# Robustness of MALA algorithms

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The standard MALA algorithms generally have improved convergence properties over the Random Walk Metropolis, however they cannot be applied for target distributions with too heavy or too light tails. Motivated by this imperfection and inspired by the recent work on Manifold Monte Carlo Methods [1], we investigate properties of a more general version of MALA based on

$$d\theta(t) = \left( \frac{\sigma^2(\theta(t))}{2} \mathcal{L}(\theta(t))' + \sigma(\theta(t))\sigma(\theta(t))' \right) dt + \sigma(\theta(t)) db(t). \tag{1}$$

By choosing $\sigma^2(\theta) = |1/(\mathcal{L}(\theta))''|$ we arrive at manifold MALA with the metric tensor $G(\theta)$ defined as the observed Fisher information matrix plus the negative Hessian of the log-prior (c.f. Section 4.2 and 5 of [1]).

We analyze it for the standard benchamrk family of targets $p(\theta) \propto \exp\{-|\theta|^\beta\}$. The results are summarized in Table 1, together with the respective properties of manifold MALA. Extensions towards more general distributions will follow.

| algorithm | $0 < \beta < 1$ | $\beta = 1$ | $1 < \beta < 2$ | $\beta = 2$ | $2 < \beta$ |
|---|---|---|---|---|---|
| RWM | N | Y | Y | Y | Y |
| MALA | N | Y | Y | Y | N |
| MMALA | Y | | Y | Y | Y |

Table 1. Geometric Ergodicity of random walk Metropolis (RWM), MALA [3, 2] and manifold MALA (MMALA) for target $\pi(\theta) \propto \exp\{-|\theta|^\beta\}$. N = geometric ergodicity fails, Y = geometric ergodicity holds.

## References

[1] M. Girolami and B. Calderhead. Riemann manifold Langevin and Hamiltonian Monte Carlo methods (2011): *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 73: 123-214.

[2] Mengersen, K.L. and Tweedie, R.L. Rates of convergence of the Hastings and Metropolis algorithms (1996): *The Annals of Statistics* 24(1): 101–121.

[3] G.O. Roberts and R.L. Tweedie. Exponential Convergence of Langevin distributions and their discrete approximations (1996): *Bernoulli* 2(4): 341–363.