CRiSM

P@W

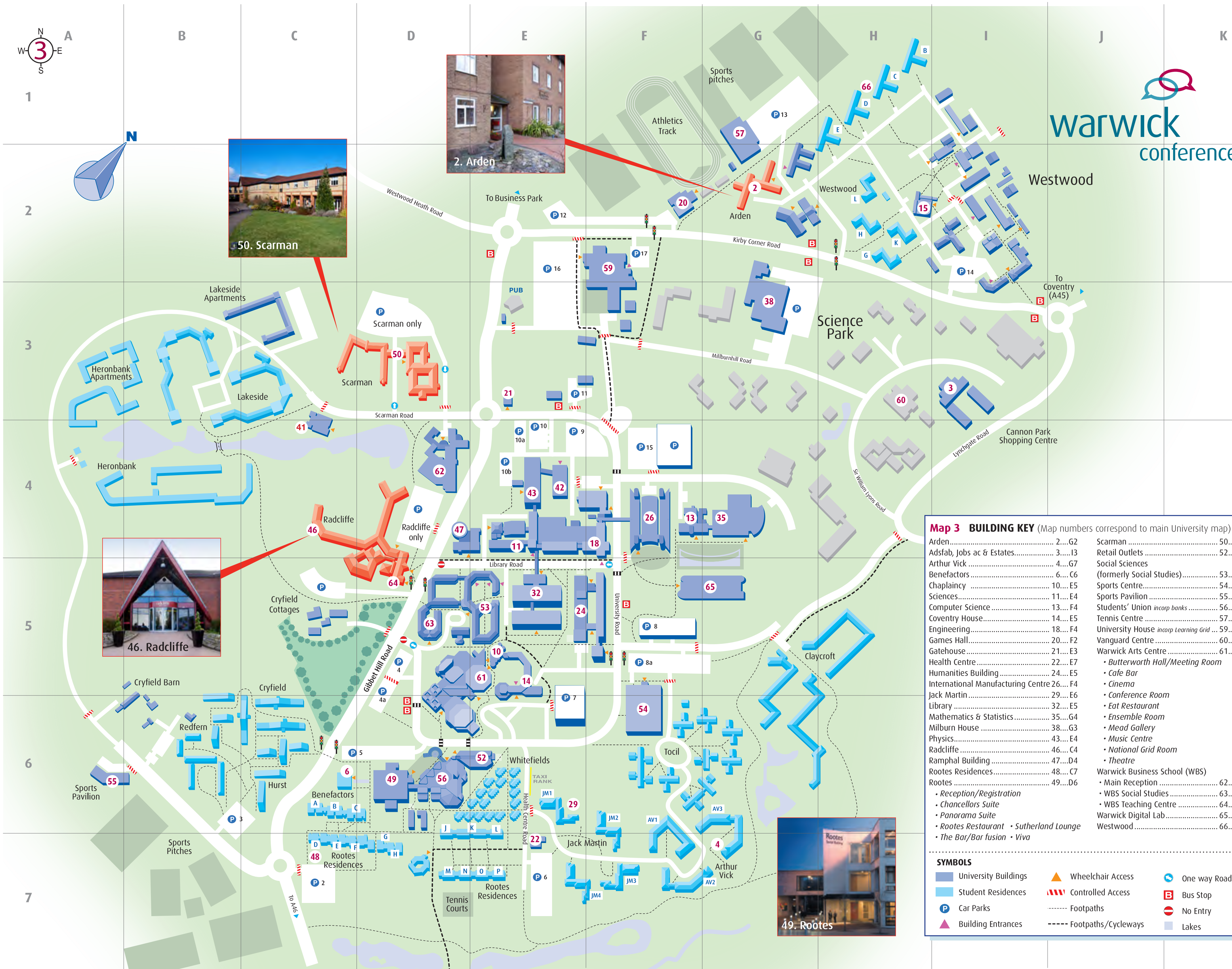
Workshops

May 2010

OPW - Orthogonal Polynomials, Applications to Statistics and Stochastic Processes

12–15 July 2010

THE UNIVERSITY OF WARWICK


warwick conference
Westwood
Science Park
Sports pitches
Athletics Track
Arden
Westwood Heath Road
To Business Park
Kirby Corner Road
Millburnhill Road
To Coventry (A45)
Cannon Park Shopping Centre
Lynchgate Road
Sir William Lyons Road
Lakeside Apartments
Heronbank Apartments
Lakeside
Heronbank
Scarman only
Scarman
Scarman Road
Radcliffe
Radcliffe only
Library Road
University Road
Gibbet Hill Road
Cryfield Cottages
Cryfield Barn
Cryfield
Redfern
Hurst
Benefactors
Sports Pavilion
Sports Pitches
Rootes Residences
Tennis Courts
Whitefields
Health Centre Road
Jack Martin
Arthur Vick
Tocil
Claycroft
To A46
PUB
TAXI RANK
2. Arden
50. Scarman
46. Radcliffe
49. Rootes
Map 3 BUILDING KEY (Map numbers correspond to main University map)
Arden 2....G2
Adsfab, Jobs ac & Estates 3.....I3
Arthur Vick 4....G7
Benefactors 6.... C6
Chaplaincy 10.... E5
Sciences 11.... E4
Computer Science 13.... F4
Coventry House 14.... E5
Engineering 18.... F4
Games Hall 20.... F2
Gatehouse 21.... E3
Health Centre 22.... E7
Humanities Building 24.... E5
International Manufacturing Centre 26.... F4
Jack Martin 29.... E6
Library 32.... E5
Mathematics & Statistics 35....G4
Milburn House 38....G3
Physics 43.... E4
Radcliffe 46.... C4
Ramphal Building 47....D4
Rootes Residences 48.... C7
Rootes 49....D6
• Reception/Registration
• Chancellors Suite
• Panorama Suite
• Rootes Restaurant • Sutherland Lounge
• The Bar/Bar fusion • Viva
Scarman 50...
Retail Outlets 52...
Social Sciences (formerly Social Studies) 53...
Sports Centre 54...
Sports Pavilion 55...
Students' Union incorp banks 56...
Tennis Centre 57...
University House incorp Learning Grid 59...
Vanguard Centre 60...
Warwick Arts Centre 61...
• Butterworth Hall/Meeting Room
• Cafe Bar
• Cinema
• Conference Room
• Eat Restaurant
• Ensemble Room
• Mead Gallery
• Music Centre
• National Grid Room
• Theatre
Warwick Business School (WBS)
• Main Reception 62...
• WBS Social Studies 63...
• WBS Teaching Centre 64...
Warwick Digital Lab 65...
Westwood 66...
SYMBOLS
University Buildings
Student Residences
Car Parks
Building Entrances
Wheelchair Access
Controlled Access
Footpaths
Footpaths/Cycleways
One way Road
Bus Stop
No Entry
Lakes

Travel Information.

Venue

The workshops will take place in the room MS03 of the

Mathematics & Statistics Building,
University of Warwick,
Coventry, CV4 7AL,
UK

The workshop venue is building no. **35** on the campus map and it is also known as "Zeeman Building".

Note:

The University is **not** in the town of Warwick (which is about 8 miles away), and it is **not** the same as Coventry University (which is a different university, located in the centre of Coventry). This is important when telling taxi drivers where you want to be!

Travelling to the University of Warwick

Information on getting to the University of Warwick from Coventry, as well as from other directions locally and further afield, can be found at `http://www2.warwick.ac.uk/fac/sci/statistics/crism/workshops/model-uncertainty/travel/`.

Parking

Parking spaces are available at the Radcliffe Hotel, which is at a walking distance from the Maths and Stats building. Please contact the Radliffe reception desk for information about parking. For people not staying at Radcliffe, Carpark 15 is the closest to the Mathematics and Statistics building.

Welcome!

Accommodation

Upon arrival, please check-in at Radcliffe Hotel (invited speakers – Building no. 46 on the campus map) or at Conference Park (other participants – building no. 40 on the campus map). These are also the places to ask about car parking, left luggage, etc.
The full addresses are :

Radcliffe Training & Conference Centre
University of Warwick,
Gibbet Hill Road
Coventry, CV4 7AL
UK
Tel: +44 (0) 24 7647 4711
Fax: +44 (0) 24 7669 4282
Email: radciffe@warwick.ac.uk

Conference Park
Rootes Building
The University of Warwick
Coventry, CV4 7AL UK
Tel: +44 (0) 24 7652 2280
Fax: +44 (0) 24 7652 4887
Email: conferences@warwick.ac.uk.

Workshop registration

Registration for the workshop will take place at the main atrium of the Mathematics & Statistics Building (no. 35 on the map). The registration time is Monday 12 July 2010, 8:00-9:00 am.

Computing facilities

If you have a WiFi enabled laptop you may access basic internet services[1] from within the Mathematics & Statistics Building by connecting to the "hotspot" wireless network and starting up a web browser. The login details for wireless are:

Username	Password
statsusr	*ready2go*

Meals

During the workshops, coffee and tea breaks will be served in the main atrium of the Mathematics & Statistics Building.
Lunch buffets (for participants who have registered) will be served in the main atrium of the Mathematics and Statistics building no. 35 on the map).
Dining options include the *Bar Fusion* in Rootes, The *Rootes Restaurant* or the *Eat* Restaurant in the Warwick Arts Centre. The *Varsity Pub* (coordinates E3 in the campus map) is a valid alternative near Scarman. A number of Bars and Cafés are at a walking distance from both Radcliffe and Conference park as well as from the Mathematics and Statistics Building. Please visit `http://www2.warwick.ac.uk/services/foodanddrink/` for all on-campus options.

[1] but note that sending email via SMTP is not allowed

Organisers

CRiSM administrator

- ▷ Paula Matthews (paula.matthews@stats.warwick.ac.uk)

Orthogonal Polynomials, Applications in Statistics and Stochastic Processes

- ▷ Persi Diaconis (diaconis@math.stanford.edu)
- ▷ Bob Griffiths (griff@stats.ox.ac.uk)
- ▷ Dario Spanò (d.spano@stats.warwick.ac.uk)
- ▷ Jon Warren (j.warren@warwick.ac.uk)
- ▷ Nikos Zygouras (n.zygouras@warwick.ac.uk)

ORTHOGONAL POLYNOMIAL WORKSHOP.

Timetable Days 1-2

	Event	Time	Details
Monday 12 July	*Registration*	08.00–08.45	
	Session 1.1	08.45–09.00	*Welcome*
		09.00–10.00	Mourad Ismail
		10.00–11.00	Gerard Letac
	Coffee	11.00–11.30	*in the main atrium, Maths & Stats*
	Session 1.2	11.30–12.30	Efoevi Koudou
		12.30–13.00	Peter Jacko
	Lunch	13.00–14.30	*in Scarman House (Building 50)*
	Session 1.3	14.30–15.30	Bob Griffiths
		15.30–16.00	Raffaello Seri
	Tea	16.00–16.30	*in the main atrium, Maths & Stats*
	Session 1.4	16.30–17.30	Jacek Wesolowski

	Event	Time	Details
Tuesday 13 July	Session 2.1	09.00–10.00	Arno Kuijlaars
		10.00–11.00	Evgeny Strahov
	Coffee	11.00–11.30	*in the main atrium, Maths & Stats*
	Session 2.2	11.30–12.30	Neil O'Connell
		12.30–13.00	Ivan Corwin
	Lunch	13.00–14.30	*in in the main atrium, Maths & Stats*
	Session 2.3	14.30–15.30	Patrik Ferrari
		15.30–16.00	Steven Delvaux
	Tea	16.00–16.30	*in the main atrium, Maths & Stats*
	Session 2.4	16.30–17.30	Eric Rains

Notes

▷ The main atrium of the Mathematics & Statistics Building is the open area as you move from the main entrance to the statistics department (where the mural with vertical lines is).

ORTHOGONAL POLYNOMIAL WORKSHOP.

Timetable Days 3-4

	Event	Time	Details
Wednesday 14 July	Session 3.1	09.00–10.00 10.00–11.00	Kshitij Khare Michael Voit
	Coffee	11.00–11.30	*in the main atrium, Maths & Stats*
	Session 3.2	11.30–12.30 12.30–13.00	Igor Borisov Dong Wang
	Lunch	13.00–14.30	*in the main atrium, Maths & Stats*
	Session 3.3	14.30–15.30 15.30–16.00	Persi Diaconis Nick Bingham
	Tea	16.00–16.30	*in the main atrium, Maths & Stats*
	Session 3.4	16.30–17.00 17.00–17.30	Hao Ni Tony Gomis
	Conference dinner	19.30	*in Conference Park*

	Event	Time	Details
Thursday 15 July	Session 4.1	09.00–10.00 10.00–11.00	Steven Evans Ryszard Szwarc
	Coffee	11.00–11.30	*in the main atrium, Maths & Stats*
	Session 4.2	11.30–12.00 12.00–12.30	Weijun Xu Ahmed El Ghini
	Lunch	12.30–14.30	*in the main atrium, Maths & Stats*
	Session 4.3	14.30–15.00 15.00–15.30 15.30–16.00	Yang Zou Wojciech Matysiak Ewart Shaw

Notes

- ▷ The main atrium of the Mathematics & Statistics Building is the open area as you move from the main entrance to the statistics department (where the mural with vertical lines is).

Abstracts

Invited Talks.

Orthogonal series and asymptotic analysis of canonical U-statistics based on dependent observations.
by Igor Borisov (Sobolev Inst.)

Let $\{X_i;\ i\in\mathbb{Z}\}$ be a stationary sequence of r.v.'s. in a separable metric space $\mathfrak{X}$, and let P be the Borel distribution of X_1. We study normalized m-variate U-statistics

$$U_n(f) := n^{-m/2}\sum_{1\le i_1\ne\ldots\ne i_m\le n} f(X_{i_1},...,X_{i_m}),\quad m\ge 2,$$

where $f\in L_2(\mathfrak{X}^m,P^m)$ is *canonical*, i.e., $\mathbf{E}f(t_1,\ldots,t_{k-1},X_k,t_{k+1},\ldots,t_m)=0$ for every $k\le m$ and all $t_j\in\mathfrak{X}$.

Let $\{e_i(t);\ i\ge 0\}$ be an orthonormal basis of the separable Hilbert space $L_2(\mathfrak{X})$ such that $e_0(t)\equiv 1$. Every canonical kernel from $L_2(\mathfrak{X}^m,P^m)$ admits the representation

$$f(t_1,...,t_m)=\sum_{i_1,...,i_m=1}^{\infty} f_{i_1\ldots i_m}e_{i_1}(t_1)...e_{i_m}(t_m), \tag{1}$$

where the multiple series $L_2(\mathfrak{X}^m,P^m)$-converges. Notice that the multiple sum in (1) does not contain the element $e_0(t)$. We also introduce the following restriction on all m-dimensional distributions of $\{X_i\}$:
(AC) *For all natural $j_1<...<j_m$, the distribution of the vector $(X_{j_1},...,X_{j_m})$ is absolutely continuous relative to the distribution P^m.*

Theorem. *Let $\{X_i\}$ be a φ-mixing stationary sequence with $\sum_k\varphi^{1/2}(k)<\infty$. Let a canonical kernel $f(t_1,...,t_m)$ satisfy the conditions: $\sum_{i_1,...,i_m=1}^{\infty}|f_{i_1\ldots i_m}|<\infty$ and $\sup_i\mathbb{E}|e_i(X_1)|^m<\infty$. If, in addition, (**AC**) is fulfilled then*

$$U_n(f)\xrightarrow{d}\sum_{i_1,...,i_m=1}^{\infty} f_{i_1\ldots i_m}\prod_{j=1}^{\infty}H_{\nu_j(i_1,...,i_m)}(\tau_j), \tag{2}$$

where the multiple series in (2) converges almost surely, $\{\tau_i\}$ is a Gaussian sequence of centered random variables with covariance matrix

$$\mathbb{E}\tau_k\tau_l=\mathbb{E}e_k(X_1)e_l(X_1)+\sum_{j=1}^{\infty}\big[\mathbb{E}e_k(X_1)e_l(X_{j+1})+\mathbb{E}e_l(X_1)e_k(X_{j+1})\big],$$

$\nu_j(i_1,...,i_m):=\sum_{k=1}^m\delta_{i_k,j}$ *(here $\delta_{i,j}$ is the Kronecker symbol), and $H_k(x):=(-1)^ke^{x^2/2}\frac{d^k}{dx^k}e^{-x^2/2}$ are the Hermite polynomials.*

In the case of i.i.d. $\{X_i\}$, an analog of this theorem was proved by H. Rubin and R. A. Vitale (*Ann. Statist.*, 1980, **8**(1), 165-170). Notice that, in general, the restriction of the Theorem on the coefficients $\{f_{i_1\ldots i_m}\}$ as well as condition (**AC**) cannot be omitted.

We also discuss the Hoeffding-type exponential inequalities for the distribution tails of the canonical U-statistics under consideration.

(Random) tri-diagonal matrices, alternating permutations, and orthogonal polynomials.
by Persi Diaconis (Stanford)

The set of $n \times n$ tri-diagonal, doubly stochastic matrices is a compact convex set (birth and death chains with a uniform stationary distribution) so it makes sense to "pick such a matrix at random." One can then ask about the distribution of entries, eigenvalues, and mixing times. This turns out to be intimately related to the set of alternating permutations and Jacobi polynomials. All of this is joint work with Philip Matchett-Wood.

Brownian motion on disconnected sets, basic hypergeometric functions, and some continued fractions of Ramanujan.
by Stephen Evans (Berkeley)

Motivated by Lévy's characterization of Brownian motion on the line, we propose an analogue of Brownian motion that has as its state space an arbitrary unbounded closed subset of the line: such a process will a martingale, has the identity function as its quadratic variation process, and is "continuous" in the sense that its sample paths don't skip over points. We show that there is a unique such process, which turns out to be automatically a Feller-Dynkin Markov process. We find its generator, which is a natural generalization of the operator $f \mapsto \frac{1}{2}f''$.

We then consider the special case where the state space is the self-similar set $\{\pm q^k : k \in \mathbb{Z}\} \cup \{0\}$ for some $q > 1$. Using the scaling properties of the process, we represent the Laplace transforms of various hitting times as certain continued fractions that appear in Ramanujan's "lost" notebook and evaluate these continued fractions in terms of basic hypergeometric functions (that is, q-analogues of classical hypergeometric functions). The process has 0 as a regular instantaneous point, and hence its sample paths can be decomposed into a Poisson process of excursions from 0 using the associated continuous local time. Using the reversibility of the process with respect to the natural measure on the state space, we find the entrance laws of the corresponding Itô excursion measure and the Laplace exponent of the inverse local time – both again in terms of basic hypergeometric functions. By combining these ingredients, we obtain explicit formulae for the resolvent of the process. We also compute the moments of the process in closed form. Some of our results involve q-analogues of classical distributions such as the Poisson distribution that have appeared elsewhere in the literature.

This work is joint with Shankar Bhamidi, Ron Peled and Peter Ralph.

Universality: from stochastic particle systems to random matrices.
by Patrik Ferrari (Bonn)

In the last decade non-Gaussian distributions discovered in random matrices (e.g., the Tracy-Widom distributions) were proven to describe limit laws of fluctuations in apparently unrelated probabilistic models, like the last passage percolation or the asymmetric exclusion process. It is believed that these distributions are universal, that is, they appear independently of the details of the model (under mild assumptions). Similarly, the extension to joint distribution has lead to the discovery of new universal limit processes (e.g., the Airy processes). The simplest example of universality is given by the central limit theorem and a universal process is Brownian motion. I will present some of these developments by focusing in one of the probabilistic model.

Exchangeable pairs of Bernoulli random variables, Krawtchouk polynomials and Ehrenfest Urns.
by Bob Griffiths (Oxford)

Joint research with Persi Diaconis. Geoff Eagleson's (1969) characterization of bivariate Binomial (N, p) distributions of exchangeable random variables (X, Y) which have Krawtchouck polynomial eigenfunctions is that the eigenvalues of the distribution have the form, for $p \geq 1/2$, as a mixture of Krawtchouck polynomials, scaled to be unity at zero,

$$\rho_k = \mathbb{E}\Big[Q_k(Z; N, p)\Big],$$

for some random variable Z on $\{0, 1, \ldots, N\}$.
We show that this characterization is equivalent to

$$X = \sum_{i=1}^{N} \xi_i, \ \ Y = \sum_{i=1}^{N} \eta_i,$$

where $\{(\xi_i, \eta_i)\}_{i=1}^N$ are Bernoulli pairs with random correlation coefficients, which are conditionally independent given these coefficients.
Another equivalent pretty characterization is that the conditional distribution of Y given X is the same as the transition distribution in a generalized Ehrenfest Urn model.

Sources of Orthogonal Polynomials.
by Mourad Ismail (City U. Hong Kong and King Saud University)

We discuss spectral problems that lead to orthogonal polynomials in a very natural way. In particular we indicate how birth and death processes and Schroedinger operators (through the J-matrix method) are rich sources of orthogonal polynomials. We will indicate how all known orthogonal polynomials, with one exception, arise either from birth and death processes or from the application of the J-matrix technique to a potential problem. We will also mention some open problems.

Rates of convergence for some multivariate Markov chains with polynomial eigenfunctions.
by Kshitij Khare (USF)

In my talk I will present examples of multivariate Markov chains for which the eigenfunctions turn out to be well-known orthogonal polynomials. This knowledge can be used to come up with exact rates of convergence for these Markov chains. The examples include the multivariate normal autoregressive process and simple models in population genetics. The examples are taken from joint work with Hua Zhou.

A survey of Lancaster probabilities.
by Efoevi Koudou (Nancy)

Lancaster probabilities on $\mathbb{R}^2$ are (weak limits of) probability measures whose density with respect to the product of the margins is of the form $\sum \rho_n P_n(x) Q_n(y)$, where (P_n) and (Q_n) are the sequences of orthonormal polynomials with respect to the margins. We recall some facts about these distributions. In particular we discuss the problem of their characterization, giving some examples where this characterization is available. We point out some open problems, for instance the issue of a better understanding of the link between Lancaster probabilities and exponential families with quadratic variance function.

Multiple orthogonal polynomials in random matrix theory.
by Arno Kuijlaars (Louvain)

Multiple orthogonal polynomials (MOPs) are a generalization of orthogonal polynomials where the orthogonality is distributed among a certain number of orthogonality weights. MOPs are related to certain determinantal point process in the same way that usual orthogonal polynomials are related to eigenvalues of unitary invariant random matrix ensembles. I will present examples arising from non-intersecting paths, random matrices with external source, and coupled random matrices. The large n-behavior of these models is described in certain special cases by a vector equilibrium problem.

Jacobi polynomials and joint distributions in Rn with Beta margins and prescribed correlation matrices.
by Gerard Letac (Toulouse)

If one calls copula in R^n any joint distribution of X in R^n such that the margins of X are uniform on $(0,1)$ it is a challenge to prove that for any correlation matrix R of order n then there exists a copula with this correlation matrix R. We shall prove that this conjecture is correct for $n < 6$ even while replacing the uniform distribution by a beta distribution. We use for this Jacobi polynomials, a famous result due to Gasper (1971) and the work of Angelo Koudou on Lancaster probabilities. The difficulty lies in the lack of an easy characterization of the extreme points of the convex set of correlation matrices of order n.

Directed polymers and the quantum Toda lattice.
by Neil OConnell (Warwick)

We relate the partition function associated with a certain Brownian directed polymer model to a diffusion process which is closely related to a quantum integrable system known as the quantum Toda lattice. This result follows from a variant of the Robinson-Schensted-Knuth correspondence and is completely analogous to the relationship between the longest increasing subsequence in a random permutation and the Plancherel measure on the dual of the symmetric group.

Lozenge tilings and elliptic biorthogonal functions.
by Eric Rains (CalTech)

One of the many combinatorial models in which fluctuations are controlled by random-matrix-related distributions is that of uniform random lozenge tilings of hexagons (or, equivalently, uniform random plane partitions in a box). These exhibit an "arctic circle" phenomenon–an inscribed ellipse outside of which the tiling can be reliably predicted–and the fluctuations in said ellipse are controlled by the Tracy-Widom distribution. The analysis of the uniform case rests on the fact that the tiling model can be viewed as a determinantal process with kernel expressed via Hahn polynomials. I'll discuss a recent generalization of this (joint with Borodin and Gorin), in which a suitable weighting of the probabilities by elliptic functions gives a process related to elliptic biorthogonal functions and degenerations thereof (e.g., q-Racah polynomials).

Representation theory of the infinite symmetric group and Pfaffian point processes.
by Evgeny Strahov (Jerulasem)

Integrable ensembles of random matrix theory can be divided into three symmetry types: unitary, orthogonal, and symplectic. The first symmetry type leads to determinantal point processes, the ensembles of orthogonal and symplectic symmetry types define the Pfaffian point processes. The aim of my talk is to describe Pfaffian point processes of random matrix type arising in the representation theory and the harmonic analysis on the infinite symmetric group.

Uniform convergence of Fourier series with respect to orthogonal polynomials.
by Ryszard Szwarc (Wroclaw)

Let $s_n(f)$ denote the nth partial sum of the classical Fourier series of a continuous 2π periodic function $f(\theta)$. We know that the quantities $\|s_n(f)\|_\infty$ need not to be uniformly bounded since the Lebesgue numbers $\int_0^{2\pi} |D_n(\theta)|d\theta$ behave like constant multiple of $\log n$. Therefore $s_n(f) \not\rightrightarrows f$ for some $f \in C_{per}(\mathbb{R})$. The question arises: Do there exist a measure space and an orthogonal system such that the partial sums are uniformly bounded in $\|\cdot\|_\infty$ norm ? We answer this question in the positive. This is a joint work with Josef Obermaier from Munich.

Some limit theorems for radial random walks on spaces with growing dimensions.
by Michael Voit (Dortmund)

Consider a series $X_p := G_p/K_p$ of homogeneous spaces with compact subgroups K_p of locally compact groups G_p with dimension parameter p such that the double coset spaces $G_p//K_p$ can be identified with some fixed space X. Fix some probability measure ν on X and consider the associated K_p-biinvariant "radial" distributions ν_p on G_p with "radial part" ν as well as associated radial random walks $(S_n^p)_n$ on the G_p. We now ask for limit theorems S_n^p when the time n as well as the dimension parameter p tend to ∞ possibly in a coupled way.

This problem is in particular studied in the literature for compact examples in connection with the cutoff phenomenon. In this talk we shall focus on a few non-compact examples. In particular, we give a survey about existing limit results for the Euclidean spaces $X_p = \underline{R}^p$ (with $K_p = SO(p)$ and $X = [0, \infty[)$) and their matrix space extensions as well as the hyperbolic spaces X_p of dimensions p over $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$. Especially, for the Euclidean case, there exist quite satisfying central limit theorems for $n, p \to \infty$. The results are partially joint work with W. Grundmann and M. Rösler.

Quadratic harnesses and Askey-Wilson polynomials.
by Jacek Wesolowski (Warsaw)

Let $X = (X_t)_{t\geq 0}$ be a real square integrable stochastic process with

$$\mathbb{E}\,[\,]\, X_t = 0 \quad \text{oraz} \quad \mathbb{E}\,[\,]\, X_s X_t = s, \quad 0 \leq s \leq t.$$

Let $\mathcal{F}_{s,u} = \sigma(X_t,\, t \notin (s,u))$, $0 \leq s < u$. The process X is called a **quadratic harness** if for any $0 \leq s < t < u$

$$\mathbb{E}\,[(]\, X_t|\mathcal{F}_{s,u}) = aX_s + bX_u \tag{1}$$

and

$$\mathbb{E}\,[(]\, X_t^2|\mathcal{F}_{s,u}) = AX_s^2 + BX_sX_u + CX_u^2 + DX_s + EX_u + F \tag{2}$$

where $a, b, A, \ldots, F$ are nonrandom functions of s, t, and u.

Examples include: Lévy'-Meixner process (eg. Wiener, Poisson and gamma processes), free Brownian motion (classical version), q-Gaussian processes, free Poisson process, q-Meixner processes ([3]), and bi-Poisson processes ([2]).

Rather unexpectedly functions $A, ..., F$ are uniquely determined by five numerical parameters. Actually, these parameters uniquely determine the whole process! It appears that all known quadratic harnesses are Markov, [1].

Description and construction of quadratic harnesses is heavily based on properties of polynomials which are supposed to be orthogonal with respect to marginal and transition (conditional) distributions. Moreover, these polynomials are also martingales. Apparently, there is a deep algebraic relation (not fully understood yet) between quadratic harnesses and orthogonal polynomials. Of particular importnace are, so called, projection formulas and formulas for connection coefficients. Recently, we have understood that the leading role belongs here to the **Askey-Wilson system of orthogonal polynomials**. In a sense, conditions (1) and (2), through a q-commutation equation (to be explained in the talk), lead to a quadratic algebra of operators, which is also related to the Askey-Wilson system. In special cases the algebra of operators has an explicit representation among operators acting on formal power series.

The talk will be based on a recent paper [4], joint with Wlodek BRYC (Univ. of Cincinnati).

Contributed Abstracts.

Free Energy of Random Polymers in 1+1 dimensions
by Ivan Corwin (Courant)

We consider the solution of the stochastic heat equation with delta function initial condition whose logarithm, with appropriate normalizations, is the free energy of the continuum directed polymer, or the solution of the Kardar-Parisi-Zhang equation with narrow wedge initial conditions. We obtain explicit formulas for the one-dimensional marginal distributions – the crossover distributions – which interpolate between a standard Gaussian distribution (small time) and the GUE Tracy-Widom distribution (large time). The proof is via a rigorous steepest descent analysis of the Tracy-Widom formula for the asymmetric simple exclusion with anti-shock initial data, which is shown to converge to the continuum equations in an appropriate weakly asymmetric limit. The limit also describes the crossover behaviour between the symmetric and asymmetric exclusion processes. This is based on joint work with Gideon Amir and Jeremy Quastel, both of the University of Toronto.

Critical behavior of non-intersecting Brownian motions at a tacnode.
by Steven Delvaux (Leuven)

We study a critical model of non-intersecting Brownian motions, where the paths asymptotically fill two touching ellipses in the time-space plane. The limiting particle density at the critical time consists of two touching semicircles, possibly of different sizes. We show that in an appropriate scaling limit of the parameters, the correlation kernel has an integrable form related to a new Riemann-Hilbert problem of size 4×4. We prove existence of the Riemann-Hilbert problem and obtain a remarkable connection with the Hastings-McLeod solution to the Painlevé 2 equation. We show that this function also shows up in the critical limits of the recurrence coefficients of the multiple Hermite polynomials that are associated to the non-intersecting Brownian motions.

Estimation of moving average models generated by non Gaussian and dependent signals.
by Ahmed El Ghini (Lille)

Estimation of the autoregressive moving average (ARMA) parameters of a stationary stochastic process is a problem often encountered in the signal processing literature and computer vision. It's well known that estimating the moving average (MA) parameters is usually more difficult than estimating the autoregressive (AR) part, especially when the model is derived by non Gaussian and dependant sequence. In this paper we present the MA parameter estimation algorithm given by the orthogonal estimates of the inverse autocovariance function. The statistical properties of the algorithm are explored and used to show that it is asymptotically efficient. The performance of the estimators is evaluated via simulation and compared with the asymptotic theory. Joint work with Rmi Auguste, Marius Bilasco and Chaabane Djeraba.

Orthogonal Polynomials, Adomian Decomposition Method,and Alienor Transformations:Theories and Frontiers Physical Applications.
by Antoine Gomis (NBI)

This talk will present and critically review powerful and complexity-reducing Adomian and Cherruault theories and solutions to frontier problems in a wide spectrum of disciplines and their related applications, including statistical physics, biology, ecology, economics and finance.

Analysis of Markov chains with Pentadiagonal Matrix via Mobius Transformation and Lucas Sequences.
by Peter Jacko (BCAM)

Spectral analysis has proven to be useful in solving recurrences appearing in the class of admission control problems with birth-death dynamics since it gives rise to tridiagonal matrices. Grunbaum (2008) extended the approach to Markov chains with pentadiagonal transition matrices, puzzling for interpretation of problems in which such matrices arise. We present an admission control problem with delayed state observation, which naturally leads to a pentadiagonal transition matrix. We further present an algorithm solving this problem in linear time, thus matching the complexity of solving its variant with no observational delay. We show how to elegantly solve the recurrences by Mobius transformation, which leads to a particular Lucas sequence with a well-known solution.

Free Quadratic Harness
by Wojciech Matysiak (Warsaw)

Quadratic harnesses are a class of stochastic processes with a special conditional structure. A description of the class and an outline of the general theory of quadratic harnesses can be found in the abstract of invited talk of Jacek Wesolowski. We will focus on a particular example of quadratic harness - free quadratic harness - and present its construction, showing some techniques that had made it possible to build a number of other quadratic harnesses. The construction is based on two families of polynomials, orthogonal with respect to one-dimensional and conditional distributions of the process. The results that will be presented were obtained in a joint work with Wlodek Bryc (University of Cincinnati) and Jacek Wesolowski (Politechnika Warszawska).

Statistical Properties of Generalized Discrepancies on the Hypersphere.
by Raffaello Seri (U. Insubria)

We consider a class of generalized discrepancies on the hypersphere encompassing the statistical tests of uniformity on the circle, on the sphere and on the hypersphere of Ajne, Beran, Bingham, Gin, Prentice, Pycke, Rayleigh and Watson. These discrepancies are defined through spherical harmonics and their strong and weak convergence properties involve this set of orthonormal functions. We first consider the asymptotic distribution under the null: apart from the well-known results, we also provide some less classical asymptotic theorems. Then we turn to the distribution of the points under the alternative. We provide two different kinds of results. First of all, we give conditions under which the scaled and normalized statistic is asymptotically normal under fixed alternatives. This needs some nontrivial preliminary results on the integral of products of Gegenbauer polynomials. Then we provide an approximation theorem (together with a bound on the uniform distance between the true distribution and the approximating one) holding for fixed and varying alternatives. This theorem is clearly much more flexible than the previous one, but its formulation is more involved since the approximating distribution is a weighted sum of noncentral chi-square random variables. In both cases we consider two different situations: the former covers very general densities, while the latter covers densities constant on hyperspheres of smaller dimension. Computations on the sphere are affordable and involve the real Gaunt coefficients.

Statistical Inference, Orthogonal Polynomials and Electrostatics.
by Ewart Shaw (University of warwick)

Links between electrostatics and the zeros of univariate orthogonal polynomials are described, focussing on the Hermite polynomials because of their importance in, for example, asymptotic approximations and numerical integration for Bayesian inference. Extensions to the multivariate case are investigated.

Hermitian matrix model with spiked external source
by Dong Wang (U. Michigan)

The Hermitian matrix model with spiked (i.e., finite rank) external source has been extensively studied when the potential is Gaussian or Laguerre, and an interesting phase transition phenomenon has been observed. We will show that such phase transition phenomenon is universal for a large class of potentials, including all convex potentials, and also show new phase transition patterns for other potentials. We utilize an integrable structure in the proof. This is joint work with Jinho Baik.

The Signature of a Path, and Inversion.
by Weijun Xu (Oxford)

Hambly and Lyons introduced the notion of the signature of a path, and proved that paths of finite length are uniquely determined by their signatures up to tree-like equivalence. Based on that, I will also discuss how to reconstruct a lattice path from its signature.

Multiple Integrals and Multivariate Polynomials.
by Yang Zou (TU Kaiserslautern)

We introduce an orthogonal system of the stochastic polynomials with respect to compensated Levy jump measures by constructing multivariate polynomials which we call multivariate Charlier polynomials. This orthogonal system turns out to have properties similar to the ones of the Hermite polynomials of Gaussian variables. We give the relationship between the multivariate Charlier polynomials and the multiple integrals with respect to the compensated Levy jump measures and also discuss their derivatives corresponding to the stochastic integral by using the feature of these multivariate polynomials.

Sponsors

Orthogonal Polynomials, Applications in Statistics and Stochastic Processes