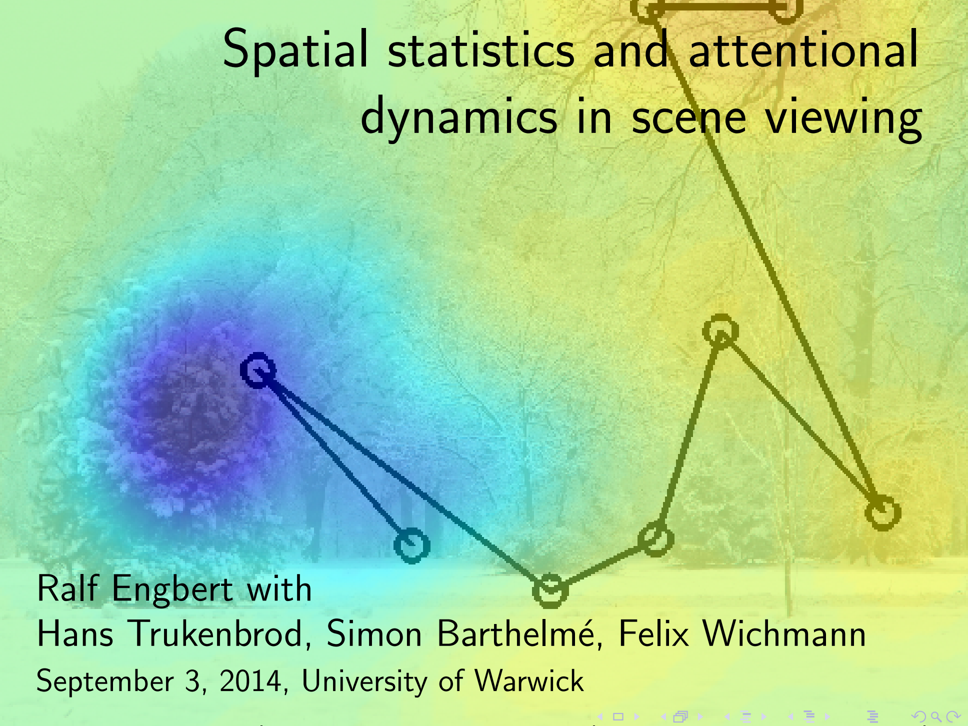
Spatial statistics and attentional dynamics in scene viewing



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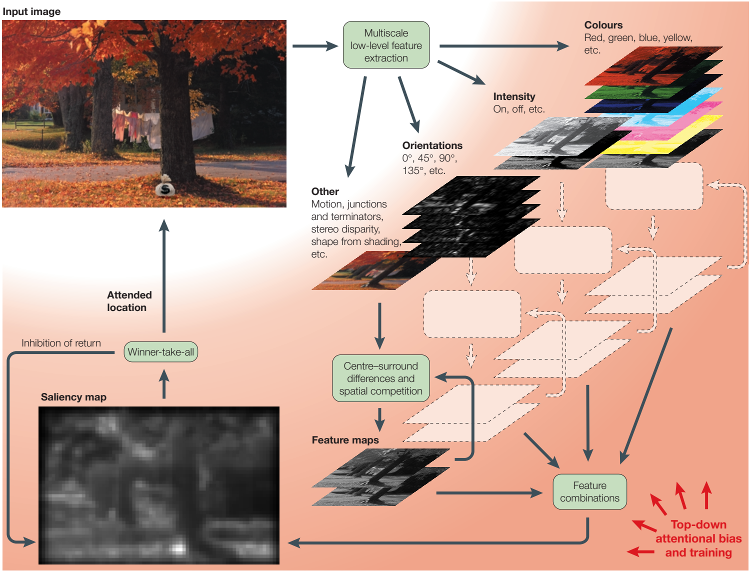
September 3, 2014, University of Warwick

Gaze control in natural scenes

(from: Henderson, 2003, *Trends Cogn Sci*)

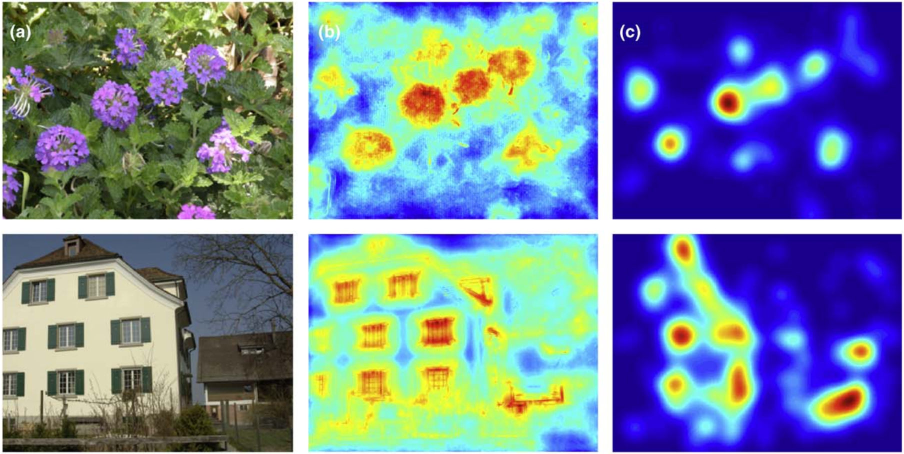
Computational modeling of visual attention



(from: Itti & Koch, 2001, *Nat Rev Neurosci*)

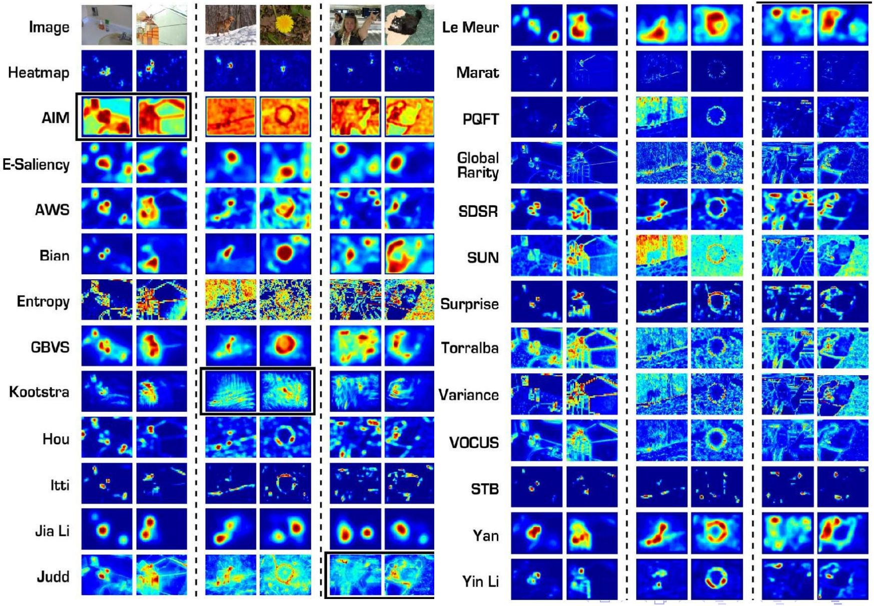
Examples of saliency computations

Stimulus display | Saliency map | Human fixations

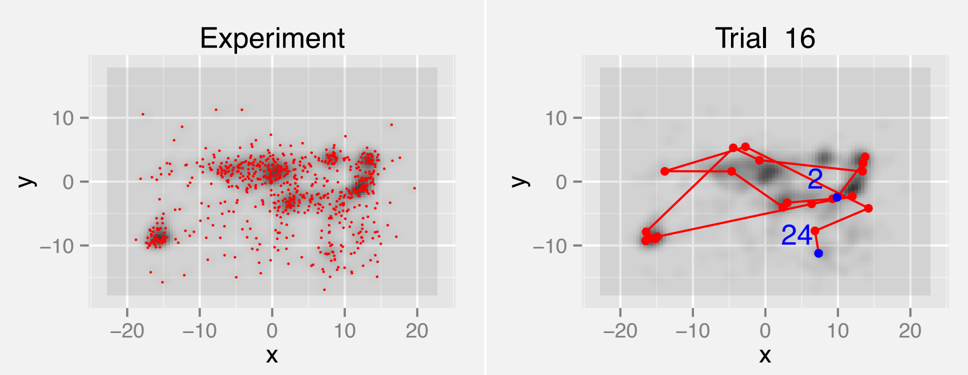


(from: Einhäuser & König, 2010, *Curr Opin Neurobiol)*

Examples of saliency computations (cont'd)



Application of the theory of spatial point processes



- red: fixation locations recorded from human observers
- shading: local intensity $\lambda(x)$, bandwidth $\sigma = 3^\circ$
- red lines: single-trial scanpath (fixations 2 to 24)

Spatial point processes: First-order statistics

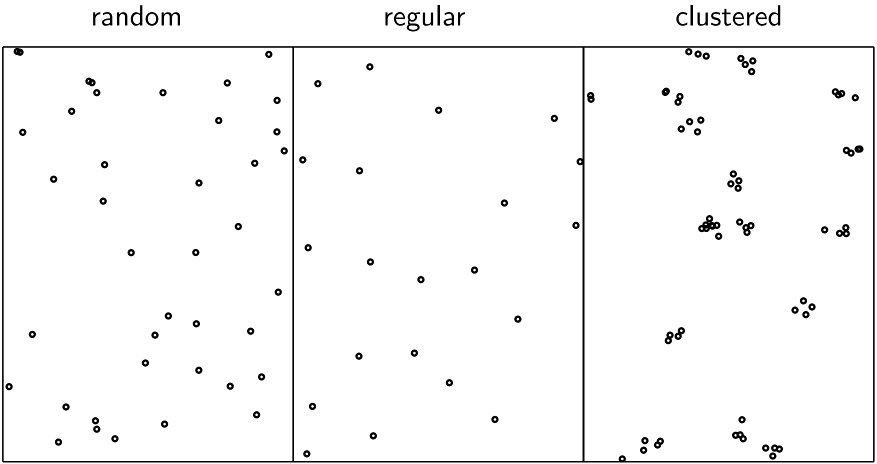
- A point process $N = \{x_1, x_2, \ldots\}$ is stationary (or statistically homogeneous) if N and the translated point process

$$N_x = \{x_1 + x, x_2 + x, \ldots\}$$

have the same distribution for all x

- For an inhomogeneous or non-stationary point process N the local intensity is a function of space, i.e., $\lambda = \lambda(x)$

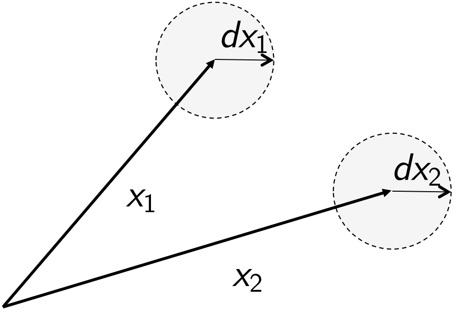
Second-order statistics for homogeneous processes



(from: Illian, Penttinen, Stoyan & Stoyan, 2008)

Second-order spatial statistics: Pair density

- Definition: $\rho(x_1, x_2)dx_1 dx_2$ = probability of points located in each of two discs b_1 and b_2 with linear dimensions dx_1 and dx_2 at positions x_1 and x_2, respectively
- $\rho(x_1, x_2)$ is typically a function of the distance $\|x_1 - x_2\|$



Inhomogeneous pair correlation function

- Inhomogeneous pair correlation function (PCF)

$$\hat{g}_{inhom}(r) = \sum_{x_1, x_2 \in W}^{\neq} \frac{1}{\hat{\lambda}(x_1)\hat{\lambda}(x_2)} \frac{k(\|x_1 - x_2\| - r)}{2\pi r A_{\|x_1 - x_2\|}},$$

where $k(.)$ is the *Epanechnikov* kernel, i.e.,

$$k(x) = \begin{cases} \frac{3}{4h}(1 - \frac{x^2}{h^2}), & \text{for } -h \leq x \leq h \\ 0, & \text{otherwises} \end{cases}.$$

and A_ξ denotes an edge correction at distance $\xi = x - y$

Interpretation of the PCF

Interpretation:

- For a random pattern without clustering: $\hat{g}_{inhom}(r) \approx 1$
- If $\hat{g}_{inhom}(r) > 1$, then pairs of fixations are more abundant than on average at a distance r
- If $\hat{g}_{inhom}(r) < 1$, then pairs of fixations are less abundant than on average at a distance r
- Thus, the pair correlation function $\hat{g}_{inhom}(r)$ measures how selection of a particular fixation location is influenced by other fixations at distance r

Estimation of the inhomogeneous PCF

Numerical estimation involves two steps:

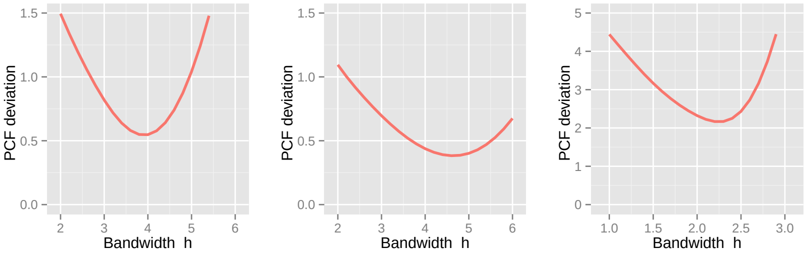
- First, we estimate the overall intensity $\hat{\lambda}(x)$ for all fixation positions obtained for a given scene. In this procedure we borrow strength from the full set of observations to obtain reliable estimates of the inhomogeneity.
- Second, we compute the pair correlation function from single-trial data to measure correlations within the scanpath.
- *Important:* The behavior of the estimator should be analyzed over a range of bandwidths h.

Dependence of PCF deviation on bandwidth

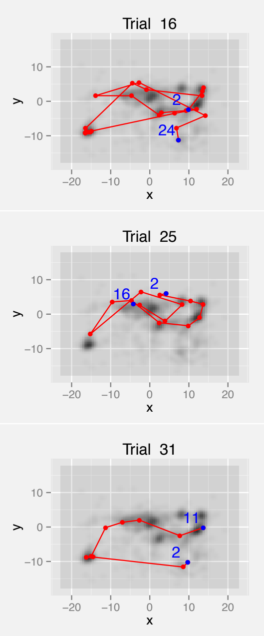
- The deviation from $g(r) = 1$ is measured by

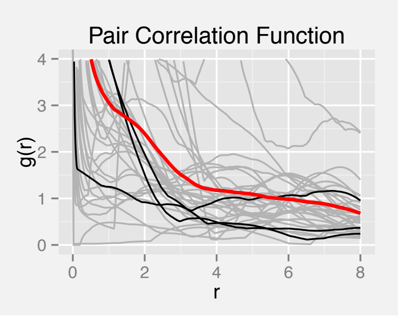
$$\Delta_g = \int_0^\infty (\hat{g}(r) - 1)^2 dr \ , \qquad (1)$$

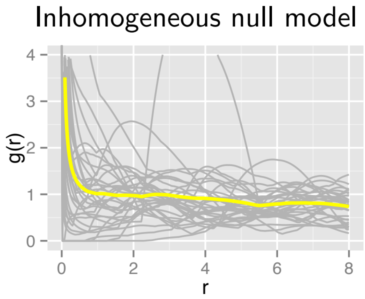
- Plot of the PCF deviation as a function of *h* for an inhomogeneous point process:



Pair correlations for gaze patterns







Replication across datasets

Image set 1

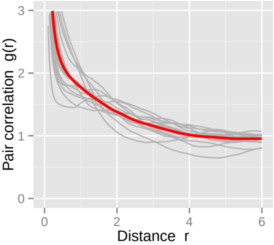
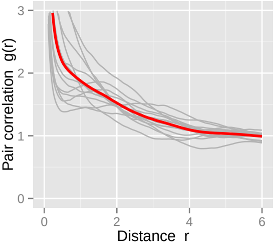


Image set 2



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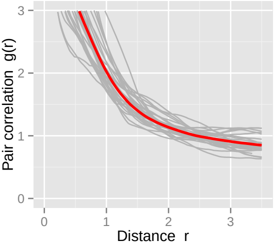

Summary & Conclusions

- We can investigate second-order statistics for gaze patterns
- Pair correlation function can be estimated in two steps using the overall (inhomogeneous) intensity and the pair density for single trials
- Null model: inhomogeneous point process
- Experimental data for gaze pattern indicate clustering for length scales $< 4^\circ$ for our data
- Scene content: no effects found
- Image size: strong effects

Thank you for your attention!

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- Preprint: `arxiv.org/abs/1405.3270`
- R-Code: `www.rpubs.com/Ralf`
- More on spatial point processes & vision: Barthelmé, S. et al. (2013). Modeling fixation locations using spatial point processes. *Journal of Vision* (doi: 10.1167/13.12.1)