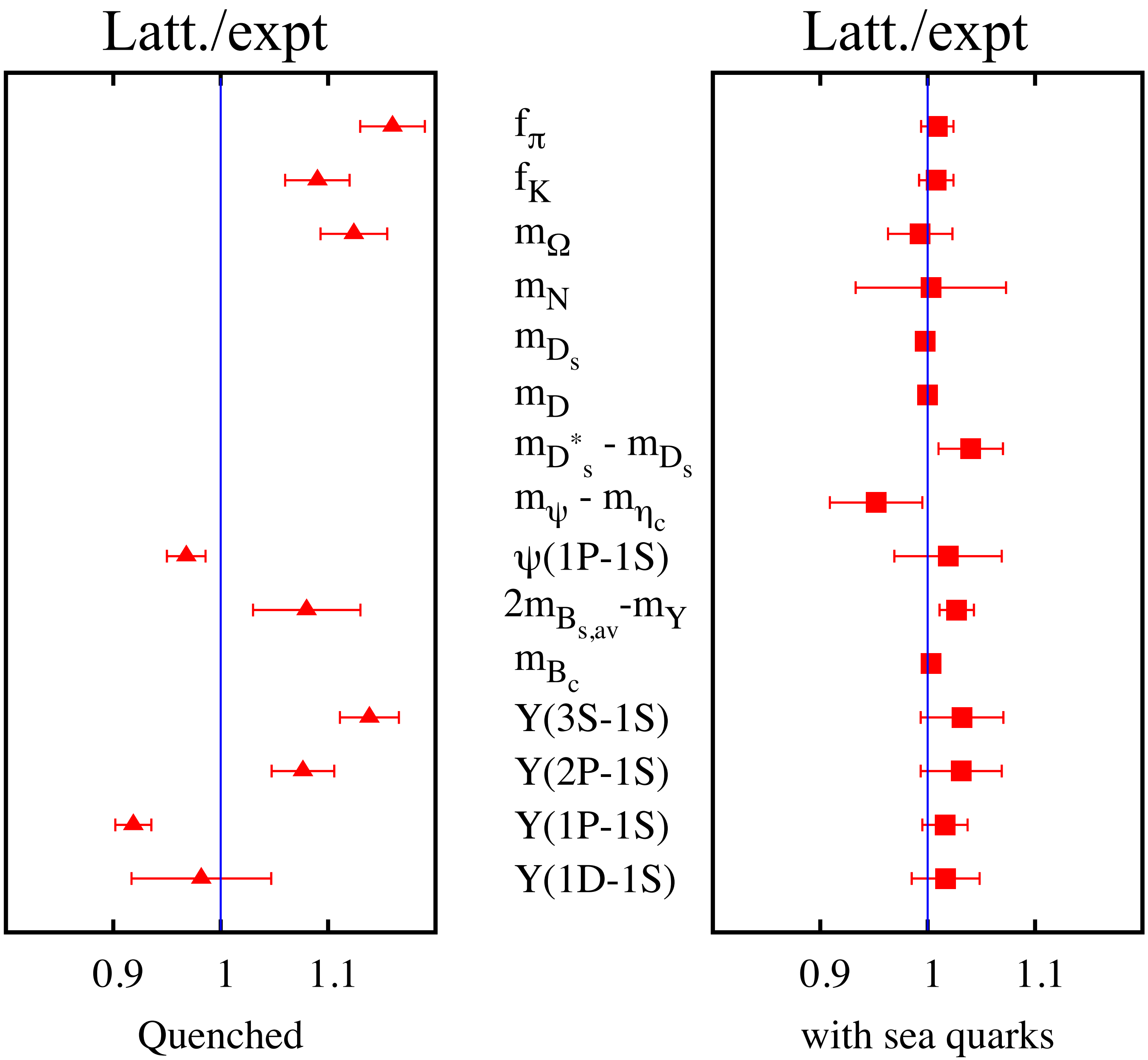
- statistical precision
- small disc. errors and several values of a
- small $m_{u/d}$

em effects must be estimated !

using HISQ charm quarks

E. Follana, CTHD et al, HPQCD, 0706.1726

Including u, d and s sea quarks is critical for accurate results, but numerically expensive - particularly light $m_{u,d}$.

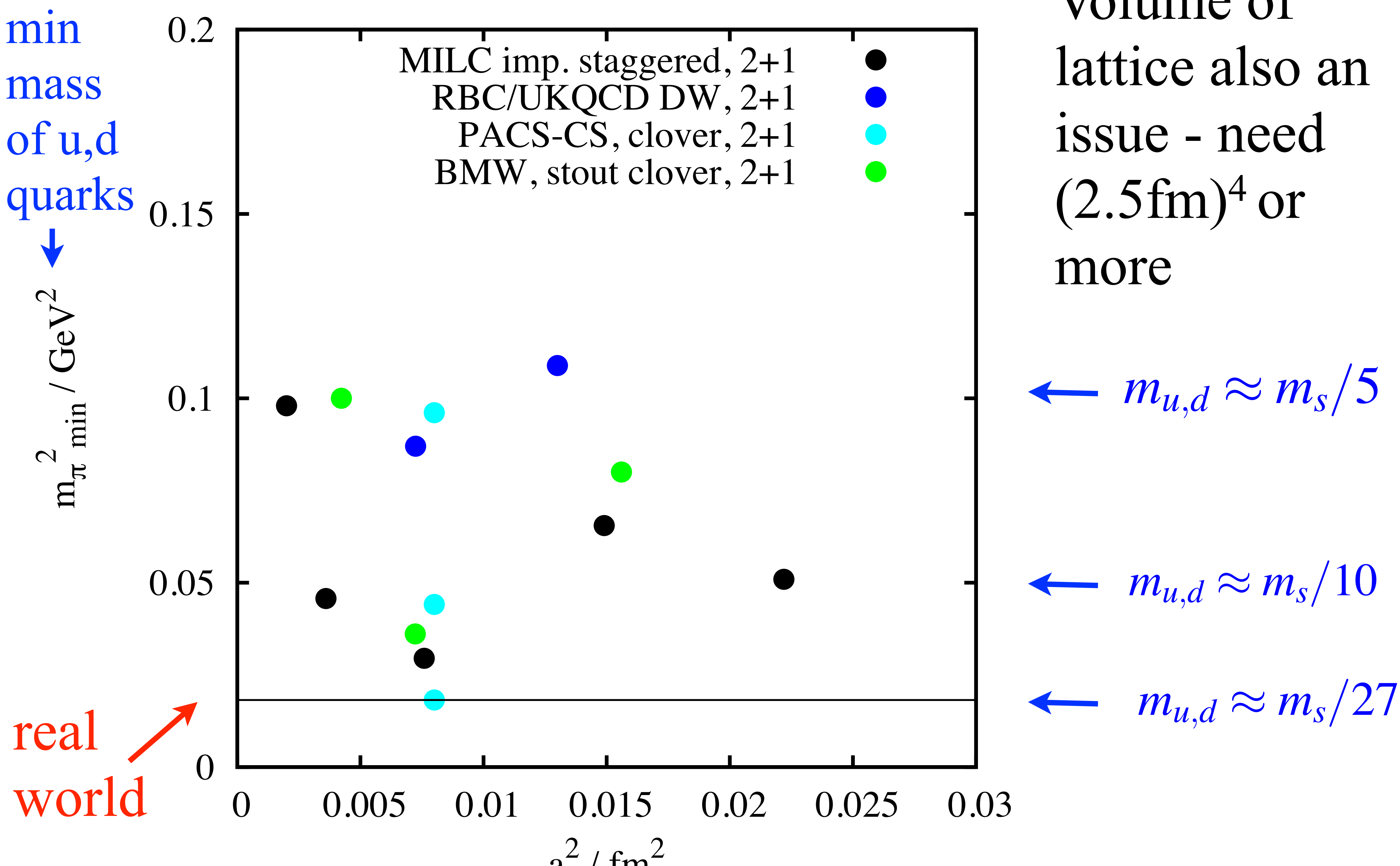


HPQCD/MILC 2008 “ratio plot”.

Multiple values of a, and of $m_{u,d}$. Extrapolate to physical point.

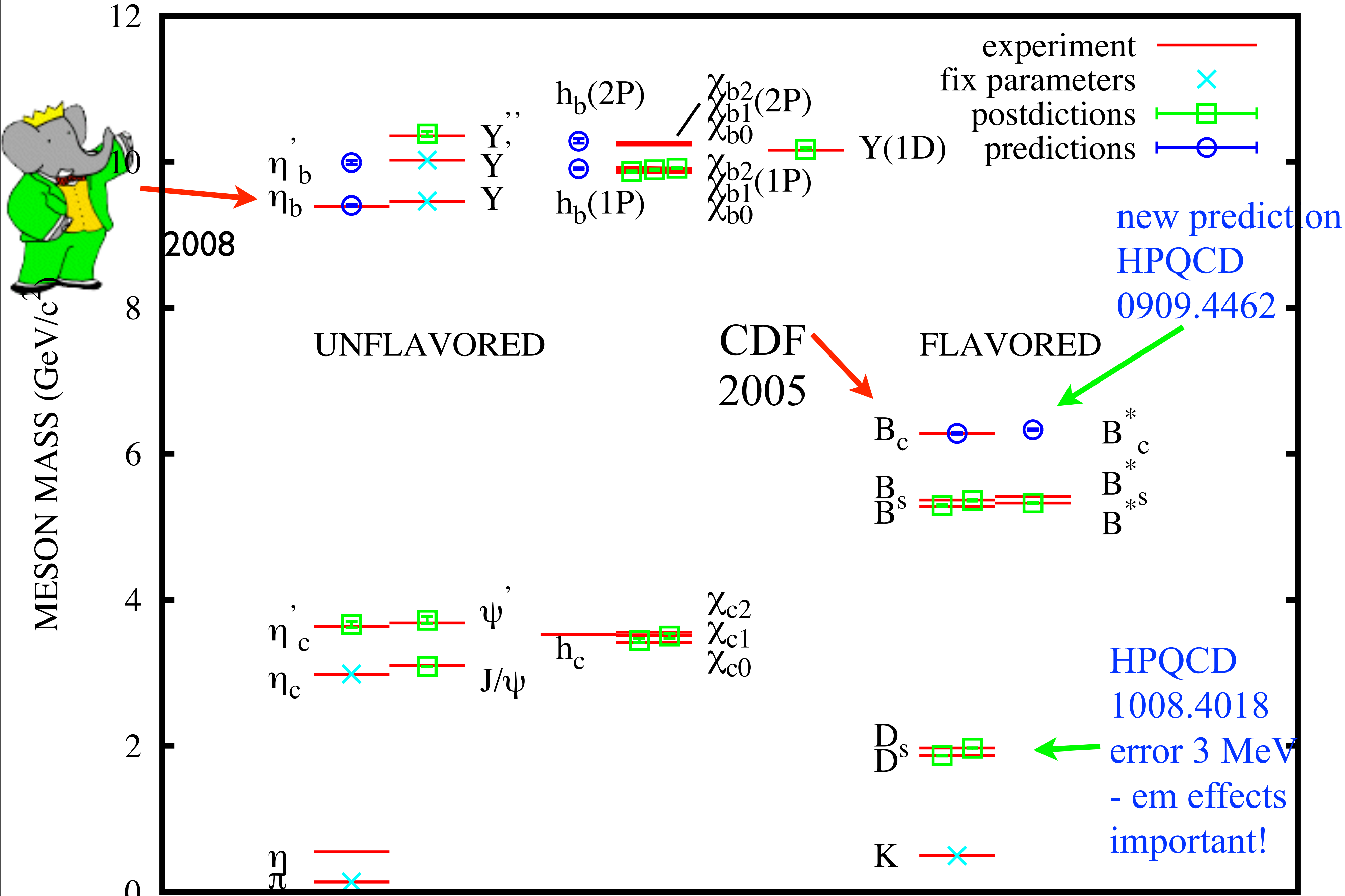
update of:
C. Davies et al, hep-lat/0304004

Example parameters for calculations now being done.
Lots of different formalisms for handling quarks.



Volume of lattice also an issue - need $(2.5\text{fm})^4$ or more

The gold-plated meson spectrum - HPQCD 2009

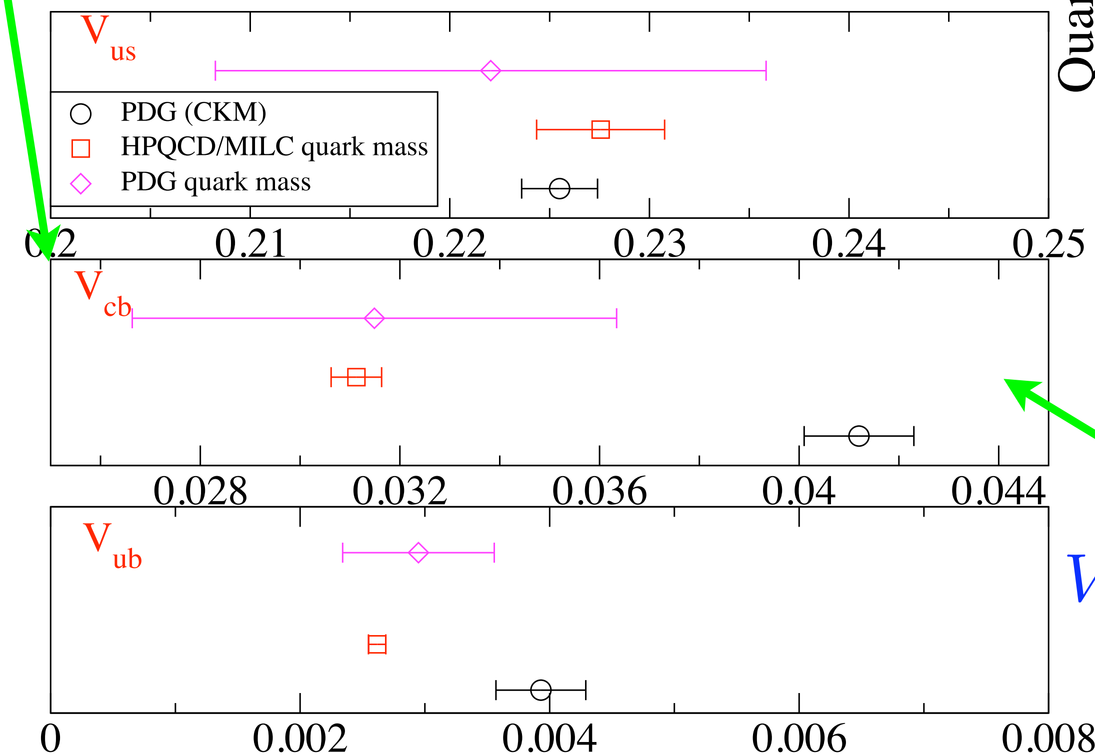


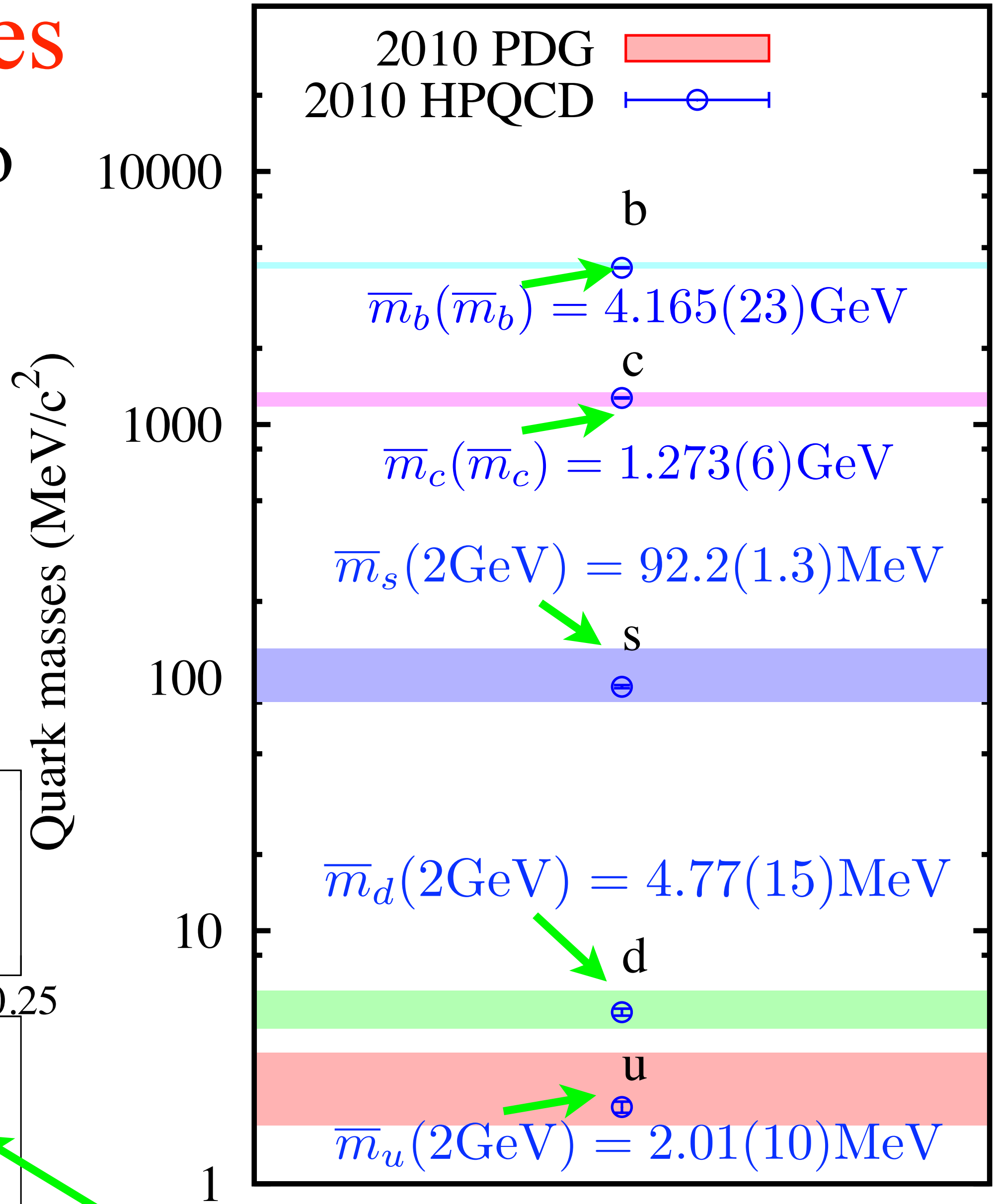
I. Allison et al, hep-lat/0411027, A. Gray et al, hep-lat/0507013

Determining quark masses

Lattice QCD has direct access to parameters in Lagrangian for accurate tuning
- issue is converting to contnm schemes such as $\overline{MS}$

Can now rule out some quark mass matrix models





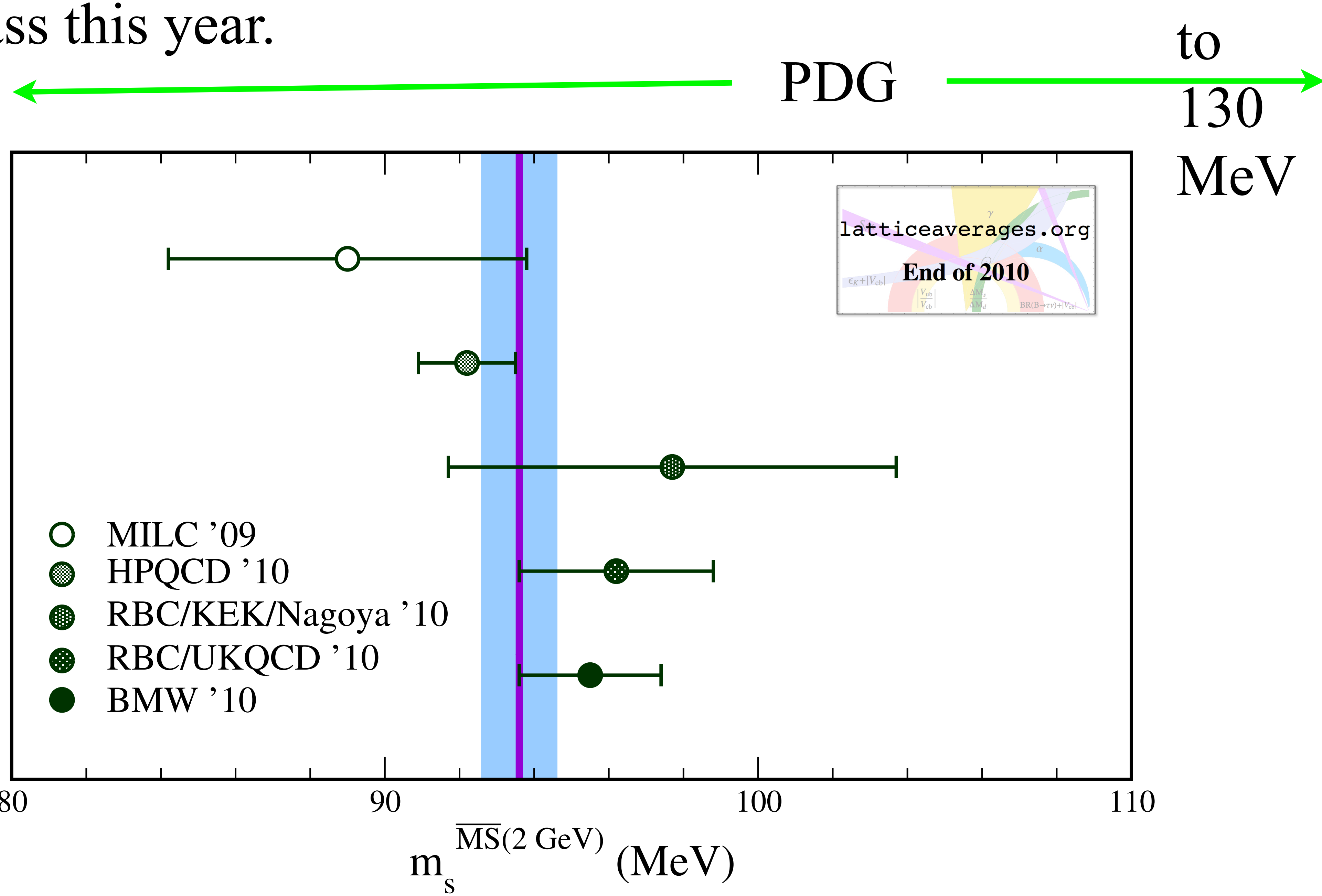
C. McNeile,CTHD, HPQCD, 1004.4285

$$V_{cb} = \sqrt{m_d/m_b}$$

vs PDG

C. McNeile, 1004.4985, model from Chkareuli+Froggatt, hep-ph/9812499

Strong convergence of lattice results for strange quark mass this year.



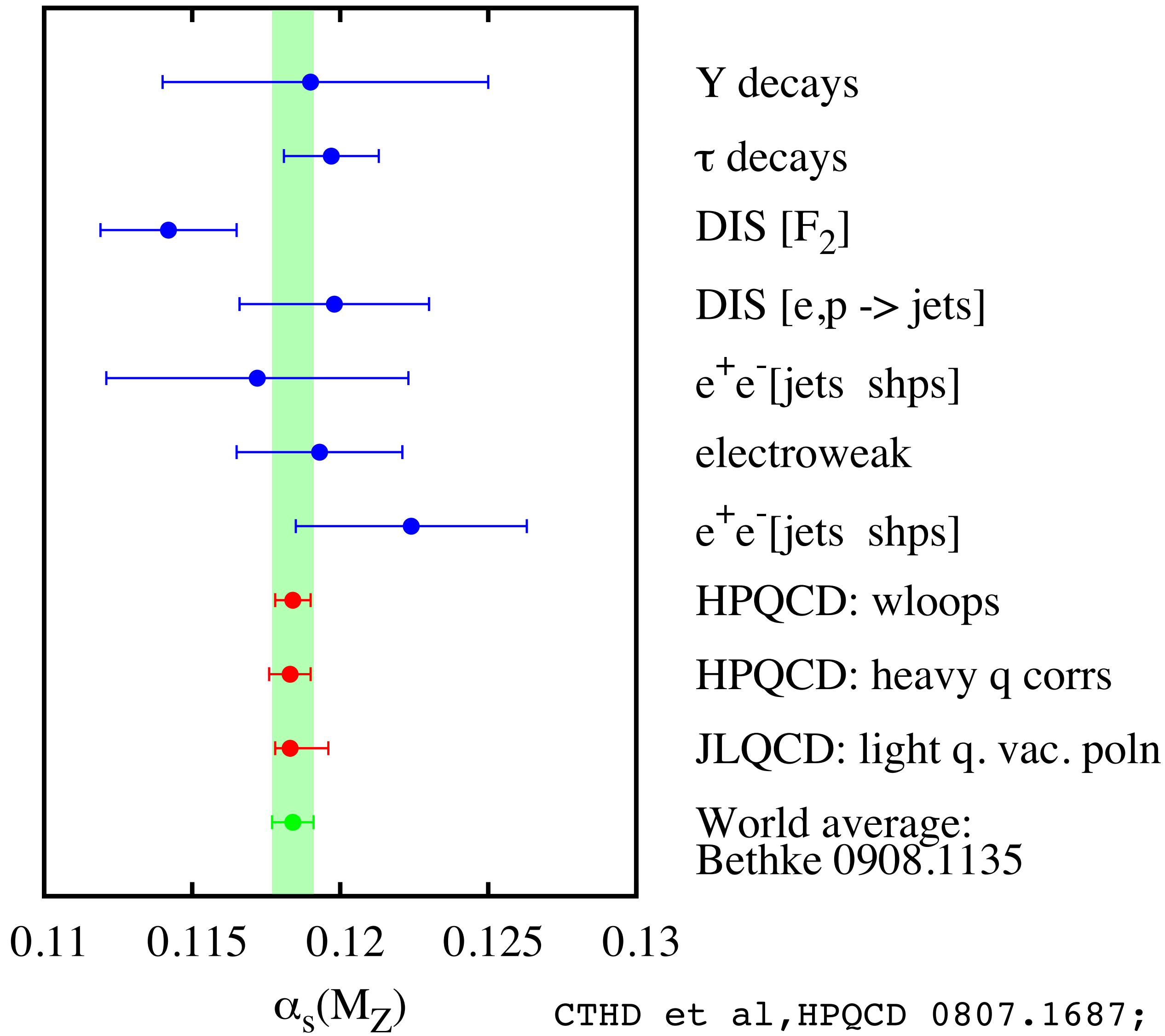
Lattice world average = 93.6(1.1) MeV

Determining α_s

Lattice QCD now has several determns of α_s to 1%.

Key points:

- high statistical precision
- high order pert. th. exists and can estimate higher orders
- higher twist not a significant issue
- approaches very different - good test

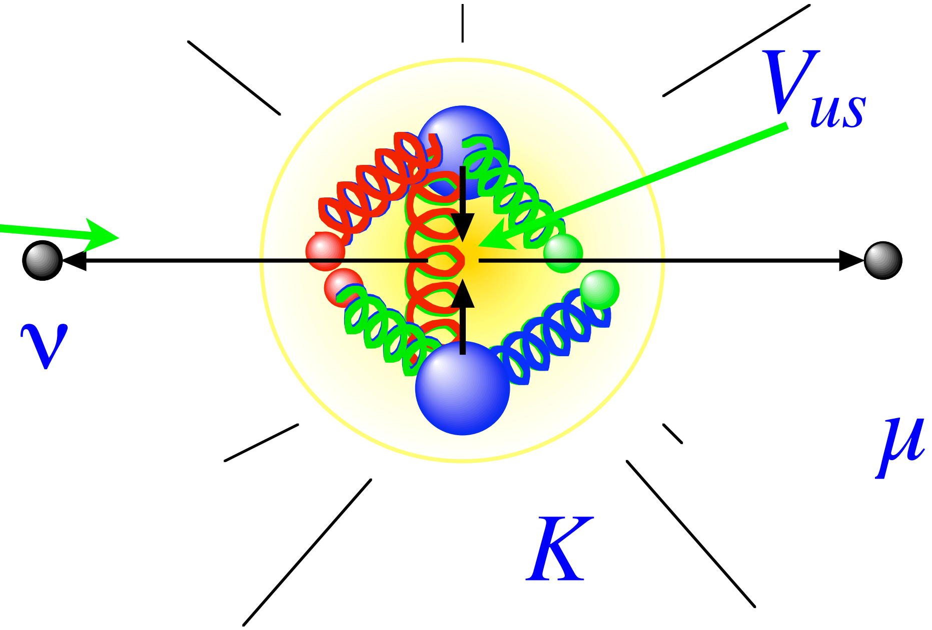


see 2011 Munich alphas workshop

CTHD et al,HPQCD 0807.1687; 1004.4285;JLQCD,1002.0371.

Determining the Cabibbo-Kobayashi-Maskawa matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to l\nu & B \to \pi l\nu \\ & K \to \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to D l\nu \\ D \to \pi l\nu & D \to K l\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & \end{pmatrix}$$

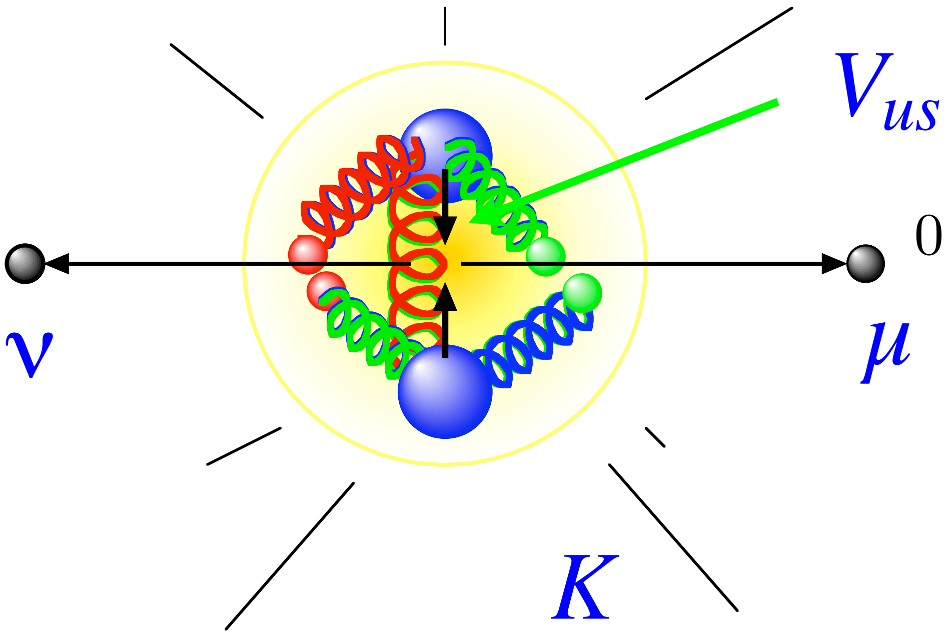


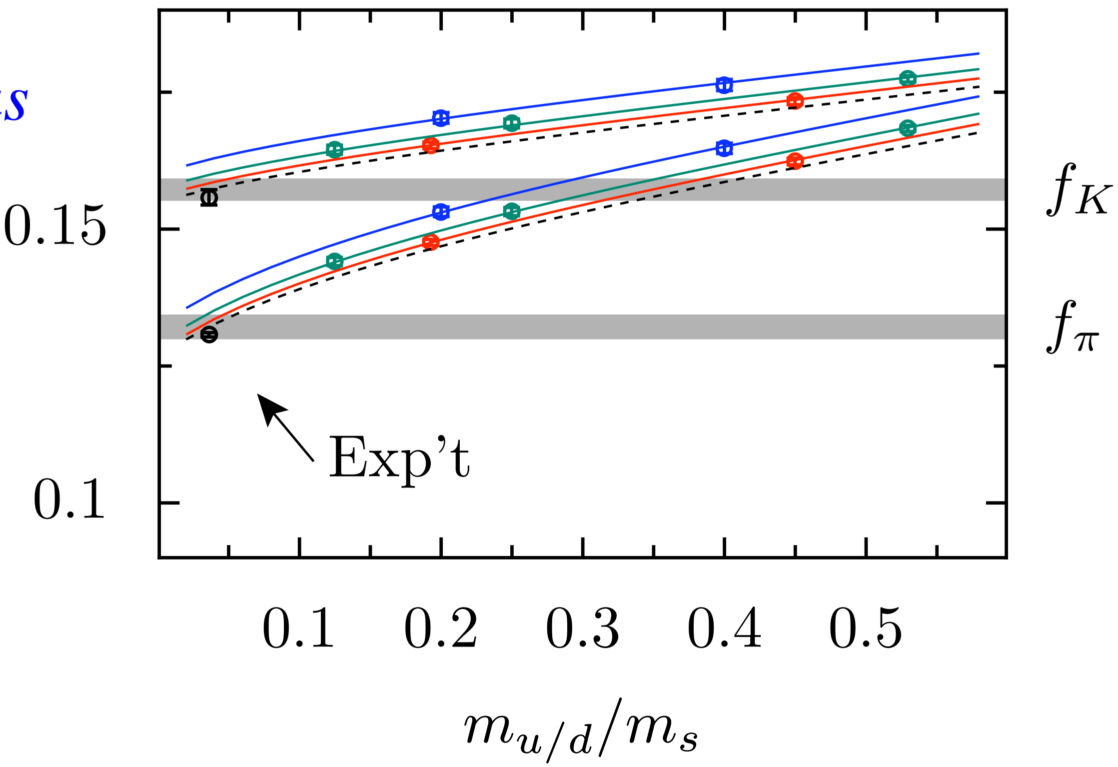
Expt = CKM x theory(QCD)

V_{ab} appears in trivial way in decay processes involving quarks a + b. Calculating QCD effects is non-trivial - need precision lattice QCD to get accurate CKM results.

If Vab known, compare lattice to expt to test QCD.

Recent CKM highlights



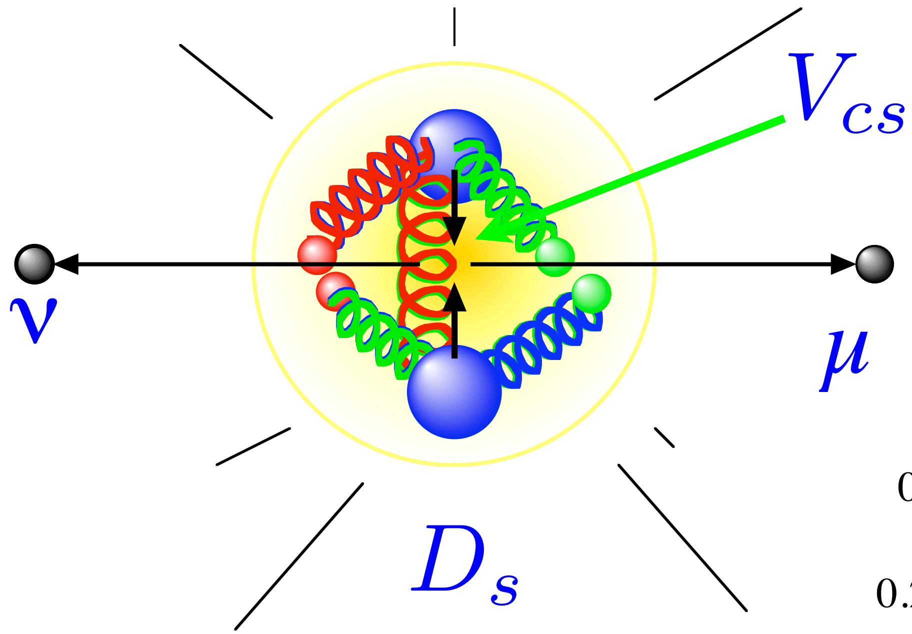


K decay constant parameterises QCD effects in leptonic decay.- calculate in lattice QCD. Combine with expt.(KLOE)

$V_{us} = 0.2262(14)$ (current world's best but theory still dominates error)

$1 - V_{ud}^2 - V_{us}^2 - V_{ub}^2 = 0.0006(8)$ test of first row unitarity of CKM matrix

Follana, CTHD et al HPQCD, 0706.1726

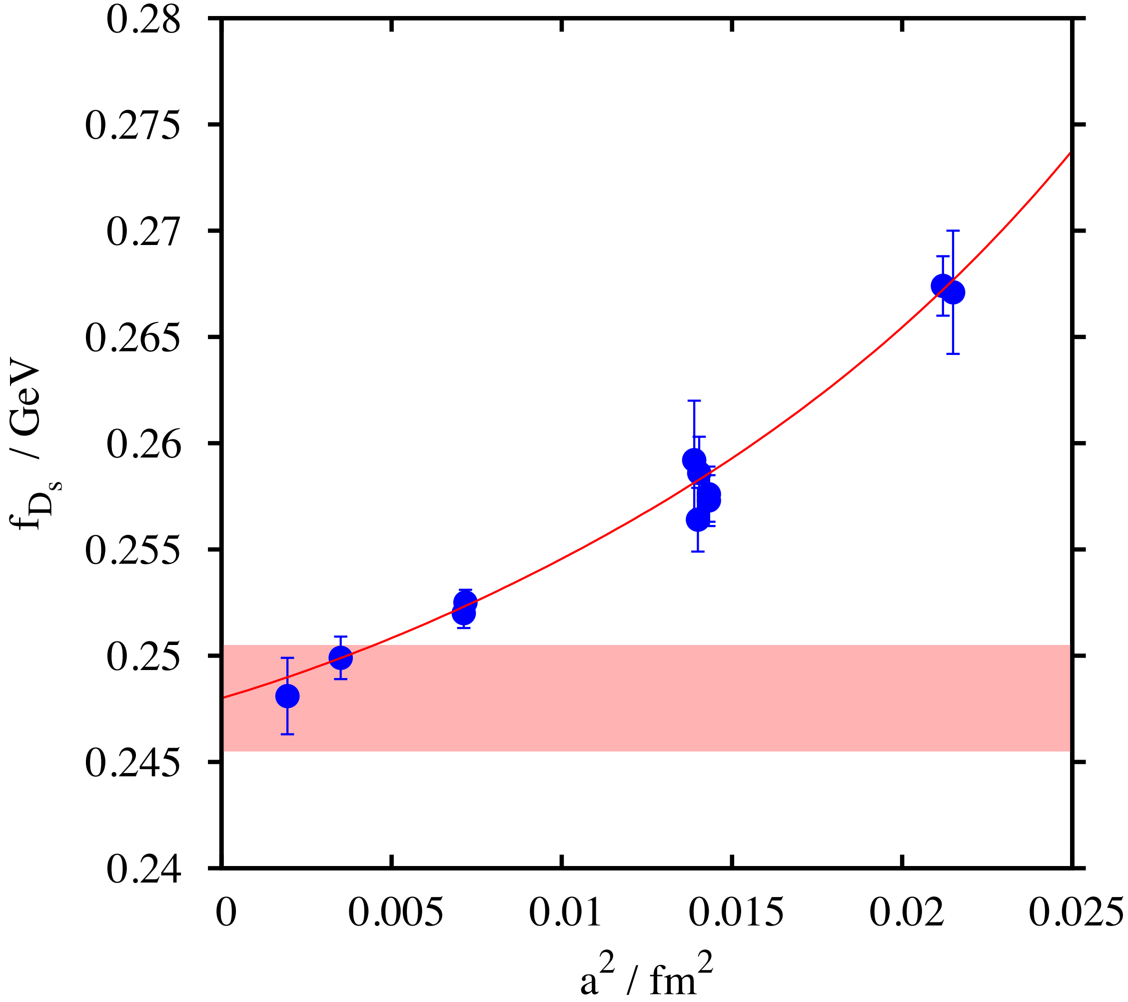


HISQ action allows us to study c meson decays also

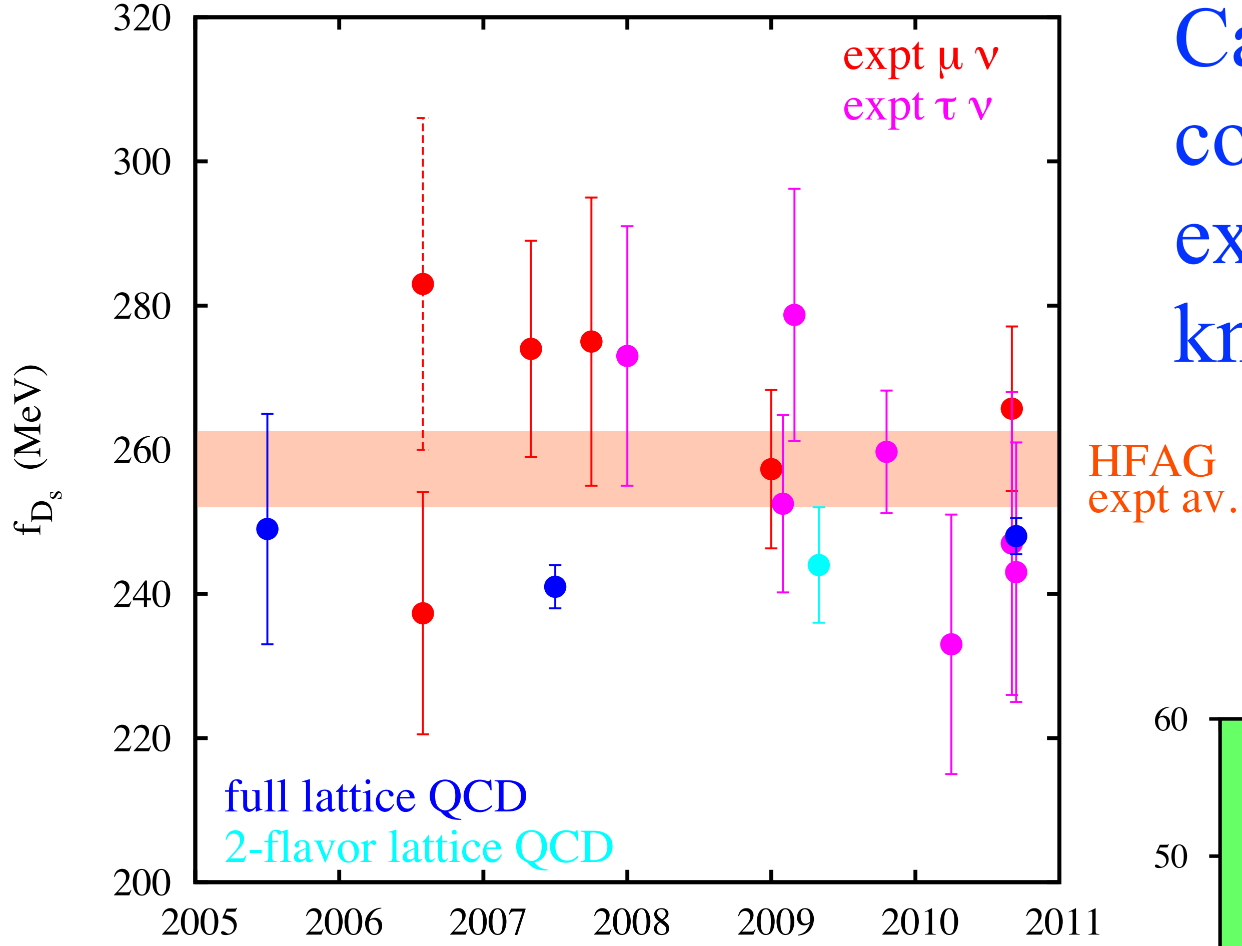
Follana, CTHD et al, HPQCD, hep-lat/0610092, 0706.1726;
CTHD et al, HPQCD, 1008.4018

$$f_{D_s} = 0.2480(25)\mathrm{GeV}$$

1% error!

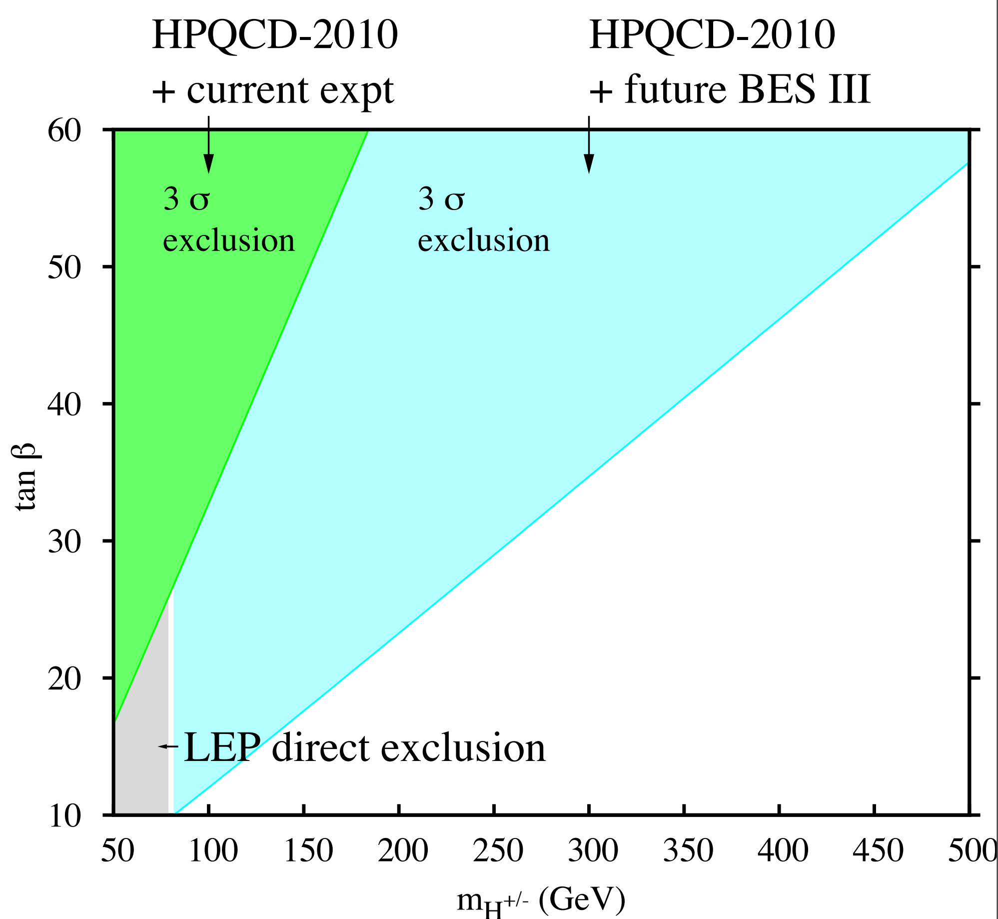


update from 2007 value of 0.241(3) GeV because of recalibration of lattice spacing



Calculate D_s decay constant - compare to experiment if assume V_{cs} known

HPQCD, 1008.4018

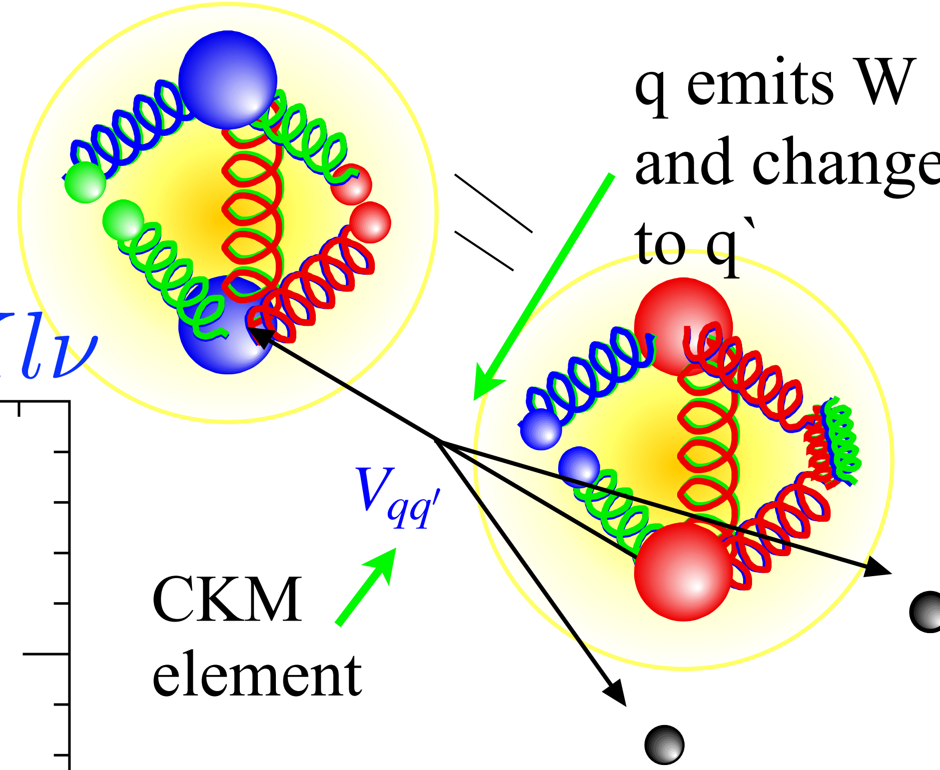


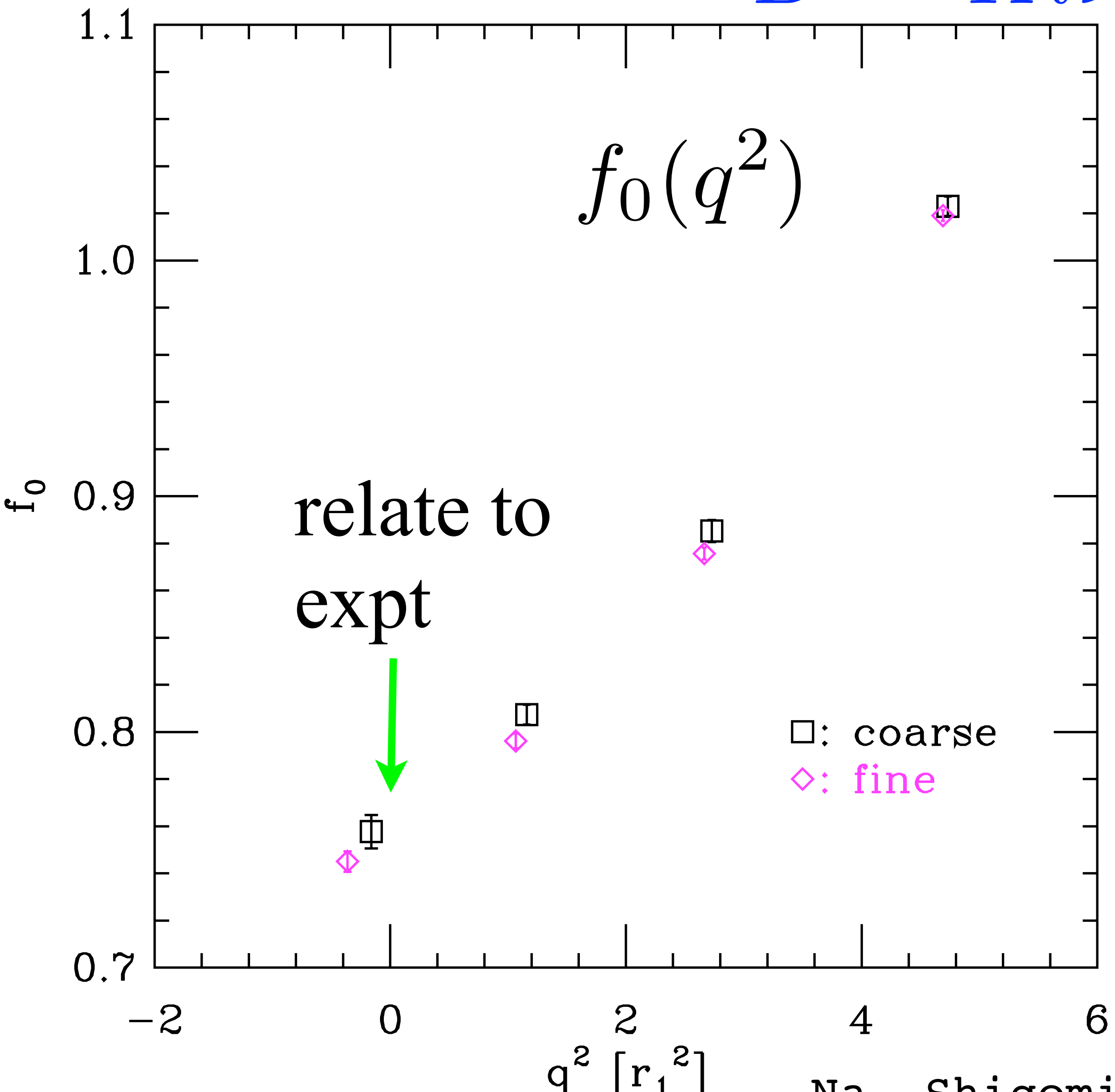
New physics would give smaller decay constant than Standard Model - allows limits on charged Higgs.

Semileptonic form factors

$$\frac{d\Gamma}{dq^2} \propto |V_{cs}|^2 f_+(q^2)$$

$D \to Kl\nu$





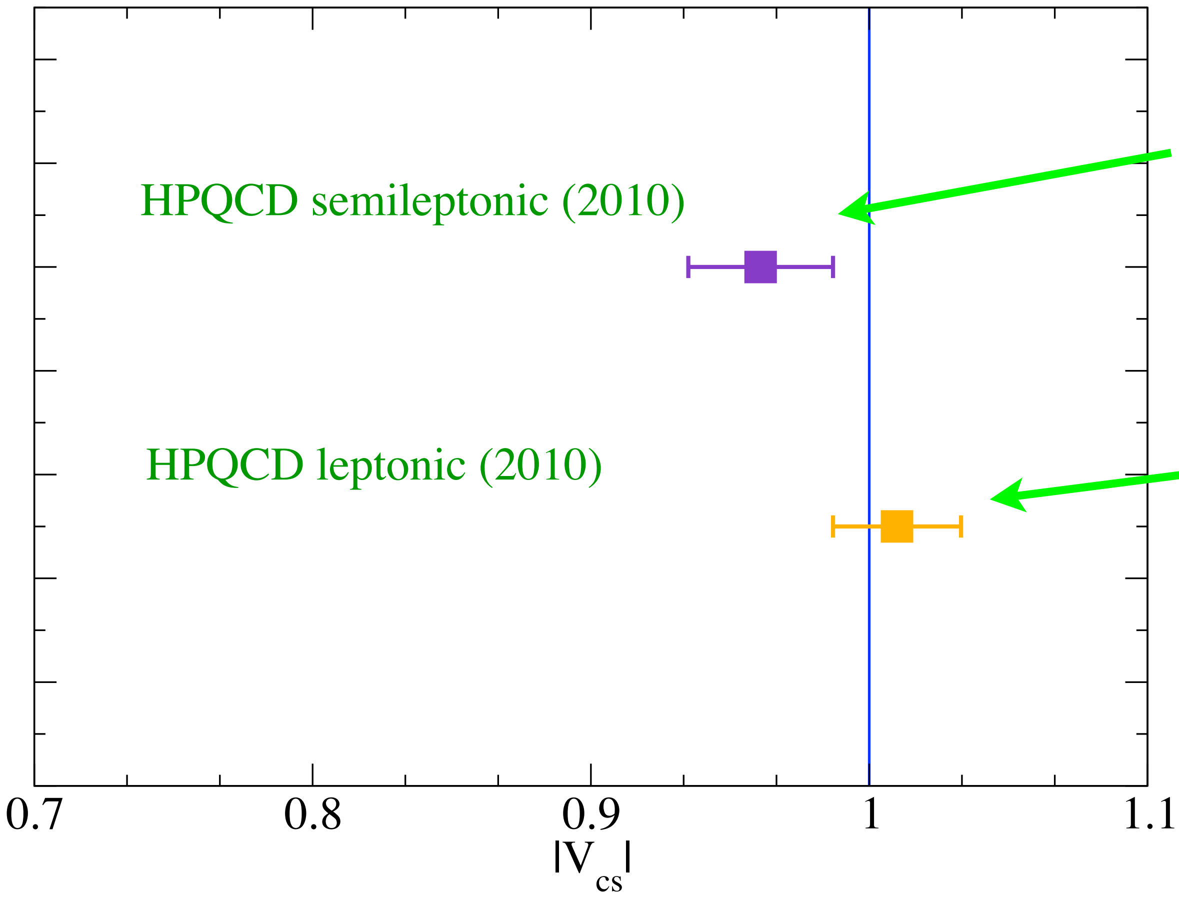
With HISQ quarks
no renormalisation issues
and good statistics
possible

$f_0(q^2=0) =$
$f_+(q^2=0) =$
$0.747(11)(15)$

2.5% !!

Na, Shigemitsu et al, HPQCD, 1008.4562

Comparison to experimental leptonic and semileptonic rates allows direct determination of V_{cs}



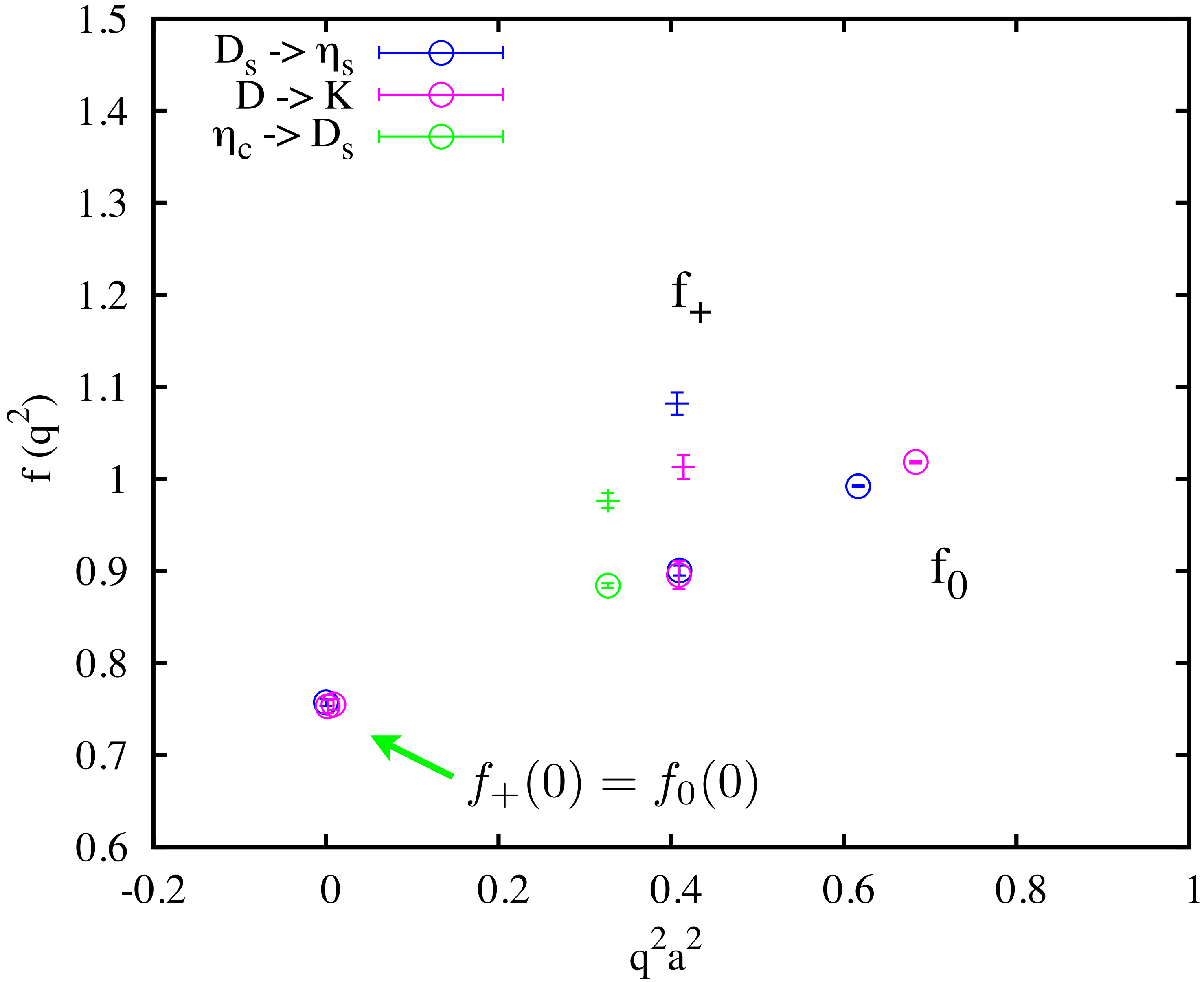
0.961(26)
error dominated by theory (now 2.5%)

1.010(22)
error dominated by expt (now 2%)

Aim: similar accuracy for CKM elements from B meson decay and mixing rates

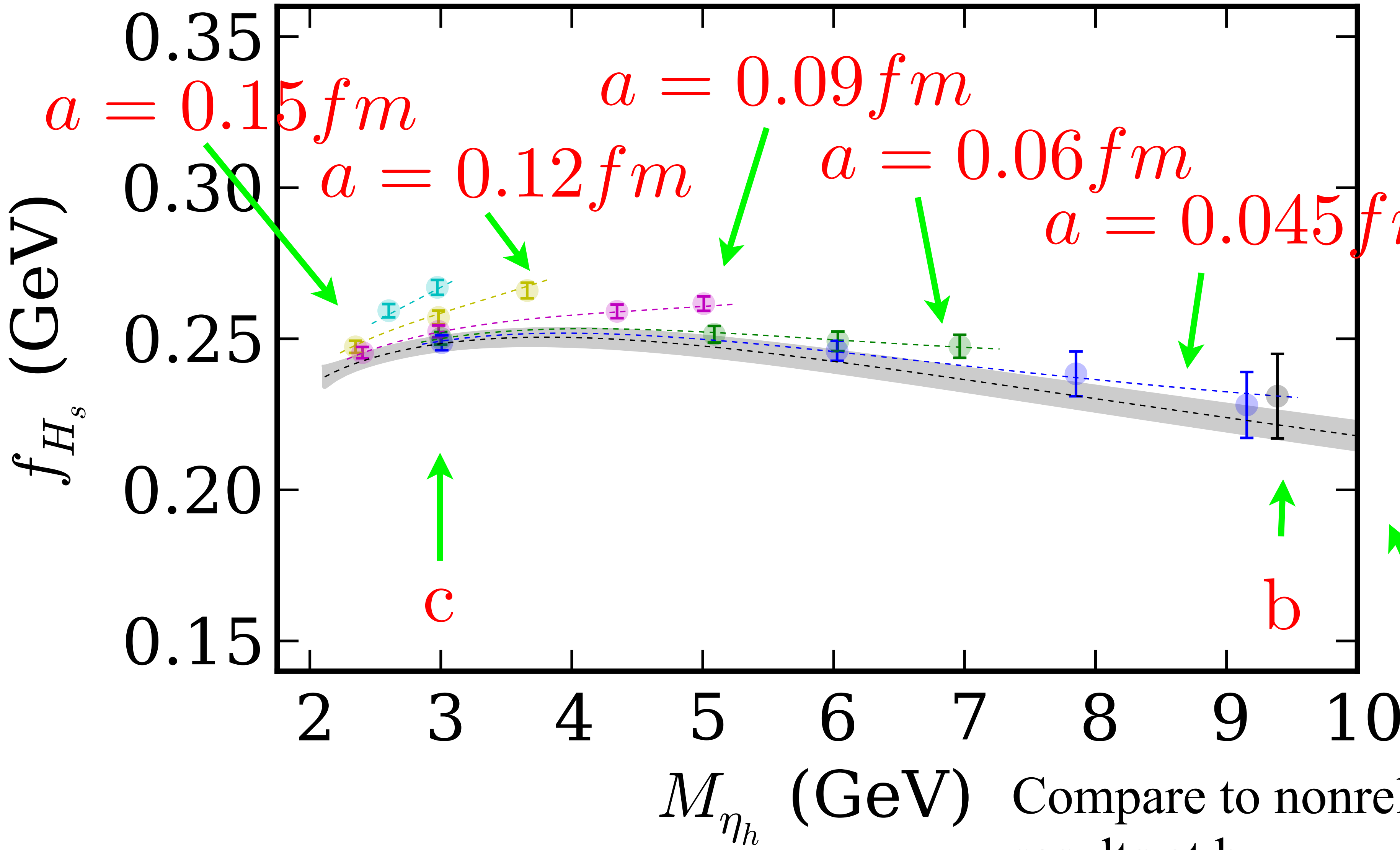
New work: calculating shape of $f_+(q^2)$

`J. Koponen, CTHD, in progress`



Comparison to expt will provide more detailed test of QCD. Note how form factor same for different processes all involving $c \to s$ decay.

New work: mapping shape of decay constants and form factors as a function of heavy quark mass from charm upwards - needs very fine lattices ...

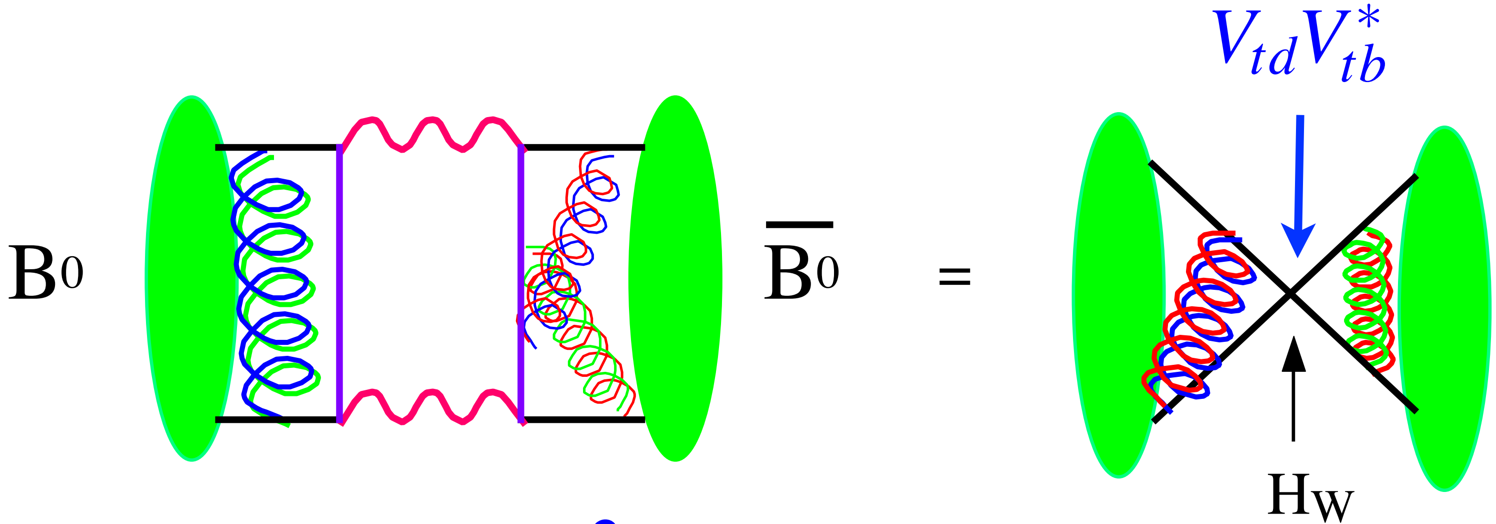


Compare to nonrelativistic results at b

McNeile, CTHD, in progress

New physics sensitivity in neutral B mixing

Neutral B (and K) mix - gives rise to 'oscillations'. Mixing determined by box diagram. Calculate in lattice QCD



Parameterise with $f_B^2 B_B$ where f_B is decay constant.

Using ratio for B_s to B_d $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$ $\rightarrow$ $|\frac{V_{td}}{V_{ts}}| = \xi\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$

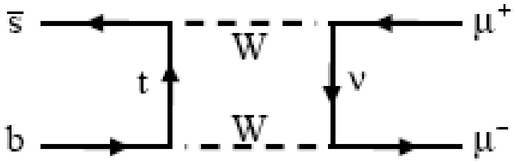
Lattice QCD results (HPQCD) using NRQCD b quarks:

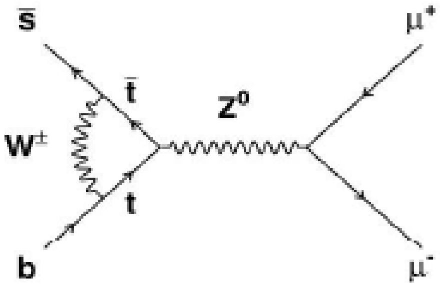
$\xi = 1.26(3)$

7% normln error cancels in ratio.

E. Gamiz, CTHD et al, HPQCD, 0902.1815

Use this to provide SM rate for LHCb of:





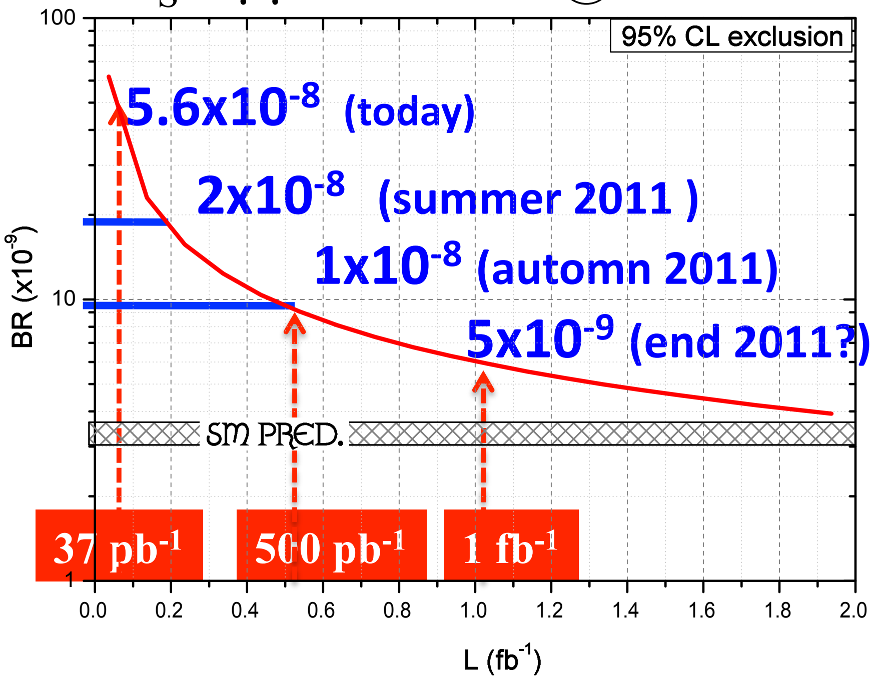
$$Br(B_s \to \mu^+\mu^-) = 3.19(19) \times 10^{-9}$$

6% error from taking ratio to mixing rate

E. Gamiz et al, 0902.1815[hep-lat]

Improvement needed, in progress ....

$B_S \to \mu\mu$ exclusion @ 95% CL

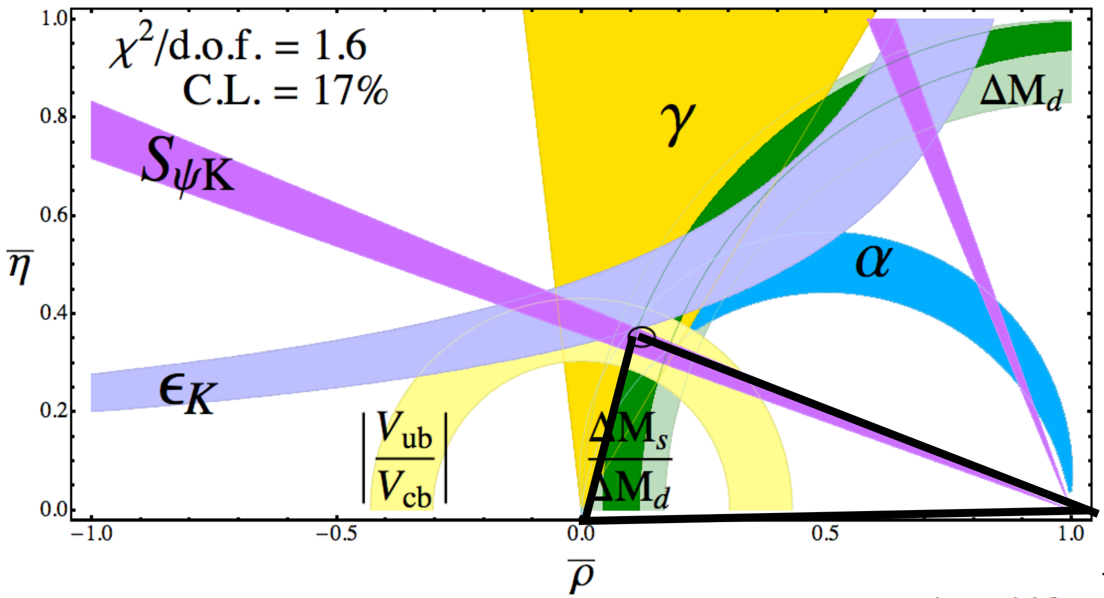


New LHCb limit

LanFranchi, LaThuile 2011

Now working to improve lattice QCD result ..

Lattice QCD calculations are key to constraining sides of CKM Unitarity triangle



current situation

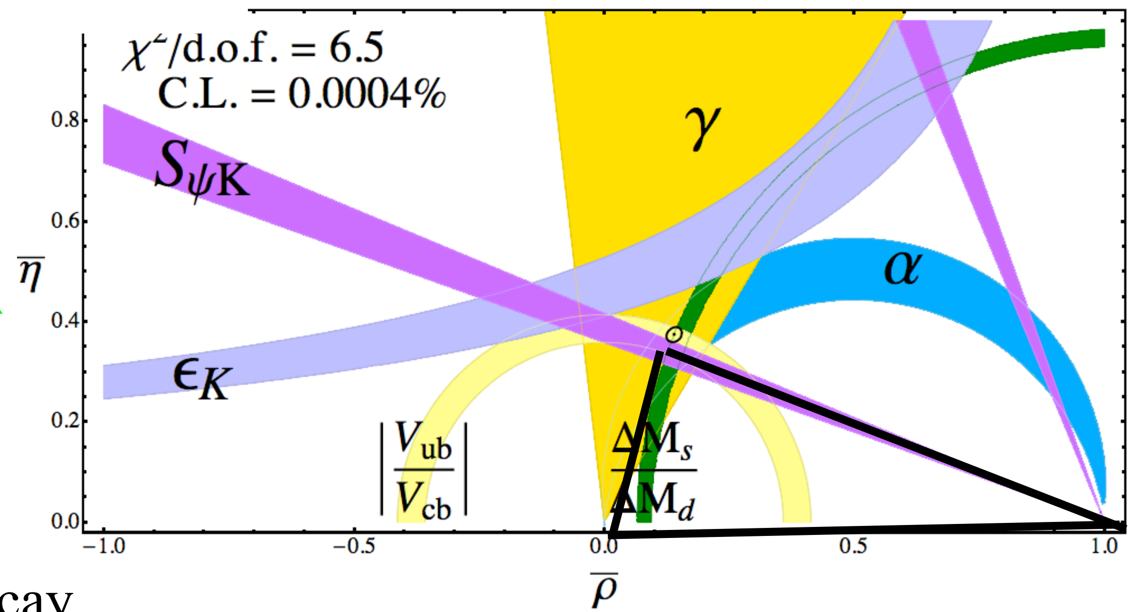
tensions developing at 2 sigma level.

Future with 1% lattice errors for

B_K, f_K

$\xi, \ f_{B_s}\sqrt{B_{B_s}}$

V_{ub}, V_{cb} from exclusive SL decay



J.Laiho et al, 0910.2928; www.latticeaverages.org

Full error budgets now available for lattice calcs

stats

tuning

chiral

continuum

$\Delta_q = 2m_{Dq} - m_{\eta c}$

	f_K/f_π	f_K	f_π	f_{D_s}/f_D	f_{D_s}	f_D	Δ_s/Δ_d
r_1 uncerty.	0.3	1.1	1.4	0.4	1.0	1.4	0.7
a^2 extrap.	0.2	0.2	0.2	0.4	0.5	0.6	0.5
Finite vol.	0.4	0.4	0.8	0.3	0.1	0.3	0.1
$m_{u/d}$ extrap.	0.2	0.3	0.4	0.2	0.3	0.4	0.2
Stat. errors	0.2	0.4	0.5	0.5	0.6	0.7	0.6
m_s evoln.	0.1	0.1	0.1	0.3	0.3	0.3	0.5
m_d, QED, etc.	0.0	0.0	0.0	0.1	0.0	0.1	0.5
Total %	0.6	1.3	1.7	0.9	1.3	1.8	1.2

→ will tell you what is possible in future
e.g. is error from disc. errors, $m_{u,d}$ extrapoln, stats ...

Conclusion

• very accurate results are available now from lattice QCD for QCD parameters and for simple hadron masses and decay matrix elements important for flavour physics.

Future

• sets of 'next generation' gluon configs will have
$m_{u,d}$ at physical value (so no extrapoln) *or*
a down to 0.03fm (so b quarks are 'light') *or*
much higher statistics (for harder hadrons)
also can include charm in the sea now.

• Pushing errors down to 1% level will mean em corrections and $m_u \neq m_d$ must be understood.

• some harder calculations (flavor singlet, excited states, nuclear physics) will also become possible