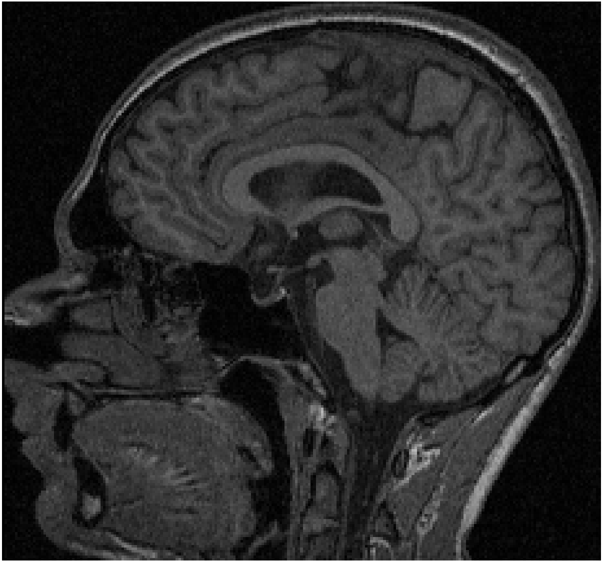
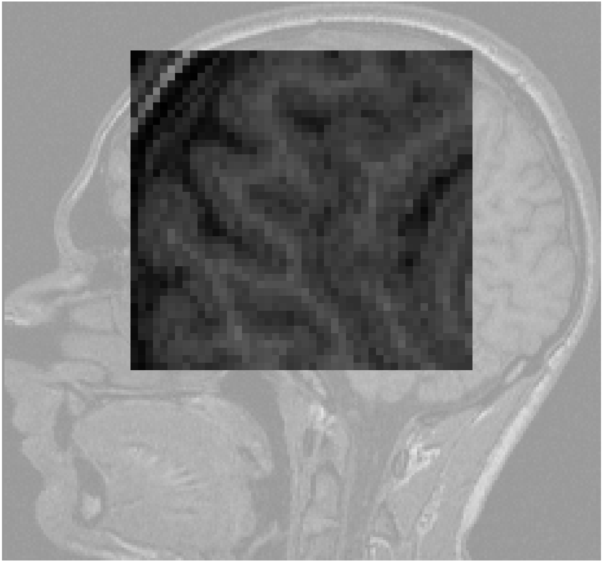
Weierstrass Institute for
Applied Analysis and Stochastics

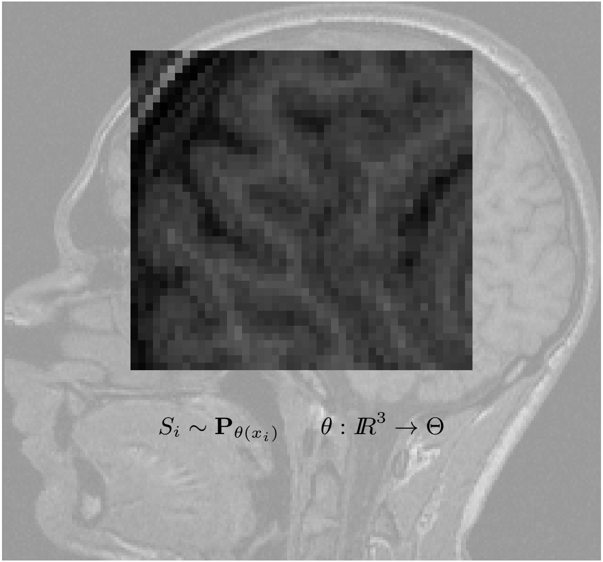
High-resolution diffusion MRI by msPOAS

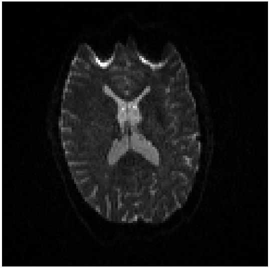
Karsten Tabelow

Images and Noise - MRI

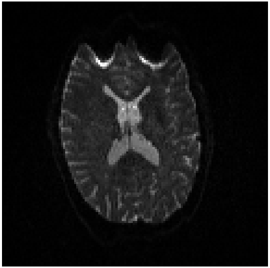





$S_i \sim \mathbf{P}_{\theta(x_i)} \qquad \theta : \mathbb{R}^3 \to \Theta$



Non-diffusion weighted S_0



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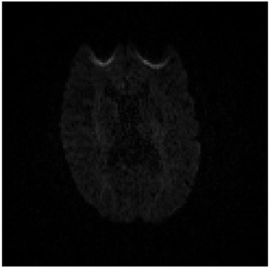
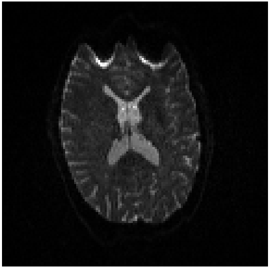


DWI: $S = S_0 \exp(-bD(\vec{g}))$



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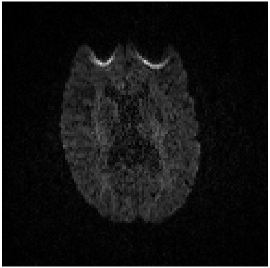

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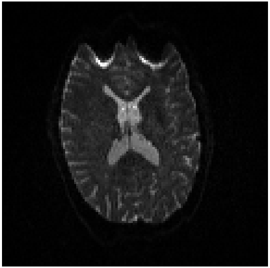

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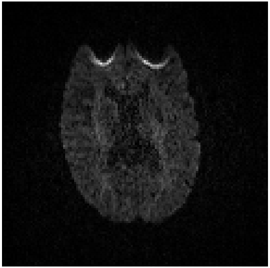

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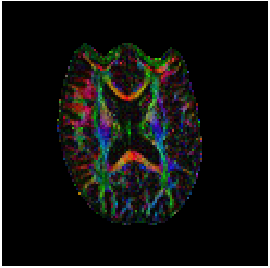
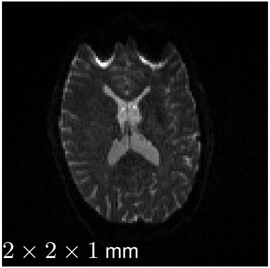
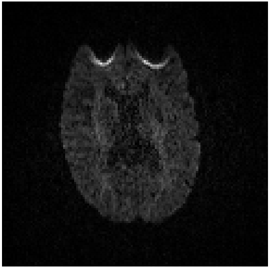
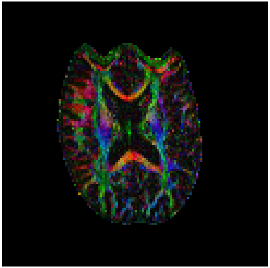


DTI: $S = S_0 \exp(-b\vec{g}\mathcal{D}\vec{g}^\top)$

Noise in diffusion MRI (dMRI)







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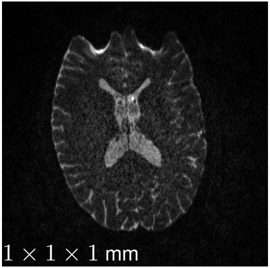

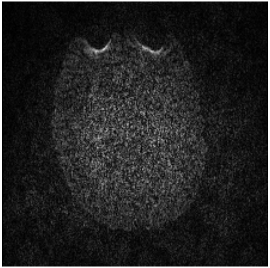

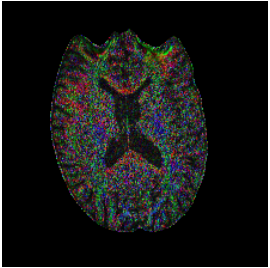

Image Denoising Methods

- Kernel estimators
- Wavelets, Curvelets, ...
- MCMC, SANN
- Diffusion methods
- Scale-space methods
- Scanner upgrade

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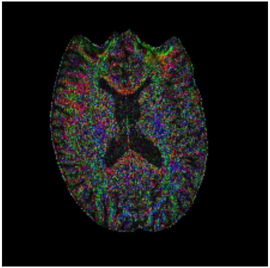
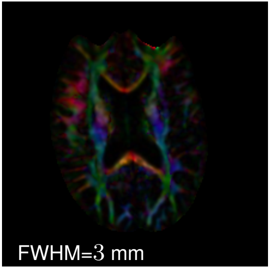
Structural Adaptive Smoothing - (Polzehl & Spokoiny, 2000, 2006)

- Dimension-free (2D - single slice, 3D - volume, 5D - dMRI, 6D - multi-shell dMRI)
- Structure preserving/enhancing

Non-adaptive weighted local mean

$$w_{ij} = K_{loc}\left(\frac{\delta(x_i, x_j)}{h}\right), \qquad \hat{Y}_i = \sum_j w_{ij} Y_j, \qquad K_{loc}(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$





From linear to anisotropic non-linear diffusion

$$\frac{\partial S(i,t)}{\partial t} = \operatorname{div}\left[D \cdot \nabla S(i,t)\right]$$

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- Stopping time?

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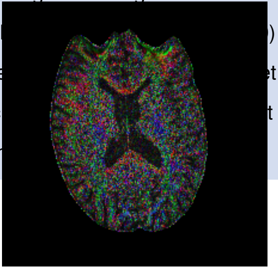

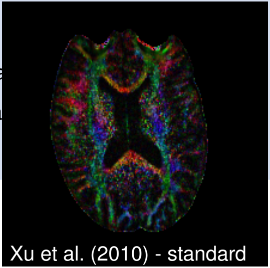

Xu et al. (2010) - standard

Wavelet filtering

- Images have sparse representations
- Wavelet filtering for MRI: Nowak (1999)
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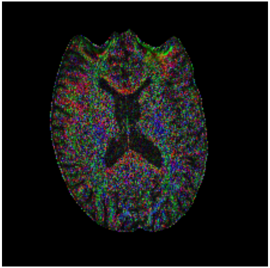

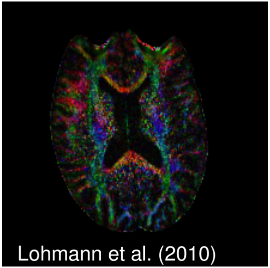

General setup / Local parametric model

- Design: $x_1, \dots, x_n \in \mathcal{X} \subseteq I\!R^p$ (or more complex space)
- Observations: $Y_1, \dots, Y_n \in \mathcal{Y} \subset I\!R^q$ (in dMRI: S)

$$Y_i \sim \mathbf{P}_{\theta(x_i)} \qquad \theta : I\!R^p \to \Theta$$

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Structural assumption

- $\exists$ Partitioning $\mathcal{X} = \bigcup_{m=1}^{M} \mathcal{X}_m$ such that

$$\theta(x) = \theta(x_i) \Leftrightarrow \exists m : x \in \mathcal{X}_m \wedge x_i \in \mathcal{X}_m$$

  i.e. θ constant on each $\mathcal{X}_m$ – local homogeneity structure
- Some components of θ may be global parameters.

Algorithm

- Choose a sequence of bandwidths: $h_0 = 1, h_{k+1} = c_h h_k$
- Initialization ($k = 0$): $w_{ij}^{(0)} = \delta_{ij}, \hat{\theta}(x_i)$ as weighted likelihood or least squares estimate.
- Adaptation (Step k): $\forall i, j$ define

$$w_{ij}^{(k)} = K_{loc}\left(\frac{\delta(x_i, x_j)}{h_k}\right) K_s\left(\frac{s_{ij}^{(k-1)}}{\lambda}\right)$$

$s_{ij}^{(k-1)}$ measures the difference between estimates $\hat{\theta}^{(k-1)}(x_i)$ and $\hat{\theta}^{(k-1)}(x_j)$

- Estimation (Step k): $\forall i$ define

$$\hat{\theta}^{(k)}(x_i) = \arg\max_{\theta} l(Y, W_i^{(k)}; \theta) \qquad \left(\text{simplest case:} \quad \hat{Y}_i^{(k)} = \sum_j w_{ij}^{(k)} Y_j\right)$$

- Iterate: Stop if $k \geq k^*$, else $k := k + 1$ and continue.

Results (Polzehl & Spokoiny, 2006; Becker, 2012)

- *Propagation* under homogeneity: If there is no structure in the image (θ equal everywhere), the result is approximately like a global non-adaptive kernel estimate.
- *Propagation* under local homogeneity: Similar results for interior points of local homogeneous regions.
- *Separation* property
- *Stability* of estimates: intrinsic stopping criterion (bounded bias)

Parameters

- λ can be selected by a *propagation condition*, independ of data
- k^* determines the maximum bandwidth
- $c_h = 1.25^{1/p}$ provides exponential growth of sum of location weights
- Kernels $K_{loc}(z) = (1-z)_+$ and $K_s(z) = \min(1, 2(1-z))_+$

Propagation condition (Becker, 2012)

- Let $\theta(x_i) \equiv \theta$ and $\bar{N}_i^{(k)}$ be the sum of nonadaptive weights in step k

$$\mathcal{Z}_\lambda(k,p) = \inf\{z > 0 : P(\bar{N}_i^{(k)} \mathcal{K}(\hat{\theta}_i^{(k)}(\lambda), \theta) > z) \leq p\}$$

  Select λ such that $\mathcal{Z}_\lambda(., p)$ is nonincreasing $\forall p > p_0$

- λ can be choosen by simulation and does not depend on the data at hand

$$w_{ij}^{(k)} = K_{loc}\left(\frac{\delta(x_i, x_j)}{h_k}\right) K_s\left(\frac{s_{ij}^{(k-1)}}{\lambda}\right)$$

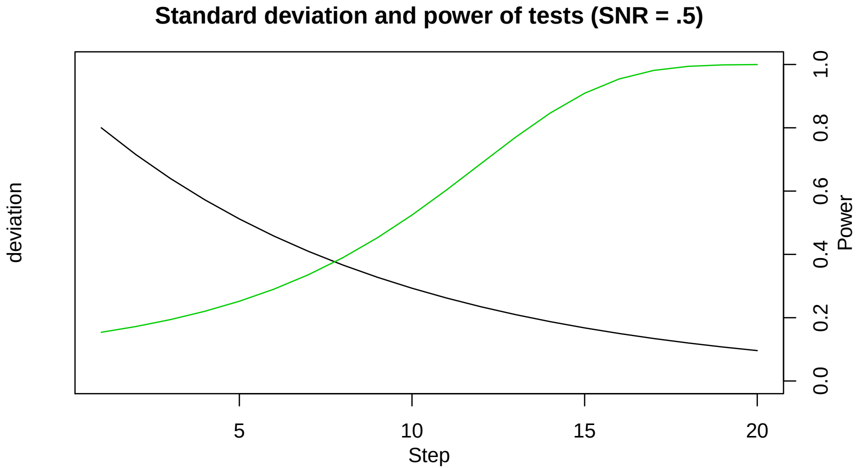
Distance in “real“ space

- $\delta(x_i, x_j)$: describes balls in the design space ($\mathbb{R}^2$, $\mathbb{R}^3$, or $\mathbb{R}^3 \times \mathbb{S}^2$, ...)

Distance in parameter space

- $s_{ij}^{(k-1)} = \sum_j w_{ij}^{(k-1)} \mathcal{KL}\left(\hat{\theta}^{(k-1)}(x_i), \hat{\theta}^{(k-1)}(x_j)\right)$
- Simple case (Gaussian distribution):

$$\mathcal{KL}\left(\hat{\theta}^{(k-1)}(x_i), \hat{\theta}^{(k-1)}(x_j)\right) = \frac{\left(\hat{\theta}^{(k-1)}(x_i) - \hat{\theta}^{(k-1)}(x_j)\right)^2}{2\sigma_i^2}$$


Standard deviation and power of tests (SNR = .5)
deviation
Power
Step
5
10
15
20
0.0
0.2
0.4
0.6
0.8
1.0

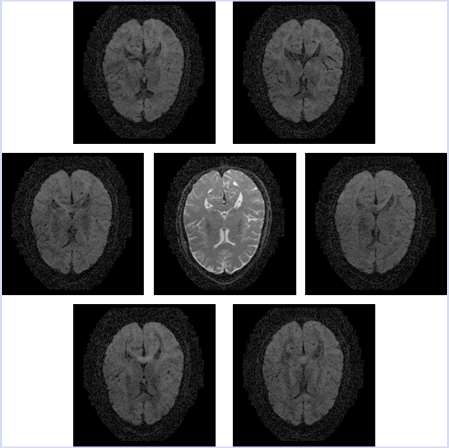
Human Eye vs. Structural adaptive smoothing

- Human eye is a very good denoiser
- Experience: Structural adaptive smoothing compares well with eyes.
- Limited use for visual inspection and for volumetric (integral) quantities

Applications

- Suitable for higher dimensional data
- Structure enhancement, structure identification.
- Imaging (Polzehl & Spokoiny, 2000)
- Functional MRI (Tabelow et al. 2006, 2009, Polzehl et al., 2010)
- dMRI (Tabelow et al. 2008, Becker et al. 2012, 2014)

dMRI data = $\mathbb{R}^3$ + S^2 (+ b-value)



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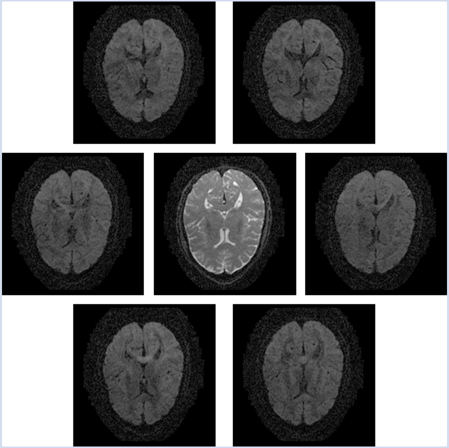

How to smooth DWI data?

- $S : \mathbb{R}^3 \to \mathbb{R}$, each image separately (inefficient!)

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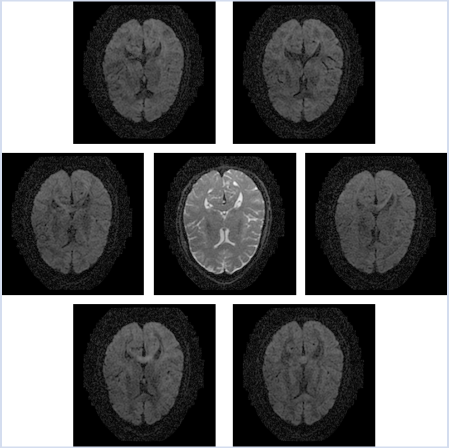

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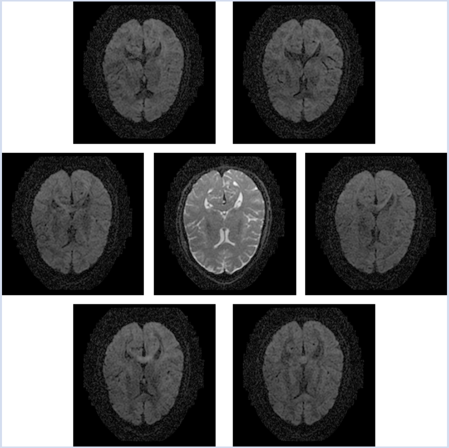

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- $S : \mathbb{R}^3 \times \mathbb{S}^2 \to \mathbb{R}$, all images jointly (POAS, Becker et al. 2012)

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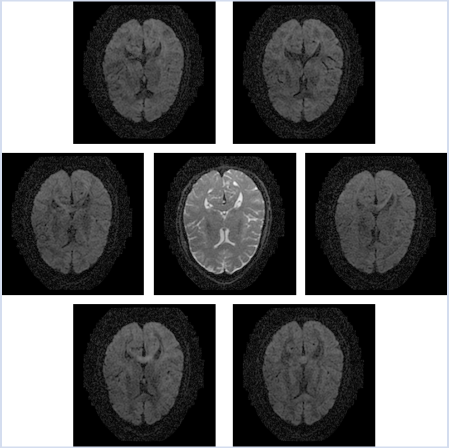

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- $S : \mathbb{R}^3 \times \mathbb{S}^2 \to \mathbb{R}$, all images jointly (POAS, Becker et al. 2012)
- $S : \mathbb{R}^3 \times \mathbb{S}^2 \to \mathbb{R}^{B+1}$, all images jointly (msPOAS, Becker et al. 2014)

Definition of the weighting scheme

- From $S : \mathbb{R}^3 \to \mathbb{R}$ ...
- ... to $S : \mathbb{R}^3 \times \mathbb{S}^2 \to \mathbb{R}^{B+1}$
- Need to define a metric on $\mathbb{R}^3 \times \mathbb{S}^2$ for the location weights

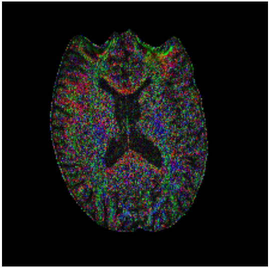
$$w_{ij}^{(k)} = K_{loc}\left(\frac{\delta(x_i, x_j)}{h_k}\right) K_s\left(\frac{s_{ij}^{(k)}}{\lambda}\right)$$

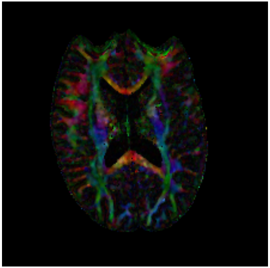
- Weights depend on position in space $\vec{v}$ and (diffusion) gradient direction $\vec{g}$ (Hagmann et al., 2006; Duits & Franken, 2011):

$$\delta(x_i, x_j) = ||\vec{v}_i - \vec{v}_j|| + \kappa^{-1} arccos| < \vec{g}_i, \vec{g}_j > |$$

- Statistical penalty $s_{ij}^{(k)}$ for non-central χ-distributed data using measurements from all shells.

Result - msPOAS





Structural adaptive smoothing dMRI

- Structural adaptive smoothing is a versatile tool for (especially high-dimensional) medical imaging data
- Structure enhancement (edge preserving)
- Structural adaptive smoothing DTI data
- Position-orientation adaptive smoothing (POAS) for dMRI data
- Multi-shell (POAS)
- R-package: **dti** (http://cran.r-project.org/package=dti)
- ACID-toolbox for SPM (http://www.diffusiontools.com)

Thanks!

Joint Work with:

- S. Becker, J. Polzehl, V. Spokoiny, WIAS
- H. U. Voss, Weill Medical College, Cornell University
- A. Anwander, R. Heidemann, MPI-CBS
- N. Weiskopf, UCL
- S. Mohammadi, UCL/Uni Hamburg

R-Community:

- CRAN Task View: Medical Image Analysis, Brandon Whitcher
- Special Volume of *Journal of Statistical Software*, MRI in R, 44 (2011).

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