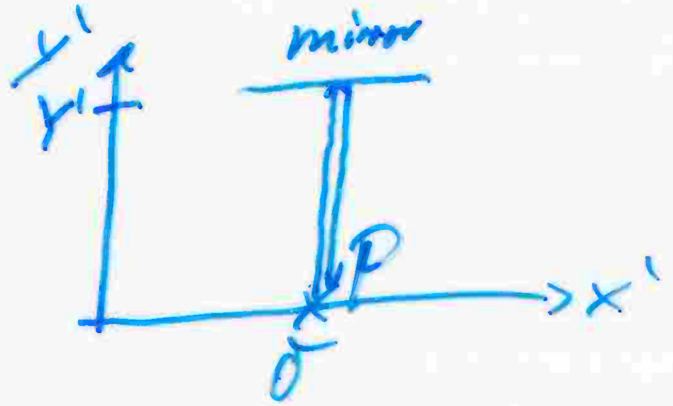
– 3 dimensions in space :

Assume again motion in x-direction with origins synchronised $t = t' = 0$

Observer stationary in S' holding a mirror at Y', start a flash at the origin O



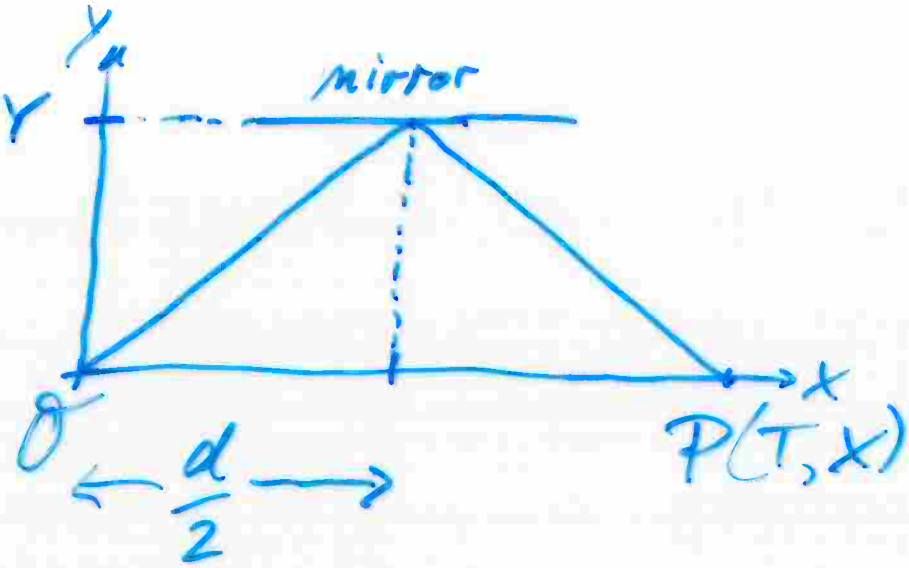
has event O at $t' = 0$; $x' = 0$

has event P at $t' = T'$; $x' = 0$

---

in S' mirror is at $Y' = \frac{c}{2} T'$

Now S moving relative to S' with speed u



has event O : $t=0$; $x=0$ by construction

event P : $t = T = \gamma T'$ // LT2 and $x'=0$

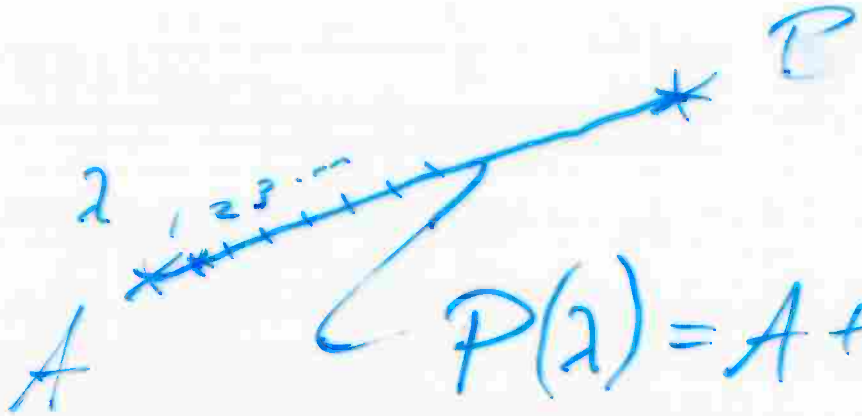
$x = X = \gamma u T'$ // LT2 and $x'=0$

Then $d = 2 \cdot \sqrt{Y^2 + \left(\frac{X}{2}\right)^2}$

and $d = c \cdot T = c \cdot \gamma T'$

light flash

Connect two points A, B and parametrise the line from A to B using a parameter λ.



$$P(\lambda) = A + \lambda(B - A)$$

Then define the vector

$$\underline{V}_{AB} = \left.\frac{dP(\lambda)}{d\lambda}\right|_{\lambda=0}$$

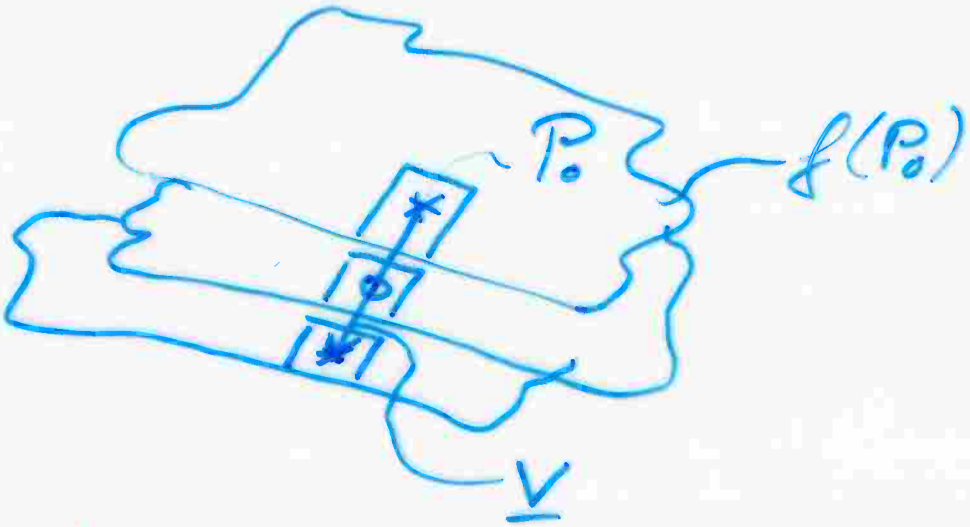
$$\Leftrightarrow \frac{d}{d\lambda}\left(A + \lambda(B-A)\right) = B - A = \text{"Tip-Tail"}$$

$\Rightarrow$ local definition at a point

$\Rightarrow$ very general

- ~~Assume~~ Now take any vector $\underline{v}$ and construct the curve $P(\lambda)$

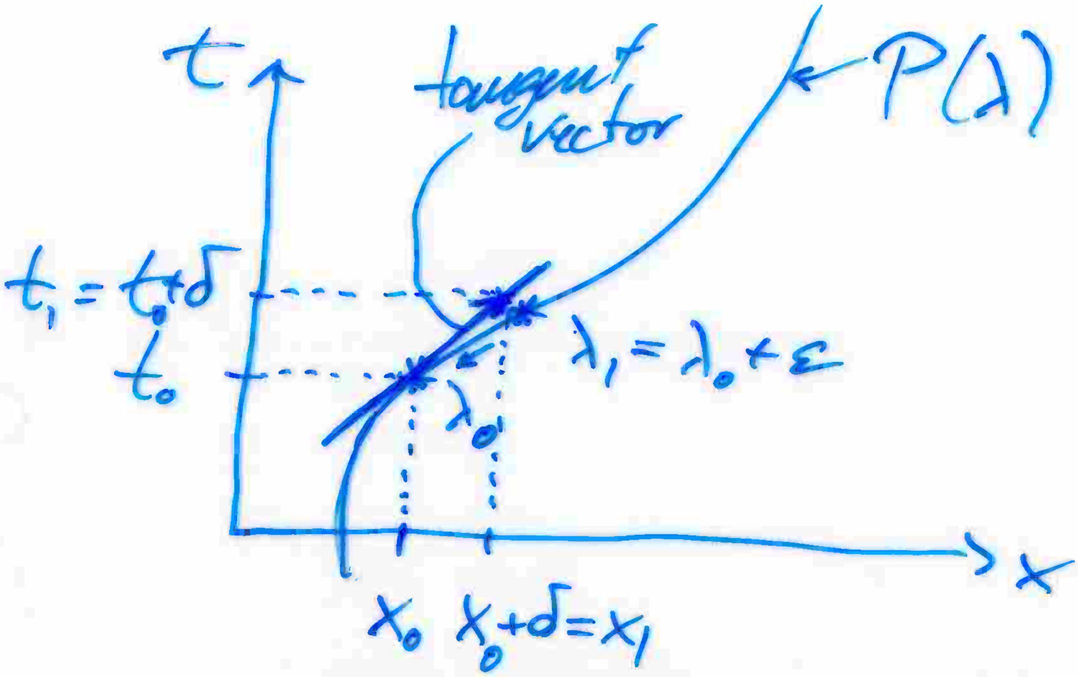
- Assume a function f of points and take a fixed point P_0 and consider all planes (3-D) of constant f in the vicinity of P_0 and $f(P_0)$:



Write to first order

$$f\big(P(\lambda)\big) = f(P_0) + \underbrace{(P(\lambda) - P_0)}_{\text{direction vector}} \cdot \underbrace{\tilde{d} \cdot f}$$

Local vector definition as tangent vector



$$\text{vector slope} = \underbrace{\dfrac{\overset{t_1 - t_0}{\cancel{x_1 - x_0}}}{\overset{x_1 - x_0}{\cancel{t_1 - t_0}}}}_{\text{in reference frame}} = \lim_{\varepsilon \to 0} \frac{P(\lambda_1) - P(\lambda_0)}{\lambda_1 - \lambda_0}$$

$$= \underbrace{\left.\frac{dP(\lambda)}{d\lambda}\right|_{\lambda_0}}_{\text{curve only, no reference frame}}$$

so the length is semi-definit, i.e. can be positive, negative or zero

$\underline{x}^2 > 0$ time-like

$\underline{x}^2 = 0$ null (light-like)

$\underline{x}^2 < 0$ space-like

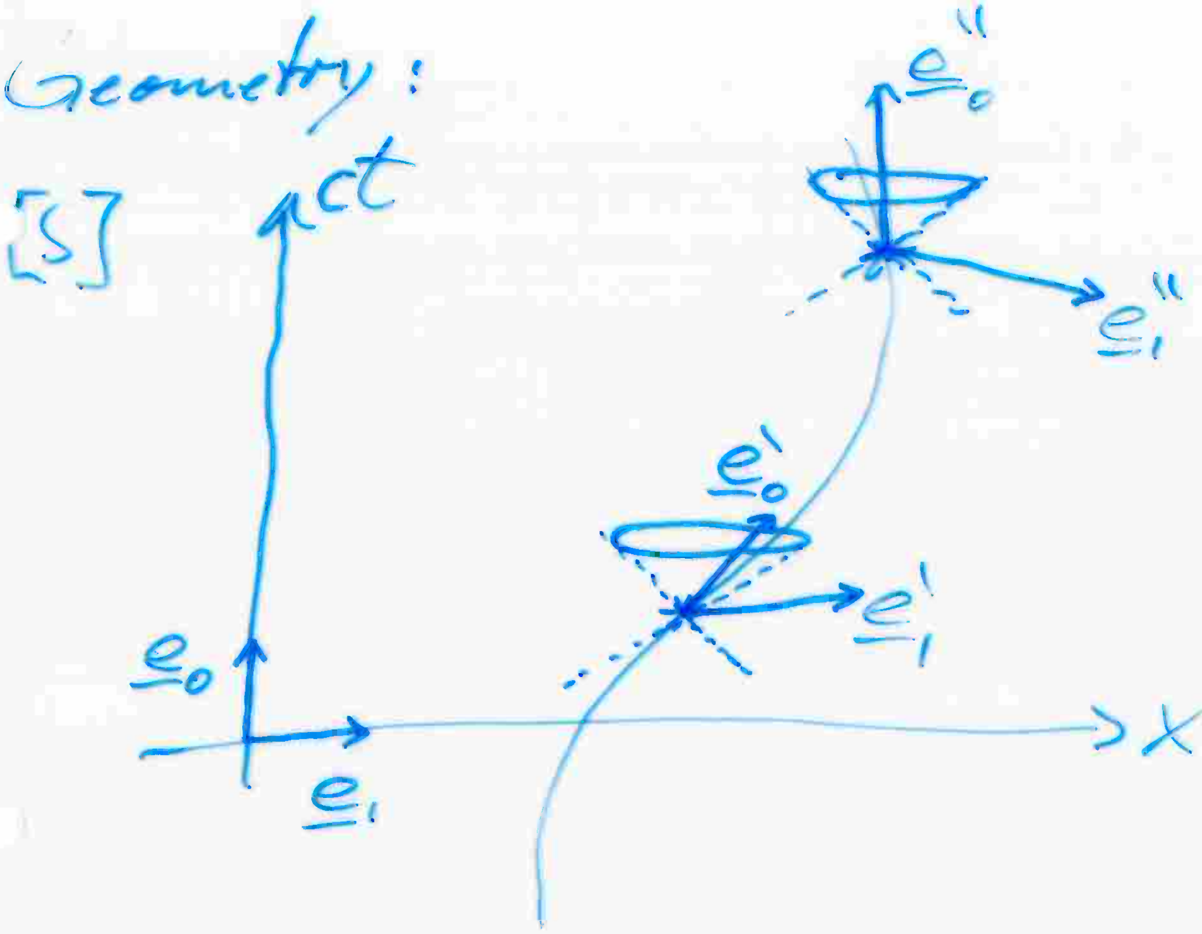
(! depends on the sign convention in textbooks)

[Exercises]

In particular:

$$u^2 = u_\mu u^\mu = \frac{dx_\mu dx^\mu}{d\tau^2} = c^2 \frac{ds^2}{ds^2} = \underline{c^2}$$

Geometry:



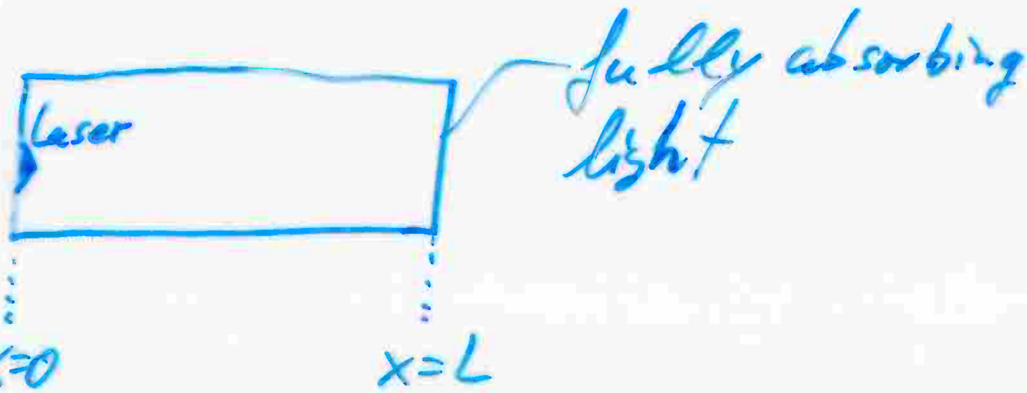
$$\frac{u^\mu}{c} = (\underline{e}_0)^\mu$$ the time-like unit vector

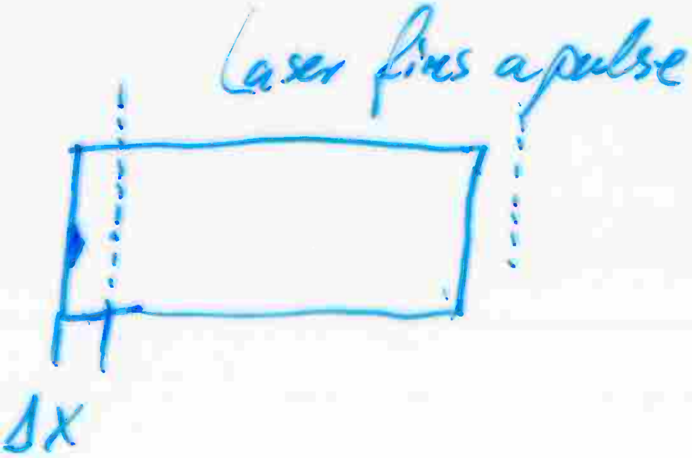
$\underline{e}_0$ is tangent on the world-line at a point.

$\Rightarrow$ get fundamental relation for relativistic particle kinematics

$$E^2 - |\underline{p}|^2 \cdot c^2 = m_0^2 \cdot c^4$$

$E = mc^2$?





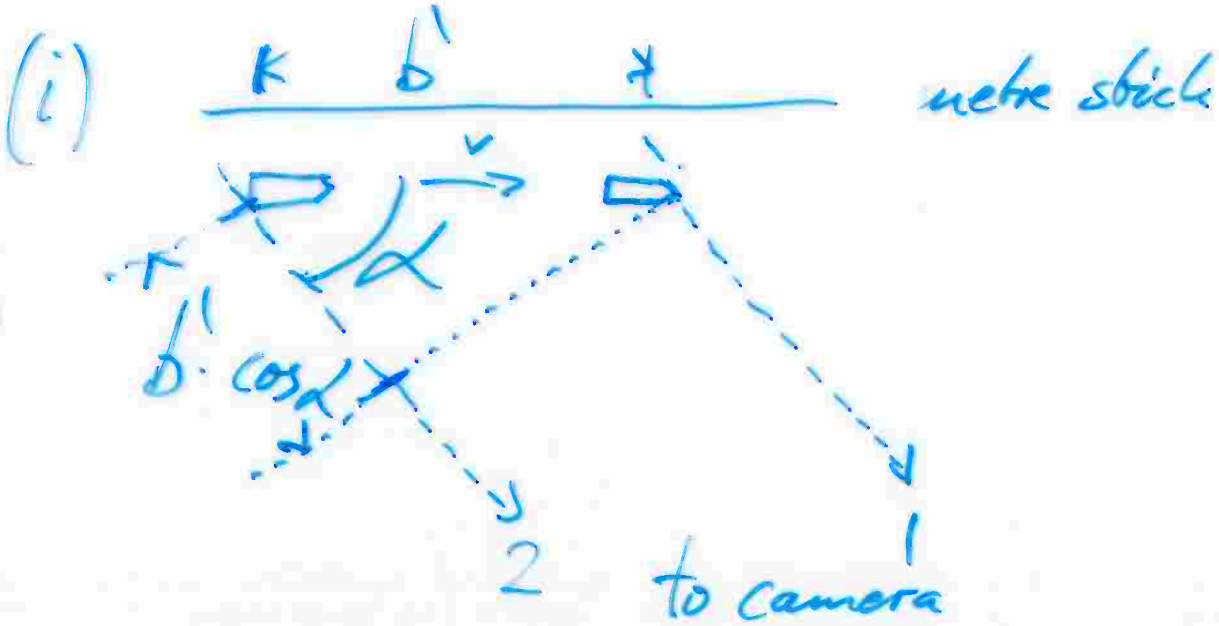
Momentum conservation: $p_{\text{light pulse}} =$

p_{box}

Mini tutorial

2010
Exam 2 (c)

(i)



– For Lorentz contraction the measurement must be made simultaneously. In this case, photons from start and end of the bullet would have to be emitted simultaneously in the lab-frame.
For a photo, they have to be received simultaneously, not emitted.

could look a bit cleaner if we choose

$$\tau = 0 \; ; \; x = \frac{c^2}{g}$$

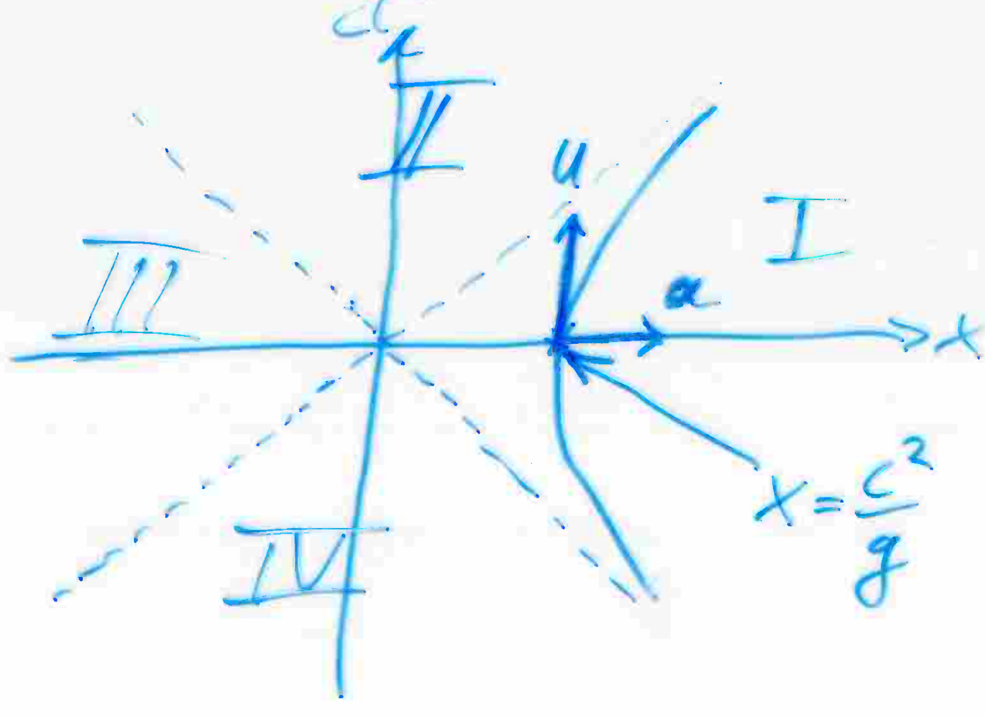
$$\Rightarrow x = \frac{c^2}{g} \cosh\left(\frac{g\tau}{c}\right)$$

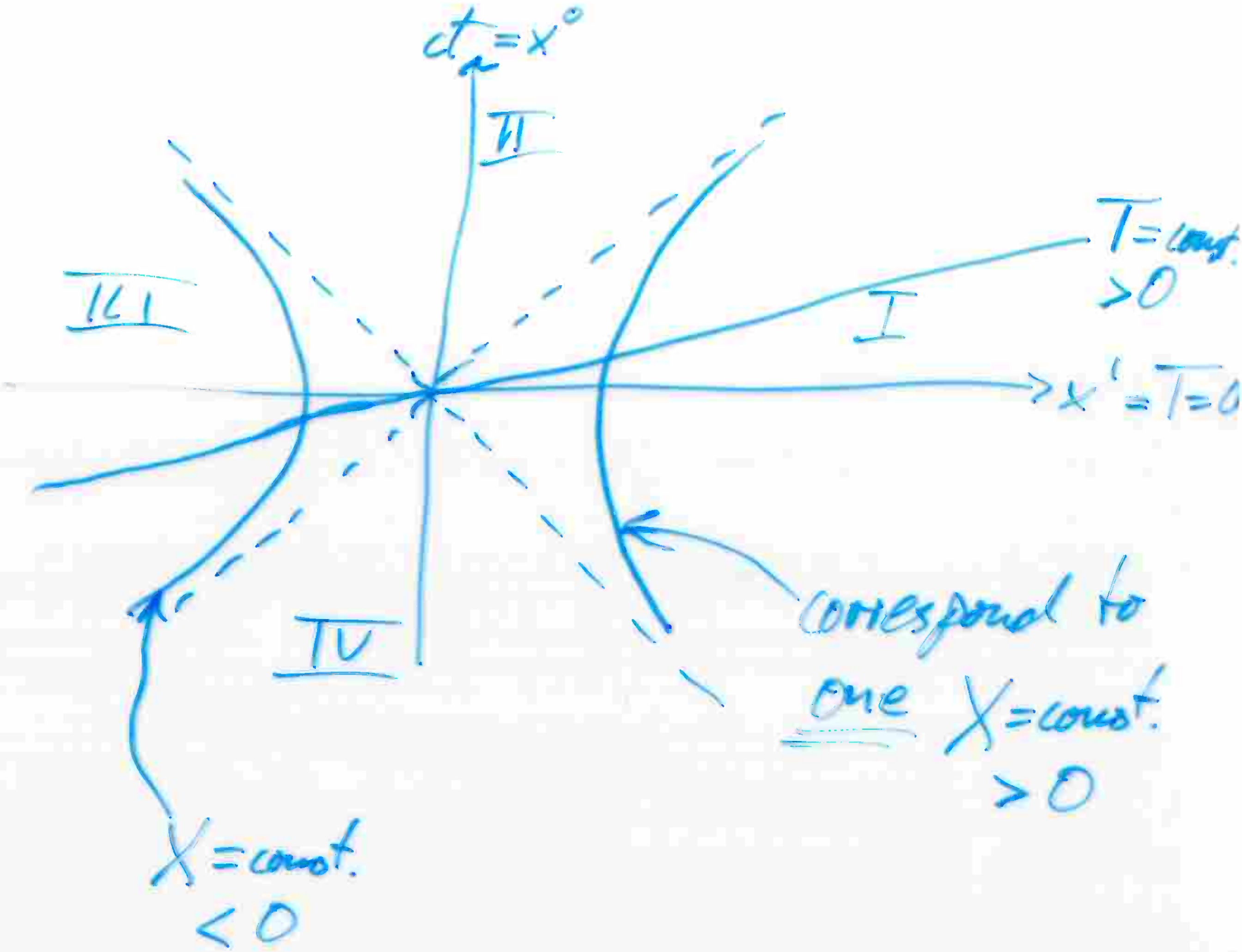
See that

$$x^2 - c^2 t^2 = \frac{c^4}{g^2} = \text{constant}$$

$$\Updownarrow$$

Hyperbola



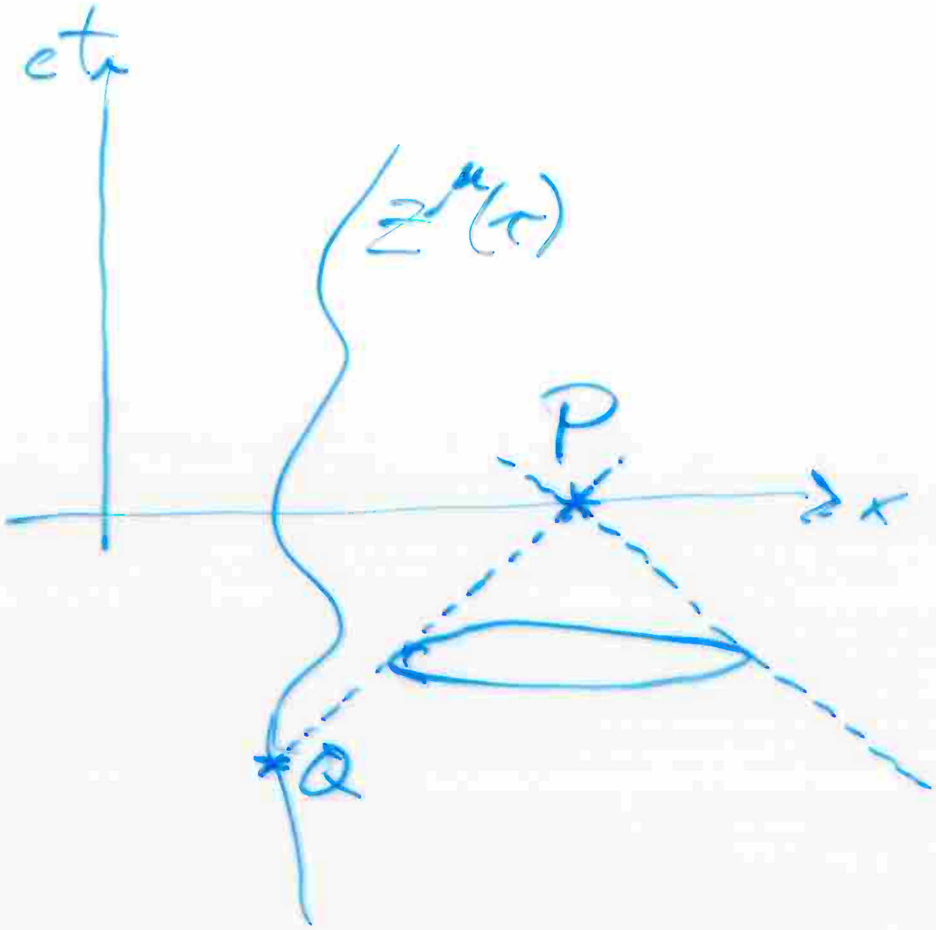


Now to the instantaneous rest frame for one object in sector I

leading to Fermi-Walker transport

Define an orthonormal tetrad, i.e. a set of basis vectors:

At a fixed proper time τ_0, a tetrad can always be built as



Geometry to algebra: world line $z^\mu(\tau)$ in a fixed Lorentz frame. Then have

$$v^\mu(\tau) = \frac{d}{d\tau} z^\mu(\tau) \quad \text{four velocity}$$

$$a^\mu(\tau) = \frac{d}{d\tau} v^\mu(\tau) \quad \text{four acceleration}$$

Put coordinates for P at x^μ

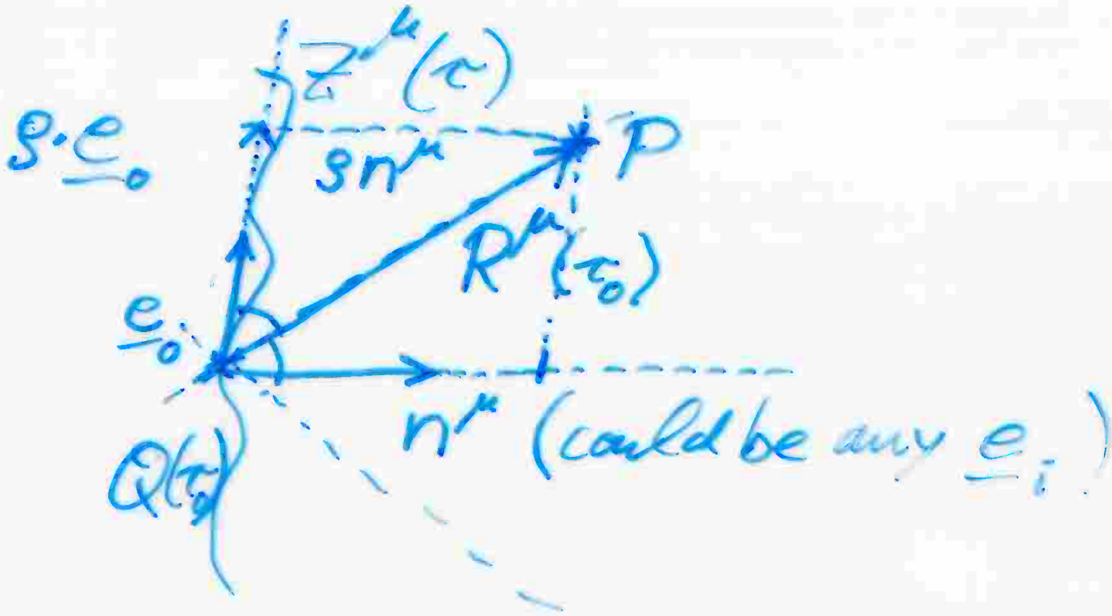
Then connect Q and P by

$$R^\mu := x^\mu - z^\mu(\tau_0)$$

or for any Q: $R^\mu = x^\mu - z^\mu(\tau)$

with $R^\mu R_\mu = 0$

Now identify an invariant distance measure between P and Q:



$$s \cdot \underline{e}_0 = s \cdot \frac{1}{c} v^\mu$$

$$n^\mu n_\mu = -1 \quad ; \; n^\mu v_\mu = 0$$

$$R^\mu = s\left(n^\mu(\tau_0) + \frac{1}{c} v^\mu(\tau_0)\right)$$

or $s = -n_\mu R^\mu = \frac{1}{c} v_\mu R^\mu > 0$

invariant retardation condition