Neural coding and information geometry

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Outline

1. Overview

2. Hierarchical model of neural interactions

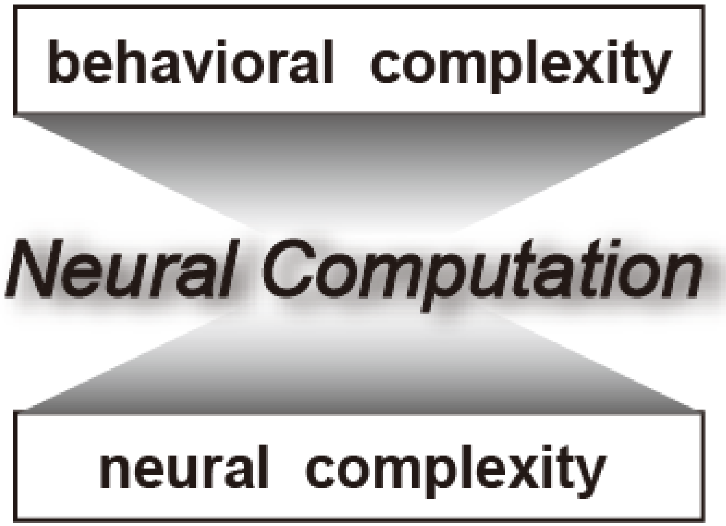
Section I

Overview

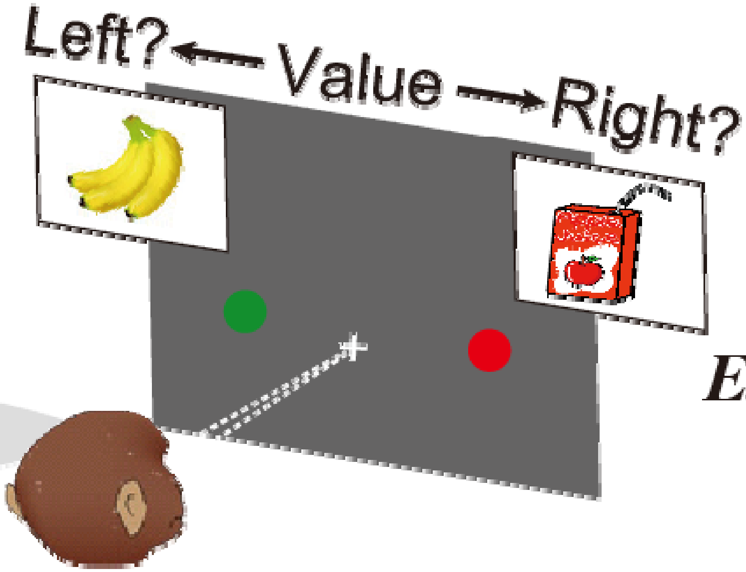
Introduction

Intelligent behavior (e.g. higher, cognitive, adaptive)

Achieved by collective neural activities



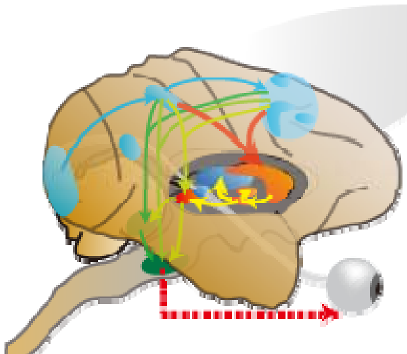
Decision making and reward-oriented behavior
— Neural computations



Expanding Frontiers

Representational learning

"Social"



* basal ganglia circuits

* neural interactions

Core Frontiers

Theory & circuit

(e.g. RL) (e.g., DA,BG)

Theoretical Foundations ***-- "Computations by collective neural activities"***

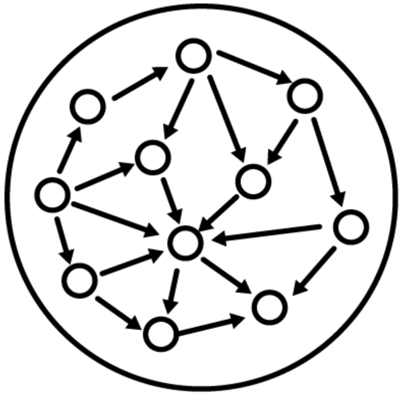
e.g. information geometry, neural coding

Introduction

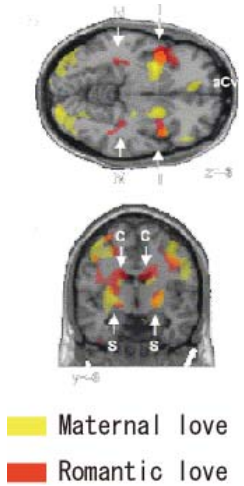
Neural computation – computational neuroscience:
At heart, interactions of neural activities

10^{15} bits/sec (#synapse x 1 bit/sec)

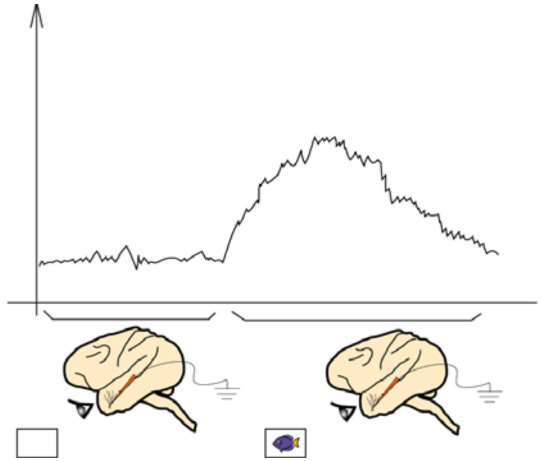
Terry Sejnowski



Most experimental findings so far
-- mostly by approach of "isolation"
(e.g., lesion, fMRI and single-unit studies)



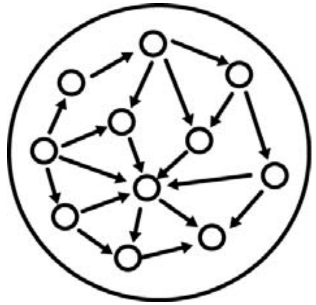
(Bartels, Zeki 04)



How to go beyond the 'isolation' approach?

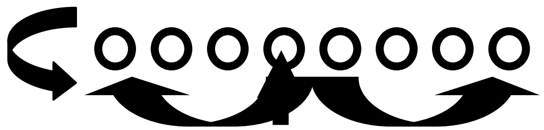
Introduction

“modeling” – rather abstract form



e.g. Amari-Hopfield model, Boltzmann machine,
gradient disecent (natural ….), em-algorithm, graphical model, etc

“modeling” – neural coding / population coding



Tuning curve (spike counts), their variability

$$r_i = \phi_i(x; c_i) + n_i(x)$$

$$n_i(x) \sim N(0, \sigma_i^2(x))$$

Q: About input x, given the activities $\mathbf{r}$?

e.g. ***Fisher information*** *etc*

Our past works

* *Faithful and unfaithful models* -- Wu et al. (’01, ’02a,b, ’04)
* *Attention modulation* -- Nakahara et al (’01, ’02)
* *Singularity in decoding* – Amari & Nakahara (’05)
* *Neural dynamics in SC* – Nakahara et al (’06)

Introduction

"data analysis" *"method of data analysis"*

Spike coding (Neural spike as binary variable)

$$\eta_i = E[x_i]$$

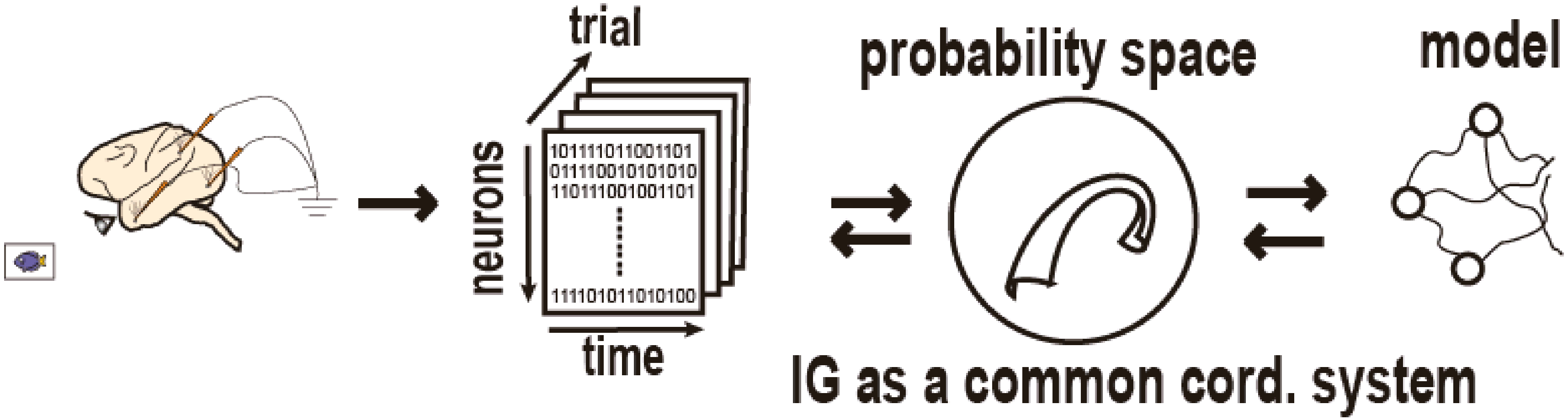
Most research *with data* so far: 1st-order or 2nd-order model

$$\log Q_{pair}(\mathbf{X}) = \sum_i \theta_i x_i + \sum_{i,j} \theta_{ij} x_i x_j - \psi$$

How can we reliably detect usefulness and meanings of higher-than-pairwise order interactions of neural activities?

Information geometry on binary random vectors

Information Geometric Approach



Basic needs : Systematic treatment for higher-order interaction

Use good properties of the dual coordinates

$$\log \mathrm{P}(\mathbf{X}) = \sum_i \theta_i x_i + \sum_{ij} \theta_{ij} x_i x_j + \sum_{ijk} \theta_{ijk} x_i x_j x_k + \ldots + \theta_{12\ldots n} x_1 x_2 \ldots x_n - \psi$$

η-coordinates: $\boldsymbol{\eta} = (\eta_{\mathrm{i}},\ \eta_{\mathrm{ij}},\ \ldots) = (\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{\mathrm{N}})$, $\quad \eta_{\mathrm{A}} = E[X_A]$

θ-coordinates: $\boldsymbol{\theta} = (\theta_{\mathrm{i}},\ \theta_{\mathrm{ij}}, \ldots) = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_{\mathrm{N}})$

k-cut mixed coord: $\boldsymbol{\varsigma}_k = (\boldsymbol{\eta}_1,\ \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{\mathrm{k}}; \boldsymbol{\theta}_{\mathrm{k+1}}, \boldsymbol{\theta}_{\mathrm{k+2}}, \ldots, \boldsymbol{\theta}_{\mathrm{N}}) = (\boldsymbol{\eta}_{\mathrm{k-}}; \boldsymbol{\theta}_{\mathrm{k+}})$

FI is given as $G_{\zeta_k} = \begin{bmatrix} A_{\zeta_k} & O \\ O & D_{\zeta_k} \end{bmatrix}$, where $G_\eta = \begin{bmatrix} A_\eta & B_\eta \\ B_\eta^T & D_\eta \end{bmatrix}$, $\quad G_\theta = \begin{bmatrix} A_\theta & B_\theta \\ B_\theta^T & D_\theta \end{bmatrix}$

$$A_{\zeta_k} = A_\theta^{-1}, \quad D_{\zeta_k} = D_\eta^{-1}$$

Pythagorian: $D[P:Q] = D[P:R] + D[R:Q]$ where $\boldsymbol{\varsigma}_k^R = (\boldsymbol{\eta}_{k-}^P; \boldsymbol{\varsigma}_{k+}^Q)$

Past works

* *IG framework for spike analysis* -- Nakahara & Amari (‘02)
* *Detection of interaction cascade* -- Nakahara et al. (‘03)
* *Emergent higher-order synchrony* -- Amari et al. (‘03)
* *Temporal domain* — Nakahara et al (’06)
* *Others etc….*

Section II

“Theory in practice?”

Hierarchical Interaction Structure of Neural Activities in Cortical Slice Cultures

(Santos, Gireesh, Plenz & Nakahara, *J. Neurosci,* 2010)

Introduction

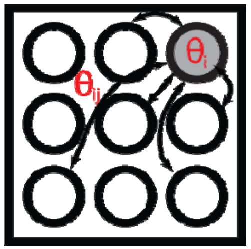
- **Previous studies suggested that 2nd-order maximum entropy model ("*pairwise* model") adequately explains activity patterns.**

$$\log \mathrm{Q}_{pair}\left(\mathbf{X}\right) = \sum_{i} \theta_i x_i + \sum_{i,j} \theta_{ij} x_i x_j - \psi$$

- **If generally true, a significant simplication for describing neural interactions**

Schneidman et al., 2006

Other works
(Shlens et al 06, 09, Tang et al 08 etc)



- **Certainly, “appropriately simple” models are crucial for improving our understanding of neural interactions and thereby functions.**

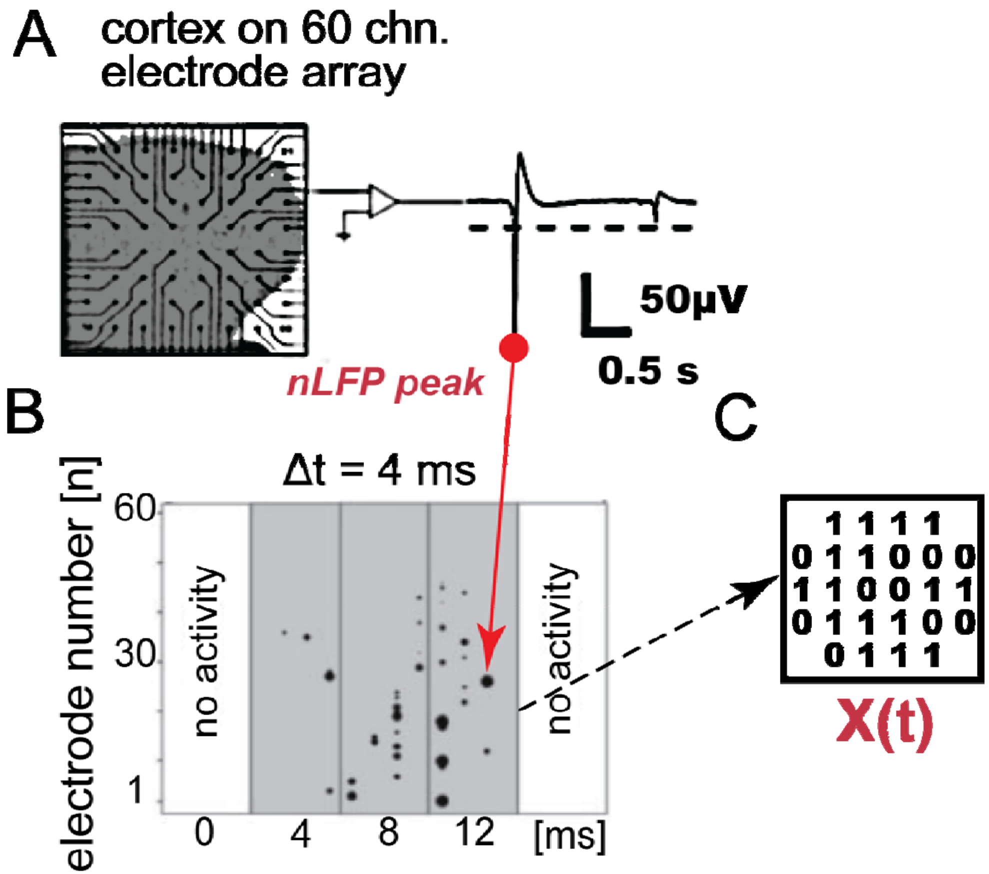
- **But is only up to 2nd-order really sufficient for any underlying network systems?**

  cf. for computations as well as scalability

- **There may be other *simplified* models that correspond better to an underlying network interaction structure, thereby being adequate for large-scale cortical activity**

  ---- *what are appropriate units for interactions?*

Our dataset (Plenz Lab, NIMH)

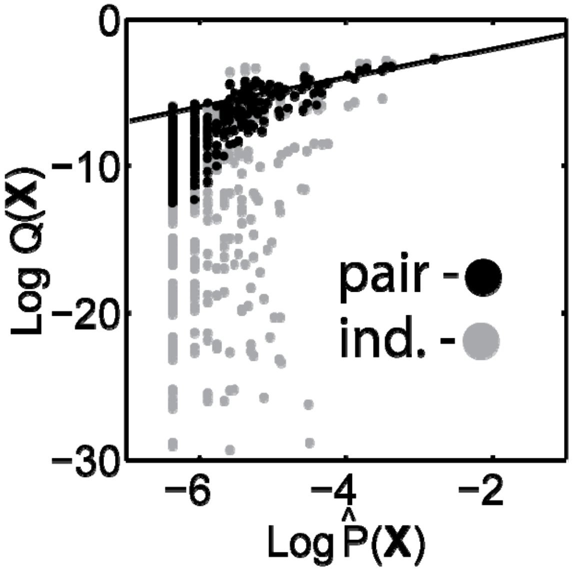


(hn note)
"neural avalanches"
power law

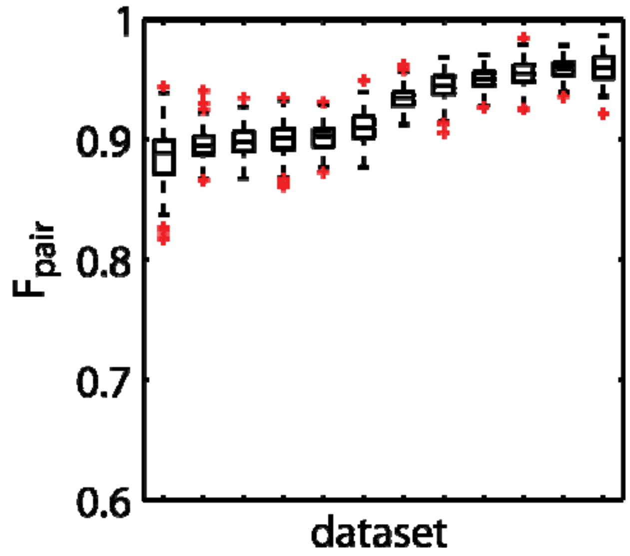
- **Cultures of coronal slices from rat somatosensory cortex and the VTA**
- **LFP signal (1-200 Hz), thresholded to obtain negative LFP (nLFP) peaks**
- **nLFP peaks binned at 4, 10, or 20 ms**

Pairwise model results

Pairwise model on a group of
10 electrodes (randomly-sampled)



Good results in 12 datasets
(200 groups / dataset)



The score called F_{pair} is frequently used :

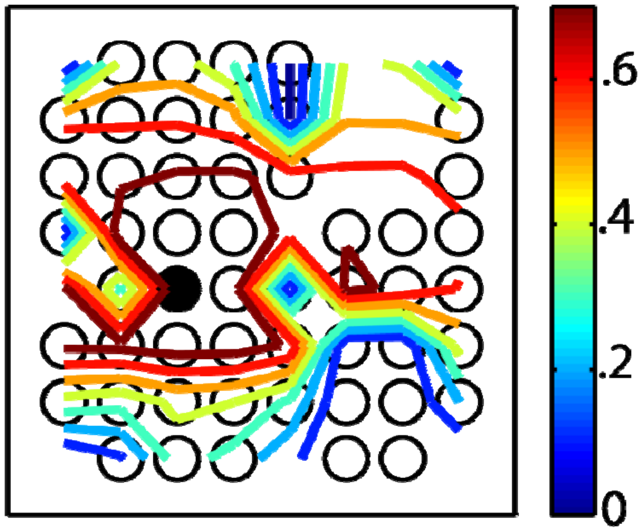
$$F_{pair} = 1 - \frac{D_{KL}\left(\hat{P}(\mathbf{X}) \| Q_{pair}(\mathbf{X})\right)}{D_{KL}\left(\hat{P}(\mathbf{X}) \| Q_{ind}(\mathbf{X})\right)}$$

Hierarchical Model: Proposal

Our proposal

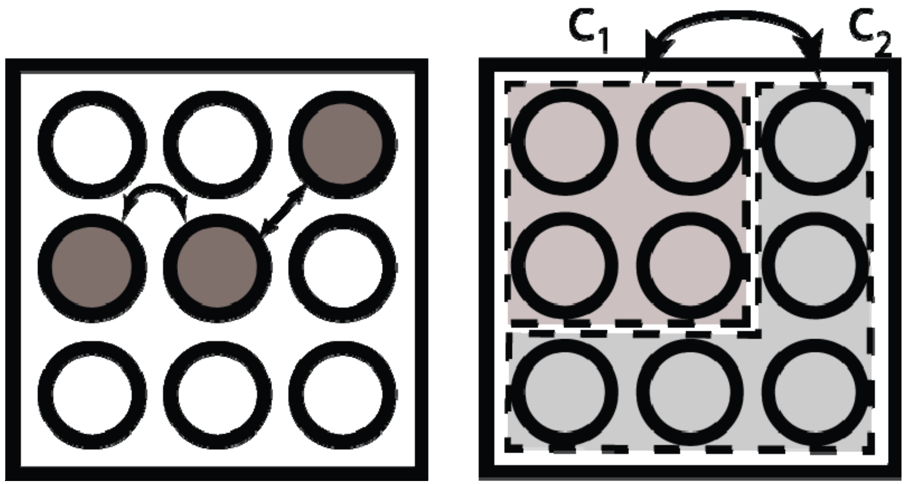
- Define a *new* functional unit for large-scale activity
- Create a hierarchical model for different spatial scales

Intuition: low resolution of correlation structure



COR of every electrode pair may not be needed

Electrodes as units | **Clusters as units**



- **How to define cluster activities?**
- **How to find clusters?**

Cluster activity = magnitude of electrode activity in cluster

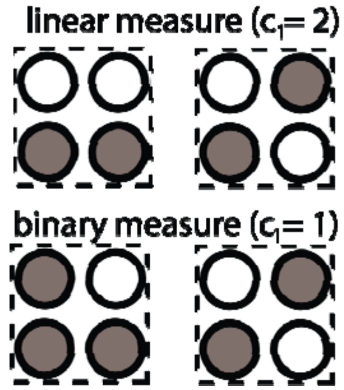
Measures of cluster activity:

Linear $c_I^{lin} = \sum_{i=1}^{n} x_i$

Log $c_I^{log} = \left\lfloor \log_2 \left(1 + c_I^{lin}\right) \right\rfloor$

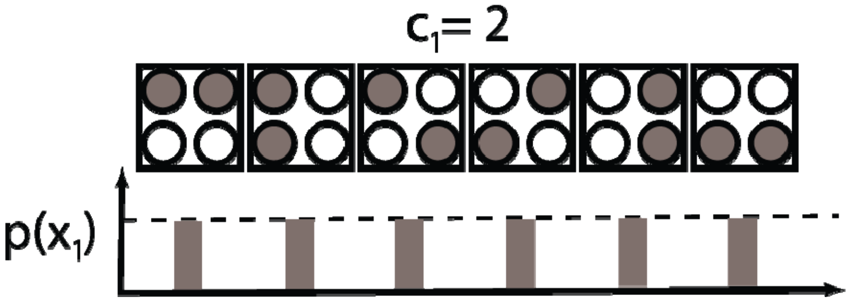
Binary $c_I^{bin} = \left(c_I^{lin} > 0\right)$

Examples



Results shown later, mostly for *log cluster activity*

Clustering criterion: homogeneity of activity

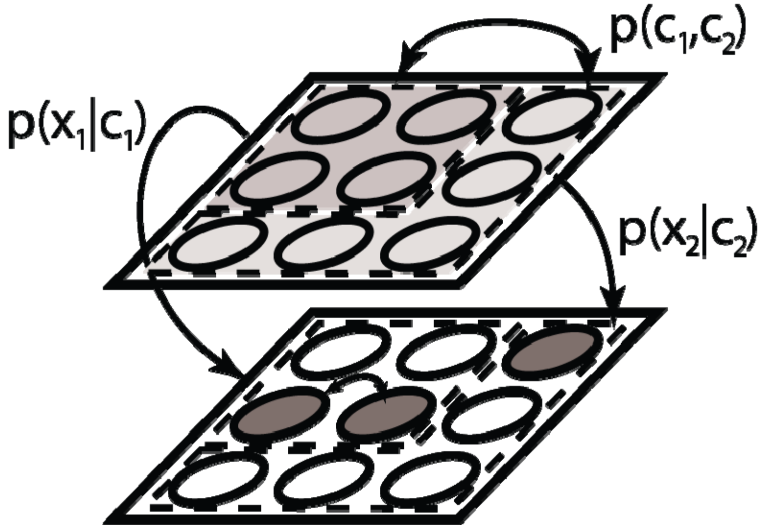


A backward searching procedure was used.

(note: caveat)

Hierarchical Model: Formulation

Hierarchical model integrates interactions over different scales



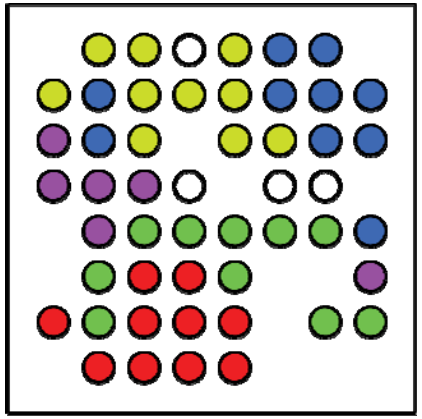
$$P(\mathbf{X}) \approx Q_{hier}(\mathbf{X}) = Q^{c}_{pair}(\mathbf{C}) \prod_{I=1}^{m} \frac{Q^{e}_{pair}(\mathbf{X}_I)}{Q^{e}_{pair}(C_I)}$$

$$\mathbf{C} = (c_1, c_2, \ldots, c_m)$$

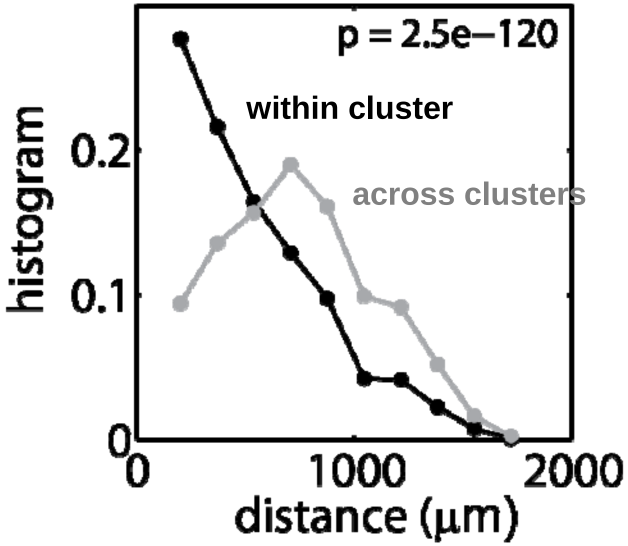
- **Pairwise models** for both ***local*** (unit = electrode) & ***long-range*** (unit = cluster) interactions, with ***conditional independence assumption on*** electrode activity across clusters given cluster activity.

Results: *clusters*

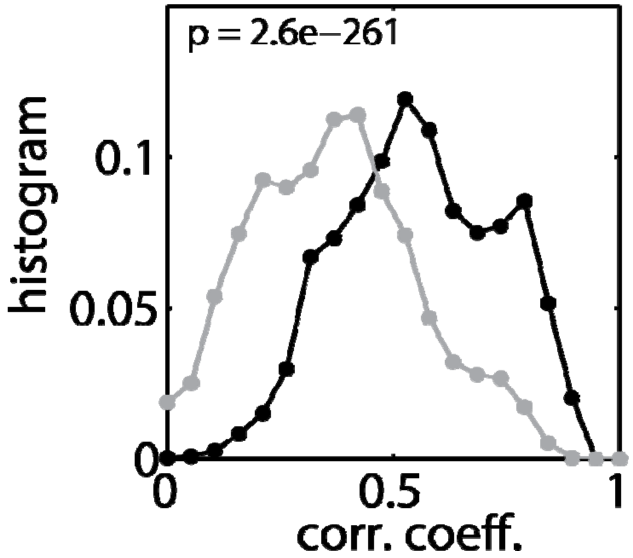
Clusters - example array:



Homogeneous clusters characterized by strong correlation and electrode proximity



• Physical distance on array

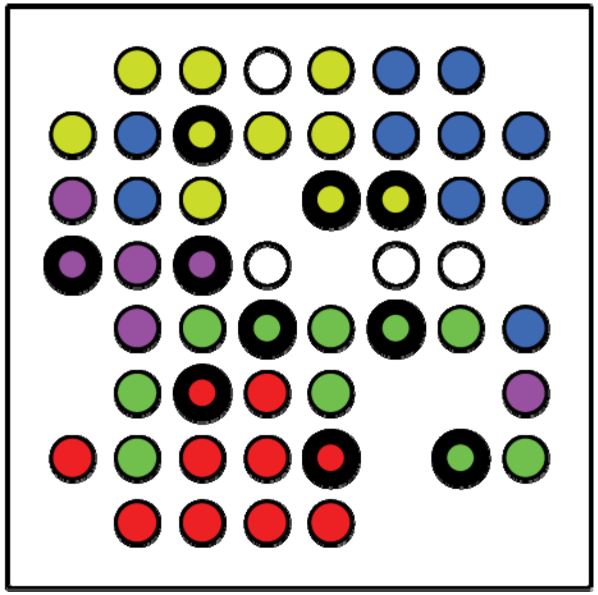


• Correlation coefficients

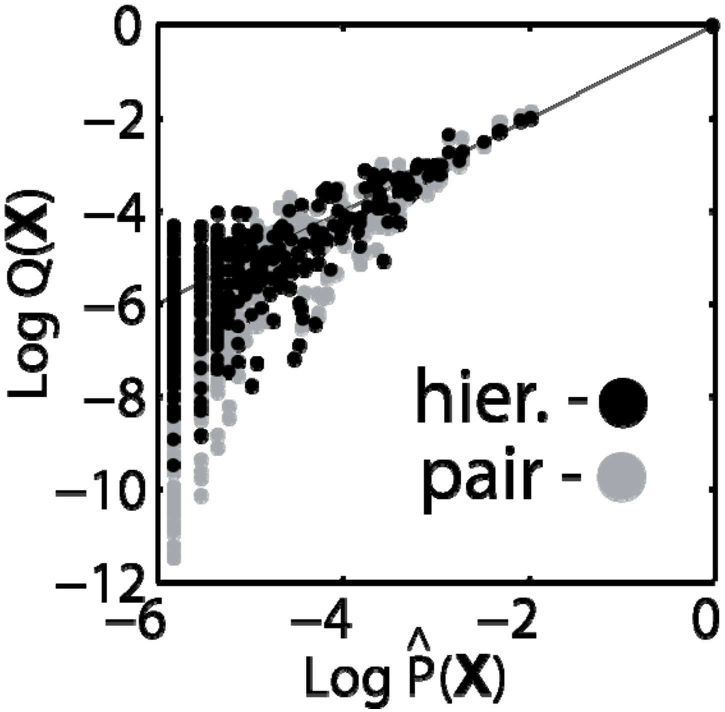
Results: *10-electrode*
– comparison with pairwise model

One example of a 10 electrode group

Cluster information used for 10-electrode groups



Example results for 1 group (F_{pair} = 0.89, F_{hier} = 0.93)

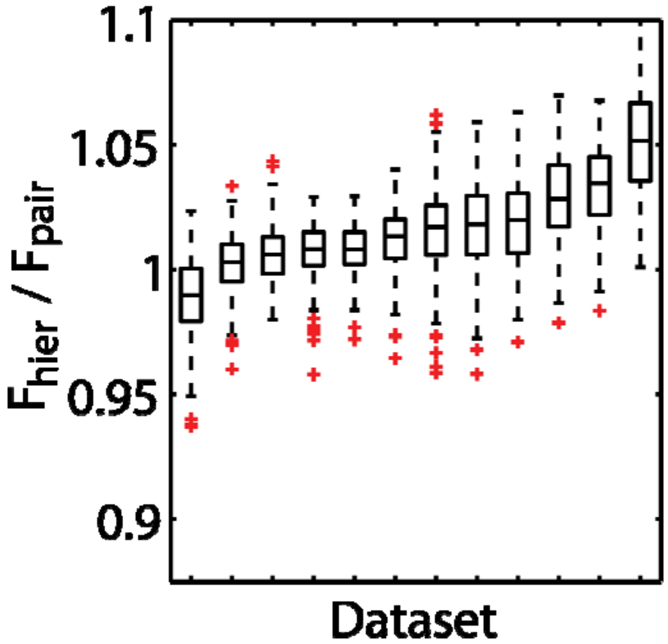


Comparison with pairwise model (10-electrode groups)

Hierarchical model has better accuracy than pairwise model, even with fewer parameters

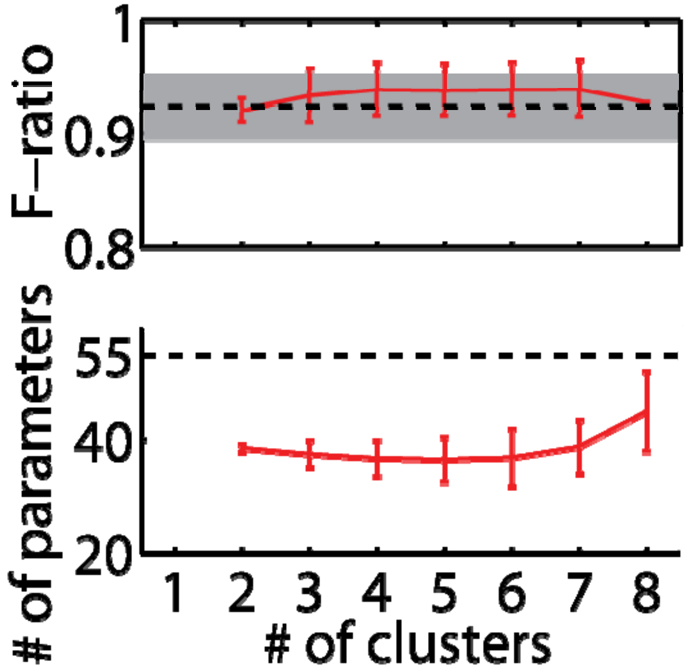
Summary

(12 datasets, 200 groups / dataset)



Accuracy and # of param

(over 12 datasets)

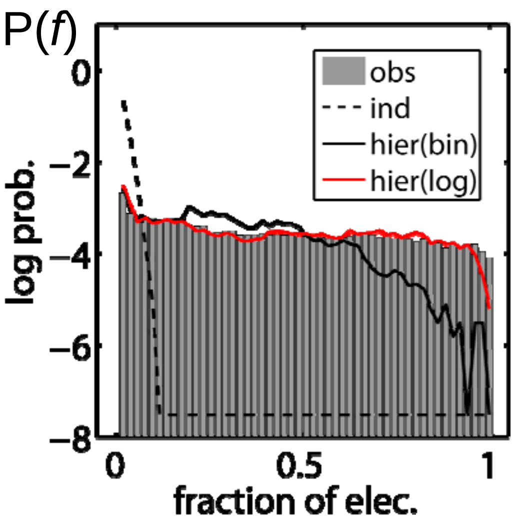


Note: also confirmed with the results using cross-validation
also confirmed w.r.t. 'shuffled' clusters

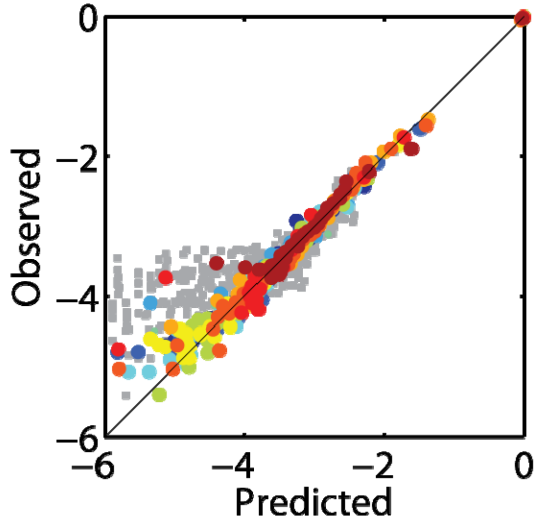
Results: *accuracy on full array* (~ 60 electrodes)

Hierarchical model with ***log*** clusters
predicts array-wide synchronized states

Example
(one dataset; *for a 'fraction'*)



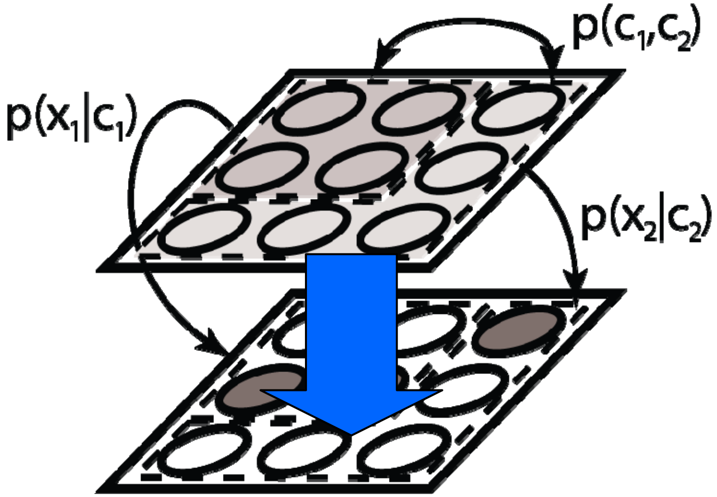
Summary
(with *log* – colored, *binary* -- gray)



Until here is in Santos et al., *J. Neurosci,* 2010

Ongoing results:

Getting back to interactions at electrode level



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$$\mathbf{C} = (c_1, c_2, \ldots, c_m)$$

Example

Original param.

$$Q^{e}_{pair}(\mathbf{x}^{\alpha}) : \{\theta^{\alpha}_{i}, \theta^{\alpha}_{ij}\}$$

$$Q^{c}_{pair}(\mathbf{c}) : \{\zeta_{\alpha|a}, \zeta_{\alpha(\beta)\alpha'(\beta)|ab}\}$$

θ – coordinates (electrode level)

$$Q_{hier}(\mathbf{x}^{\alpha}) : \{\tilde{\theta}_i, \tilde{\theta}_{ij}, \tilde{\theta}_{ijk}, \ldots, \tilde{\theta}_{123\ldots n}\}$$

(Note: possible to write analytically)

Discussion

- **The hierarchical model captures two levels of interactions using two units: electrodes and clusters.**

- **The model captures cortical LFP activity patterns better than the *pairwise model*.** → *clusters -- underlying cortical layer structure?*

→ Identifying the appropriate units of interaction of a network may enable the network interactions to be better characterized, *with better pasimony and scalability*

- **Significant higher-order interactions are embedded in a specific way. A new hierarchical model further helps us examine those properties.** cf. across/within cluster interactions

Follow-up (Q&A session)

- **I make a note here regarding "context specific graphical model" and/or "a weak definition of conditional independence", which I mentioned in Q & A session. Here is the paper I was referring to; Nakahara et al. (2003) *Bioinformatics.* (you can also download it from www.itn.brain.riken.jp). Any comments and feedbacks will be greatly appreciated.**

- **Taking advantage of adding this slide after my presentation, I would like to express my sincere gratitude to all the particiants in the workshop and the organizers who have all of us get together.**

END