The code below generates a time series of length \( n \), and then fits autoregressive models of order up to \( \text{order.max} \) (which is set to 19 below). The AIC is plotted as a function of the order, and the optimal order is tabulated.\(^1\)

```r
plot.aic <- function(fit, new=T, sd=0.1) {
  # code to plot AIC against order of AR model fitted
  if (new) plot((1:length(fit$aic))-1,fit$aic,type="l",xlab="Order",ylab="AIC") else 
    lines((1:length(fit$aic))-1,fit$aic,type="l")
  points(rnorm(1,fit$order,sd=sd),rnorm(1,sd=sd),pch=16,col="red")
}

# generates data from an autoregressive process, by default of order 1
sim.y <- function(n, model=list(ar=c(0.9))) arima.sim(model=model, n)

n <- 25  # length of time series
R <- 1000 # number of replicates

# first dataset to get things started
y <- sim.y(n,list(ar=c(0.5,0.1)))
fit <- ar(y,order.max=19)
plot.aic(fit)

# we will store the orders chosen using AIC, BIC, and AICC
AIC.order <- NULL
BIC.order <- NULL
AICC.order <- NULL

# Now make R replicates, plot the corresponding AIC curves
for (i in 1:R ) {
  fit <- ar( sim.y(n, model=list(ar=c(0.5,0.1))), order.max=19 )
  plot.aic(fit, new=F)
  AIC.order <- c(AIC.order, fit$order)
  # The next two lines should be uncommented and modified to give the
  # optimal orders when BIC and AICC are used for order selection
  # BIC.order <- c(BIC.order, NA)
  # AICC.order <- c(AICC.order, NA)
}

# tabulate the order of the chosen model
table(AIC.order)
```

\(^1\)The code is available from the APTS website.
(a) Try seeing how AIC performs as a basis for model selection for $n = 25, 50, 100, 500$.

(b) Vary the simulation model, using, for example, `model=list(ma=0.9)` in the `arima.sim` function, to see how well AIC works when the data are not generated by an autoregressive model.

(c) Modify the code above to compute the values of BIC and AIC$_c$, where

$$\text{AIC} = 2(p + 1 - \hat{\ell}), \quad \text{BIC} = p \log n - 2\hat{\ell}, \quad \text{AIC}_c = n - \frac{n + p}{n - p - 2} - 2\hat{\ell},$$

and $p$ is the order of the fitted model. Assess how well these criteria perform as bases for model selection, for $n = 25, 50, 100, 500$.

Hint: write BIC and AIC$_c$ as functions of AIC. You may find it useful to use `?ar` to access the help file for the `ar` function for details of the returned elements (or use `ls(fit)` to list them).