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The impact of the market portfolio on the valuation, incentives and optimality of executive stock options

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This paper examines the effect on valuation and incentives of allowing executives receiving options to trade on the market portfolio. We propose a continuous time utility maximization model to value stock and option compensation from the executive's perspective. The executive may invest non-option wealth in the market and riskless asset but not in the company stock itself, leaving them subject to firm-specific risk for incentive purposes. Since the executive is risk averse, this unhedgeable firm risk leads them to place less value on the options than their cost to the company.

By distinguishing between these two types of risks, we are able to examine the effect of stock volatility, firm-specific risk and market risk on the value to the executive. In particular, options do not give incentive to increase total risk, but rather to increase the proportion of market relative to firm-specific risk, so executives prefer high beta companies. The paper also examines the relationship between risk and incentives, and finds firm-specific risk decreases incentives whilst market risk may decrease incentives depending on other parameters. The model supports the use of stock rather than options if the company can adjust cash pay when granting stock-based compensation.

1. Introduction

Stock based compensation now represents a significant proportion of executive remuneration. In 2002, stock options represented 24% of CEO pay in the UK and 58% in the US. Average CEO compensation was $2.5 million in the US compared with $1.5 million in the UK†. Ferrarini *et al.* (2003) use data from 2001 on FTSE Eurotop300 companies and report that 47% (France and Germany), 38% (Italy), 41% (Netherlands) and 33% (Sweden) of CEO pay is performance related.

This paper proposes a continuous time utility maximization model to value stock and option compensation to the executives receiving it‡. The novel feature of our model is that the executive is able to invest in the market portfolio whilst being restricted from trading in the stock of his own company. The executive is exposed to total risk but can only hedge market risk. Firm-specific risk cannot be hedged, although the executive must be exposed to it for incentive purposes. It is this exposure to unhedgeable risk, combined with the risk aversion of the executive which leads to a drop in value compared with the market value. The difference in the cost to the company and the value to the executive is the loss due to non-tradeability.

This framework enables us to consider the effect of both market and firm-specific risk on the executive's valuation and incentives, with interesting findings. We find systematic relationships between different types of risk and the value of the option to the executive. First,

executives awarded options do not necessarily want to increase stock volatility in general, when correlation is held constant, since their risk aversion can outweigh the convexity effect of the options. Secondly, executives do have an incentive to increase the proportion of market versus firm-specific risk, and thus prefer high beta companies. However, if instead market risk is held constant, firm-specific risk again generally has a negative effect on option value, but not necessarily for out-of-the-money options. This depends on the importance of convexity. Finally, holding firm-specific risk constant, a generally positive relationship between market risk and option value is demonstrated, although for some parameter values it is possible to conclude the reverse. For stock, market risk does not seem to have much effect on value. All these effects can occur in the one model and indicate that it is difficult to make general statements about 'risk' and value without clarifying exactly what variables are being held constant and which vary. See section 4 for more details.

By employing a richer model, we are able to make conclusions about the relationship between firm-specific and market risk and incentives. Our first conclusion that incentives (and optimal incentives) decrease with total stock volatility (when other parameters are held constant) is consistent with principal agent theory. We find firm-specific risk decreases incentives (and optimal incentives), regardless of whether total risk or market risk are kept fixed. However, we find incentives (and optimal incentives) may increase or decrease with market risk, depending on whether total or firm risk are kept fixed. These results extend those of Jin (2002) to options. Jin (2002) examines the relationship between risks and optimal incentives for restricted stock in a static principal agent model. Our last result on market risk is in contrast to Jin (2002) who found for stock compensation, market risk has no effect on optimal incentives.

In our set-up, we can also recover a number of known features of executive stock option valuation. First, the value to the executive is much less than the Black–Scholes value or company cost of the options. The Black–Scholes value overstates the value that executives place on the options, due to restrictions they face in trading the stock of their own company. This has been highlighted in a number of papers, including Lambert *et al.* (1991), Kulatilaka and Marcus (1994), Huddart (1994), Detemple and Sundaresan (1999) and Hall and Murphy (2002). These papers argue that there is an asymmetry between the cost to shareholders and the certainty equivalent value to the executive caused by trading restrictions. Short sales are restricted and private wealth is invested in the riskless asset. Each finds that under a power

utility framework, the value to a manager can be significantly less than the cost perceived by shareholders†.

Our framework allows us to find explicitly a utility indifference, or certainty equivalence value the executive places on the stock or option compensation. This value depends primarily on the risk aversion parameter and the correlation between the company stock and the market, but also on the volatilities and expected returns of the stock and the market. As the size of the correlation between the stock and the market approaches one, the value of the stock or options to the executive tends to their market value. This is an important feature, and Meulbroek (2001) has pointed out that not all utility models recover the Black–Scholes option value. Of course, our valuation formula depends on the preference assumption of exponential utility. However, results in Henderson (2002) for power utility indicate that similar qualitative conclusions still hold.

Secondly, we can ask in our framework with the market asset, what strike the company should set on its options to maximize incentives to executives to act to raise the stock price? Hall and Murphy (2002) find in a one-asset model that companies should grant stock rather than options, if the executive's cash pay is adjusted when the stock compensation is awarded. We observe the same in our richer two-asset model. In that way, our results generalize Hall and Murphy (2002) to include the market asset. Although neither set of conclusions are emerging from a principal-agent framework, they do arise from making a fair comparison between granting stocks versus options. Given the imminent expensing of options, this is an important and timely question.

Both Detemple and Sundaresan (1999) and Cai and Vijh (2004) also study utility based valuation of executive stock options when the executive can invest in the market portfolio. However, their treatment is via binomial or trinomial trees. Trees and associated numerical schemes give these models flexibility to use alternative utility functions, and in Cai and Vijh (2004), to cope with American exercise‡. However, for ease of numerical calculations, Cai and Vijh (2004) assume the executive cannot modify his chosen investment in the market over time. This is a significant restriction and in our model the executive rebalances his portfolio continuously. The option price obtained in this paper is additionally simpler to calculate than prices in either Detemple and Sundaresan (1999) or Cai and Vijh (2004). Kahl *et al.* (2003) study valuation of restricted stock in a utility based model with power utility. Whilst they also allow for investment in the market portfolio, they solve their problem numerically and do not treat options. In a similar framework with options, Ingersoll (2002) assumes the executive holds

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†All these papers are structural models which maximize expected utility subject to restrictions. It is worth noting that another strand of literature models early exercise as an exogenous stopping time, see Carpenter (1998) and Carr and Linetsky (2000). Carpenter (1998) examines two models where non-option wealth is invested in the Merton no-option portfolio. The first has an exogenous stopping state where the executive must optimally forfeit or exercise. The second model is a utility model where the executive is offered a reward for leaving the firm each instant.

‡Detemple and Sundaresan (1999) only consider American exercise in a simpler model without the market asset.

a restricted portfolio with at least a certain fraction in the firm's stock and does not solve for this portfolio in a full utility maximization. Earlier, Meulbroek (2001) considered a two asset framework, again concentrating on restricted stock (although this is extended by Merton (2001) to options) and a mean-variance set-up. Each of these papers observes that non-tradability reduces the value of stock or options to the executive, but none investigate systematic relationships between value or incentives and different risks.

The paper is structured as follows. Section 2 sets up the model to value executive stock and option compensation. The next section uses a utility indifference approach to value options or stock using the exponential utility model. Features of this valuation are explored and numerical examples given. Section 4 separates market and firm-specific risk and examines their effect on the value to the executive of the options. Incentive benefits are introduced in section 5 and maximized for a fixed cost to the company and fixed (pre-grant) value to the executive. The relationship between risk and incentives is explored in section 6. Concluding remarks are in the final section and details of derivations are placed in the Appendix.

2. A utility maximization model for executive stock option valuation

The executive receives European call options on the company's stock S, with expiry T and strike K. These are traditional executive stock options as defined in Johnson and Tian (2000). Often in practice these options are issued at-the-money, we allow for this, but retain strike K to examine the effect of moneyness on valuation†. We can easily consider stock compensation in the model by setting $K = 0$.

We restrict ourselves in this paper to European style options which are an approximation to the American style (with vesting period) usually granted in practice. Our valuation will represent an upper bound to the true valuation of an American option with vesting, since the American feature can only reduce the option value. An American valuation model would be computationally more challenging and we would lose the pleasing representation of the option price obtained in the following section.

We make the assumption that the executive cannot trade the stock, but is able to take positions in the market portfolio, denoted by M. The dynamics for the market index M are given by a geometric Brownian motion

$$\mathrm{d}M/M = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}B \qquad (1)$$

where B is a standard Brownian motion. Assume the company's stock S also follows a lognormal process

$$\frac{\mathrm{d}S}{S} = \nu\,\mathrm{d}t + \eta\left(\rho\,\mathrm{d}B + \sqrt{1-\rho^2}\,\mathrm{d}W\right), \qquad (2)$$

where W is a standard Brownian motion independent of the Brownian motion B driving the market‡. The total variance of S is η^2 and using the above, this can be split into the *market or systematic risk* $\rho^2\eta^2$ and the remaining *firm-specific or non-systematic risk* $(1-\rho^2)\eta^2$. When $\rho = 0$, the risk is all firm risk, and when $|\rho| = 1$ the risk is solely market risk, which can be diversified away. For $|\rho| < 1$ the presence of a second Brownian motion W and the fact that no trading is allowed on S means that the firm-specific risk cannot be hedged away.

The executive is able to invest in the riskless asset r and the market M. Cash amount θ is invested in the market and the remainder earns the riskless rate, giving dynamics for the wealth X of

$$\mathrm{d}X = \theta\,\mathrm{d}M/M + r(X-\theta)\,\mathrm{d}t.$$

The aim of the executive is to maximize expected utility of wealth, where, in addition to funds generated by trading, the executive receives λ units of the call option with payoff $(S_T - K)^+$§. The value function of the executive is given by

$$V(t, X_t, S_t; \lambda) = \sup_{(\theta_u)_{u \geq t}} \mathbb{E}_t[U(X_T + \lambda(S_T - K)^+)] \qquad (3)$$

and is used in the following section to obtain a certainty equivalence or utility indifference value for the options.

In the remainder of the paper, we will consider utilities with constant absolute risk aversion of the form $U(x) = -(1/\gamma)\exp(-\gamma x)$, $\gamma > 0$. Choice of γ is discussed in the next section. Some justification for this choice is the research of Bliss and Panigirtzoglou (2004) who estimate empirically the market's or representative agent's degree of risk aversion from option prices. They conclude CARA is more consistent with the data. The lack of dependence on wealth can also be an advantage if empirical work were to be done, since the manager's wealth is unobserved. However, the main advantage gained here from CARA utility is its tractability, as it enables closed form solutions to be found.

3. Valuing stock and option compensation

Using the model outlined in section 2, we can find the amount of cash the executive would forego to receive the stock options. This certainty equivalence or utility indifference amount represents the value of the options

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†We use the terminology 'at-the-money' throughout the paper to refer to options with an exercise price equal to the grant-date stock price. These are sometimes called fair market value options. Out-of and in-the-money options are also defined with respect to the grant-date stock price.
‡We can generalize the dynamics of the stock price S to $\mathrm{d}S = \nu(S_t, t)\,\mathrm{d}t + \eta(S_t, t)(\rho\,\mathrm{d}B + \sqrt{1-\rho^2}\,\mathrm{d}W)$ provided $\nu(S, t)$ and $\eta(S, t)$ satisfy suitable boundedness and regularity conditions. Details of this extension are available from the author upon request.
§We make the standard assumption that the executive's option grant does not have an effect on the value of the company stock.

to the executive. More specifically, the idea is to compare the expected utility for an executive who does not receive any options to the expected utility of the executive who receives the calls. The adjustment to the initial wealth which makes these two values equal gives the utility indifference value. Given an initial wealth of x, the utility indifference amount p is the solution to the equation $V(0, x - p, S; \lambda) = V(0, x, S; 0)$ (see Hodges and Neuberger (1989)).

We write the value function of the executive given in (3) as

$$V(t, X_t, S_t; \lambda) = -\frac{1}{\gamma} \exp[-\gamma X_t \exp(r(T-t))] g(T-t, \log S_t)$$

where $g(0, \log s) = \exp[\lambda\gamma(S-K)^+]$. It follows (see Appendix, Derivation of value function) that

$$V(t, X_t, S_t; \lambda) = -\frac{1}{\gamma} \exp[-\gamma X_t \exp(r(T-t))]$$

$$\times \exp[-(\mu-r)^2(T-t)/2\sigma^2]$$

$$\times [\mathbb{E}^0 \exp(-\lambda\gamma(1-\rho^2)(S_T - K)^+)]^{(1-\rho^2)^{-1}}.$$

$$(4)$$

Solving $V(t, X_t - p_\lambda^e, S_t, \lambda) = V(t, X_t, S_t, 0)$ for the value p_λ^e of λ calls gives the representation

$$p_\lambda^e = -\frac{\exp[-r(T-t)]}{\gamma(1-\rho^2)} \log \mathbb{E}^0[\exp(-\lambda\gamma(1-\rho^2)(S_T - K)^+)].$$

$$(5)$$

Notice the expectation in (4) and (5) is no longer taken with respect to the original probability measure $\mathbb{P}$. Under this new probability measure denoted $\mathbb{P}^0$, the rate of return on the market is r, but the return on innovations orthogonal to the market are left unchanged. This is the minimal martingale measure of Föllmer and Schweizer (1990). Specifically, the market follows

$$\frac{dM}{M} = r\,dt + \sigma\,dB^0,$$

where $B^0 = B + [(\mu - r)/\sigma]t$, a $\mathbb{P}^0$ Brownian motion and the stock follows

$$\frac{dS}{S} = \left[r + \eta\left(\frac{\nu - r}{\eta} - \frac{\rho(\mu - r)}{\sigma}\right)\right] dt$$

$$+ \eta\rho\,dB^0 + \eta\sqrt{1-\rho^2}\,dW,$$

$$(6)$$

where we define $\delta = r + \eta[(\nu - r)/\eta - \rho(\mu - r)/\sigma] = \nu - (\mu - r)\eta\rho/\sigma$ to be the drift of the stock under $\mathbb{P}^0$. As mentioned earlier in the second footnote on p. 3, the stock dynamics can be generalized and in this case, (5) will still hold as the representation for the value to the executive.

It is important to observe that we have not priced under this measure, it is merely appearing in the expression for executive value, which was derived using utility maximization arguments. To reinforce this, we could expand the value in (5) in terms of the number of options, λ, to give

$$p_\lambda^e = \exp[-r(T-t)]\lambda$$

$$\times \left(\mathbb{E}^0(S_T - K)^+ - \frac{\lambda}{2}\gamma(1-\rho^2)\text{Var}[(S_T - K)^+] + \cdots\right).$$

$$(7)$$

The first term in the expansion is exactly the value of the option under the measure $\mathbb{P}^0$, but there are correction terms to this. We now show that when the market and stock are perfectly correlated, this valuation method gives the correct value for the option, its Black–Scholes or market value. Putting $|\rho| = 1$ into the expansion for p_λ^e above eliminates the second order term. Ignoring higher order terms, the value appears to be simply the price under the measure $\mathbb{P}^0$. However, on closer inspection, to avoid arbitrage in the model, the relationship $\nu - r = (\mu - r)\eta\rho/\sigma$ must hold, with either $\rho = 1$ or $\rho = -1$. This implies the drift δ under $\mathbb{P}^0$ is actually just the riskless rate, and hence the value becomes simply the Black–Scholes price (as $\mathbb{P}^0$ becomes the risk neutral measure). In this case, there is no firm-specific risk and the executive is fully diversified.

We have established that the limiting case of perfect correlation between the stock and the market gives the Black–Scholes value, or company cost. However, for the more realistic case of non-perfect correlation, how does the value differ from the company cost? Let us first recognize that the value p_λ^e solves a partial differential equation (pde). We write

$$p_\lambda^e(t, s) = -\frac{\exp[-r(T-t)]}{\gamma(1-\rho^2)} \log W(t, s), \quad (8)$$

where $W(t, S_t) = \mathbb{E}^0 \exp[-\lambda\gamma(1-\rho^2)(S_T - K)^+]$. Using the pde for W developed in the Appendix, we may derive a pde for the executive value p_λ^e

$$\frac{\partial p_\lambda^e}{\partial t} + \frac{1}{2}\eta^2 S^2 \left(\frac{\partial^2 p_\lambda^e}{\partial S^2} - \left(\frac{\partial p_\lambda^e}{\partial S}\right)^2 \left(\gamma(1-\rho^2)\exp[r(T-t)]\right)\right)$$

$$+ \delta S \frac{\partial p_\lambda^e}{\partial S} = r p_\lambda^e$$

$$(9)$$

with $p_\lambda^e(T, s) = \lambda(s - K)^+$. Again, we can see that taking $|\rho| = 1$ and $\delta = r$ in (9) gives back the Black–Scholes pde.

We return to the question of non-perfect correlation between the stock and the market. We can show the value p_λ^e is lower than the company cost and increasing in the size of the correlation. As correlation tends to one, the value to the executive tends to the company cost. This is shown in the Appendix, Effect of correlation on executive value.

The risk aversion parameter γ plays an important role. When the executive is awarded options ($\lambda > 0$), higher risk aversion results in him placing a lower value on them. This can be shown using a similar comparison argument to that used for the correlation.

We now illustrate some of the features of our analytical formula for executive valuation further by varying the parameters in (5). Dividing (5) by the number of units λ, gives a per-unit value p_λ^e/λ. We can observe that the parameters λ and γ only appear together in

the per-unit value, and we set $q = \lambda\gamma$. We motivate our numerical choice of this parameter by equating local absolute risk aversion with the more popular power utility with constant relative risk aversion. This is useful since many authors use values of R around 3–4 and we can adopt similar values in our numerical examples†. The equating of local risk gives the relation $\gamma = R/x$, where R is the coefficient of relative risk aversion in the power utility and x is private wealth.

Say the initial value of the stock is 100 cents. Now consider a number of scenarios concerning the ratio of options or stock to private wealth of the executive. Suppose the executive receives $25\,000$ worth of shares so $\lambda = 25\,000$. If this represents a very small fraction of his wealth, say $1/40$, then $x = \$1\,000\,000$ or $x = 10^8$ cents and a suitable risk aversion parameter is $\gamma = 4/10^8$ and $\lambda\gamma = 4 \times 25\,000/10^8 = 1/1000$. This scenario might describe a wealthy CEO. At the other extreme, if the shares represent $1/4$ of the executive's wealth, then $x = \$100\,000$, $\gamma = 4/10^7$ and $\lambda\gamma = 4 \times 25\,000/10^7 = 1/100$. This would be more suitable for firms such as Internet firms where many employees have a large part of their wealth in options or stock of the company. Other values between these two values of q that we use later are $q = 4/2500$ and $q = 4/1500$. Of course, these numbers can be scaled. The executive who receives $\$2\,500\,000$ worth of shares has $\lambda = 2\,500\,000$ and if this represents $1/40$ of his wealth, x also gets multiplied by 100. This results in the same value for $q = \lambda\gamma$. This provides some justification for our choices of risk aversion γ and shows how implicitly, this choice may reflect private wealth.

In each of the graphs (unless specified otherwise) we use parameters $T = 10\,\text{yr}$, $\rho = 0.8$, $\mu = 0.10$, $\eta = 0.45$, $\sigma = 0.35, r = 0.05$. We set the drift of S, δ under $\mathbb{P}^0$ to be r and let ν vary according to the relationship given earlier. Despite having the closed form representation in (5), we need to use numerical integration to compute values. Equivalently, we could have solved the pde in (9).

Figure 1 is a plot of the value per unit option. The Black–Scholes or company cost represented by the highest line on the graph is significantly higher than either of the executive values, the two broken lines. The executive values use two values of q. This represents two executives with the same number of options λ and different risk aversion parameters, but can also be thought of as representing two executives with the same risk aversion parameters but holding different numbers of options. As predicted from the valuation formula, as risk aversion increases (moving from the higher to lower of the two broken lines), the value placed on the options falls. A similar picture can be obtained for stock compensation.
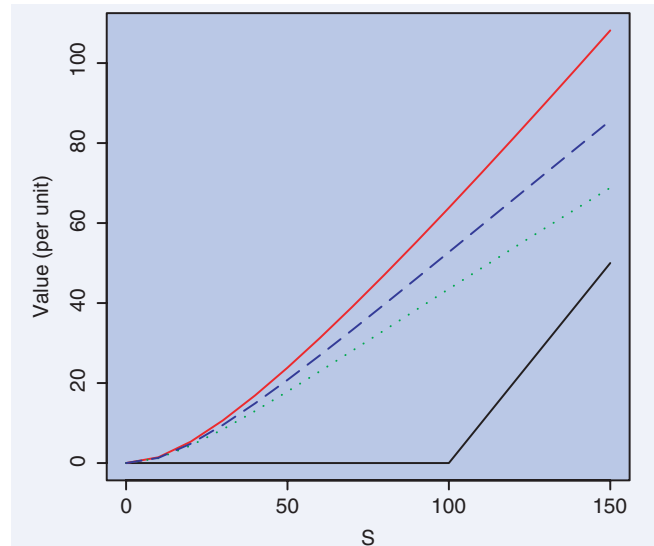

Figure 1. Per-unit option value. The top line is the risk neutral value or company cost and the lowest line is the option payoff. The two inner lines represent the value to the executive using the model, the higher line uses $q = \lambda\gamma = 1/1000$ whilst the lower line uses $q = \lambda\gamma = 4/1500$. Parameters are $T = 10$, $K = 100$, $\rho = 0.80$, $\mu = 0.10$, $\eta = 0.45$, $\sigma = 0.35$, $r = 0.05$ and $\delta = r$.

We can also examine the effect of changing the option's expiry, correlation between the market and stock and q, representing risk aversion and number of options, in a table. Table 1 considers an at-the-money option and reports the ratio of the executive value to the company cost for 3, 5 and 10 year options. We fix σ and η and vary ρ, which also changes the beta of the company. As predicted, when correlation tends to one, the ratio approaches one, as the executive value tends to the company cost. In this case, the executive is fully diversified and will not require a return premium. Higher risk aversion leads to a lower executive value. A CEO with much wealth outside the firm may value 10 year options at 80% of the Black–Scholes value (when $\rho = 0.75$), but the same options may only be worth 39% say to an Internet company employee using $q_4 = 1/100$. Qualitatively similar results are obtained by looking at stock compensation and options are less efficient than stocks in terms of their executive value versus company cost. For the same parameters (although not in the table), the stock compensation has ratio 0.97 for $T = 3$ and 0.81 for $T = 10$. This is due to the convexity of the options.

4. The effect of total, market and firm-specific risk on option value

By separating the firm-specific and market risks in the model we are able to examine the effect of each on value and gain insights into the nature of the relationship

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†Many papers, including Jenter (2001), Nohel and Todd (2001), Lambert *et al.* (1991) and Hall and Murphy (2002) use R about 3–4. Although there is a debate as to whether R should be much larger, (see the discussion of the equity premium puzzle in Cochrane (2001)) this would only magnify the effects observed in our model.

Table 1. Ratio of executive value to company cost for at-the-money option compensation. We vary expiry (in years), $q = \lambda\gamma$, and correlation between stock and market. Parameters: $q_1 = 1/1000$, $q_2 = 4/2500$, $q_3 = 4/1500$ and $q_4 = 1/100$. Take $\sigma = 0.22$, $\eta = 0.45$, risk premium $\mu - r = 0.075$ and $S_0 = K = 100$.

ρ	$T = 3$				$T = 5$				$T = 10$			
	q_1	q_2	q_3	q_4	q_1	q_2	q_3	q_4	q_1	q_2	q_3	q_4
0.25	0.92	0.87	0.81	0.55	0.85	0.79	0.71	0.43	0.68	0.6	0.5	0.26
0.5	0.93	0.90	0.84	0.60	0.88	0.82	0.75	0.48	0.72	0.64	0.55	0.30
0.75	0.96	0.93	0.90	0.71	0.92	0.88	0.83	0.60	0.80	0.74	0.65	0.39
0.90	0.98	0.97	0.95	0.85	0.96	0.94	0.91	0.76	0.89	0.85	0.78	0.56
0.95	0.99	0.98	0.97	0.91	0.98	0.97	0.95	0.85	0.93	0.90	0.86	0.68
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

between risk and option value. The analysis in this section observes how the value of an option with given moneyness varies with different exogenous levels of either stock volatility η, correlation ρ, firm-specific risk or market risk. All other parameters are held constant. Note we are not allowing the option grant to alter the firm's risk but are observing how option values change if risk is altered. This allows us to ask whether an executive would prefer a high or low exogenous risk (of the various types) to give them the highest option value. Extrapolating from this, we can decide whether an executive will want to influence the various risks to increase the value of his options, or equivalently, whether the executive has an incentive to alter risk. For instance, it is commonly believed (see, for example, Johnson and Tian (2000)) that executives have a desire to increase stock volatility. We show in this section that this is not the case, but rather that executives want to increase the proportion of market relative to firm-specific risk. These new observations can only be made in a model such as ours which considers market and firm-specific risk separately.

By varying the stock volatility η, we can observe the effect on executive value. By fixing the correlation, varying η will have an impact on both firm-specific and market risks. Some thought tells us that increasing volatility will have two effects on value. First, firm-specific risk rises, and since this is unhedgeable and executives are risk averse, value falls. But volatility also has the effect of increasing value via the convexity of the call option payoff. Therefore we can expect a mixed effect on value, depending on which dominates. This will depend on the option moneyness and other parameters.

Figure 2 depicts this using options with various degrees of moneyness. The three solid lines represent the company cost for in-the-money, at-the-money and out-of-the-money options, in order from highest to lowest. Each is increasing with volatility, since this only takes the convexity effect into account. The remaining three lines represent value to the executive for the same three options. For the out-of and at-the-money options, when η is low, the convexity effect dominates firm-specific risk and value is increasing. However, this reverses for higher volatility, leading value to decrease with volatility. The convexity is less important for the in-the-money option (the extreme case being stock with no convexity effect), and value is decreasing in volatility for the entire range.
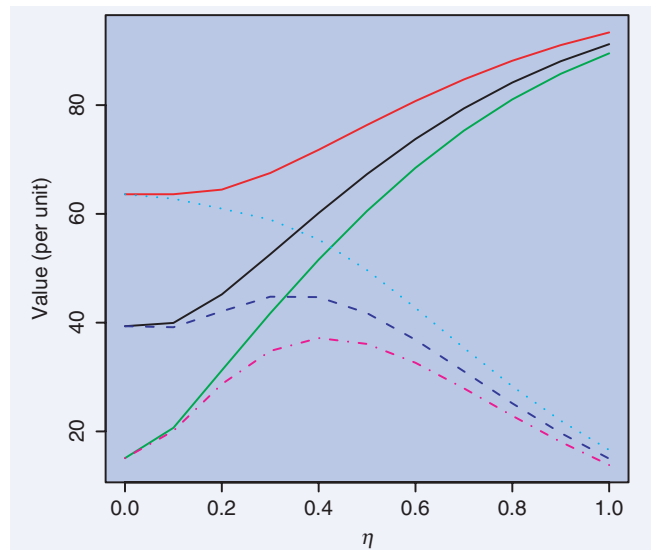

Figure 2. Effect of volatility on value, fixing $\rho = 0.8$. Graph for three options: the three increasing solid lines are, from highest to lowest, in-the-money $(K = 60)$, at-the-money $(K = 100)$, out-of-the-money $(K = 140)$ Black–Scholes values. The executive value for the same three options are represented by the dashed lines, with the same ordering highest to lowest. Risk aversion $q = \lambda\gamma = 4/1500$ is used and $S_0 = 100$. Other parameters are those used in figure 1.

At high values of total volatility, the executive's risk aversion dominates any convexity effect, resulting in falling values.

Empirical evidence supporting the idea that options do not always encourage risk taking is available but limited. Firstly, Cohen *et al*. (2000) test for evidence of risk shifting behaviour and find it to be a very small effect, regardless of direction. In the oil and gas industries, Rajgopal and Shevlin (2002) find managers are motivated to take on exploration risks but hedge oil price risk. Empirical evidence supporting our conclusion may also be found in Meulbroek (2000). She examines insider transactions in high volatility internet firms. The study finds negative information is not the primary motivation for insider selling. Rather, the evidence suggests executives place a high value on having a diversified portfolio and attribute a lower value to the options when volatility is high.

The second analysis we consider is varying correlation whilst keeping total volatility η constant. We relate the effect of correlation on value to firm-specific and
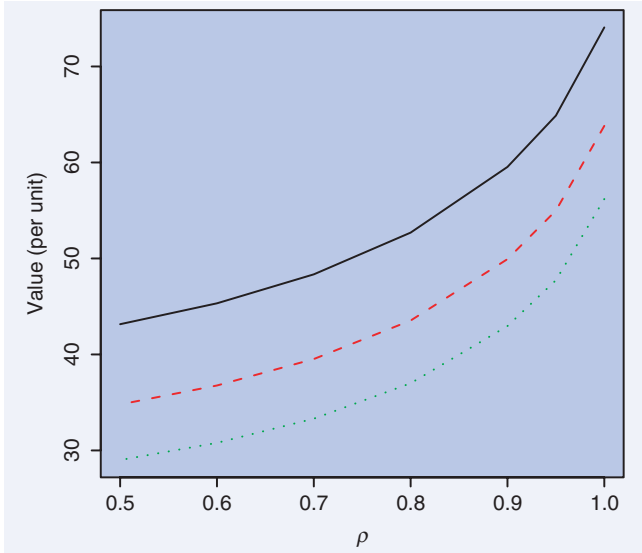
Figure 3. Effect of correlation on value to the executive, fixing $\eta = 0.45$. The three lines represent in-the-money ($S_0 = 100$, $K = 60$), at-the-money ($S_0 = 100 = K$) and out-of-the-money ($S_0 = 100, K = 140$) options from highest to lowest. Each uses $q = \lambda\gamma = 4/1500$. Parameters are $T = 10$, $\mu = 0.10$, $\sigma = 0.35$, $r = 0.05$, $\delta = r$.
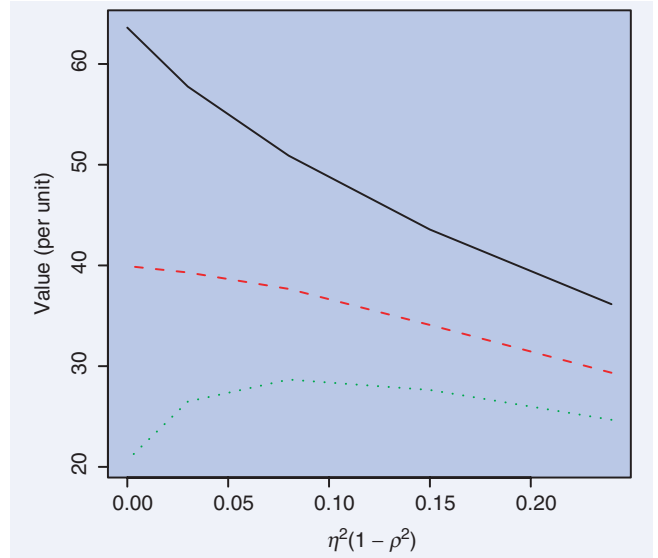

Figure 4. Effect of firm-specific risk on option value, fixing market risk. The three lines represent in-the-money ($S_0 = 100$, $K = 60$), at-the-money ($S_0 = 100 = K$) and out-of-the-money ($S_0 = 100$, $K = 140$) options from highest to lowest. Each uses $q = \lambda\gamma = 4/1500$. Other parameters are those used in figure 1.

market risks. Figure 3 plots the value (per unit option) to the executive against correlation between the market and stock. The three lines represent three different options (from highest to lowest) in-the-money, at-the-money and out-of-the-money, with parameters given in the caption to figure 3. Firstly observe that the value increases with correlation, for each option moneyness. This extends also to the case of stock. Altering correlation whilst keeping total volatility η fixed changes the proportion of firm-specific and market risks. As correlation increases, less of the risk is firm-specific and more is market risk, leading to an increase in the option value to the executive. By varying firm-specific risk as a proportion of total risk, we obtain a negative relationship between firm-specific risk and value. Varying market risk as a proportion of total risk gives a positive relationship between market risk and value. These conclusions also hold if the executive receives the stock itself. Therefore, when an executive receives options or stock, there is an incentive for him to act such that more of the total risk is market, rather than firm risk. Equivalently, the executive will act to increase the beta of the company.

We can also try to isolate the effect of firm-specific risk on value by keeping market risk constant. Here, two forces are interacting and increasing firm-specific risk is reducing value, whilst the corresponding increase in total risk η will increase value via convexity. This convexity effect should have less of an impact than in figure 2 however as market risk is constant, although the overall impact is not obvious and may depend on the parameter values. Figure 4 substantiates this claim. The lines from highest to lowest represent in-the-money, at-the-money and out-of-the-money options. The figure shows option value to the executive is generally decreasing with
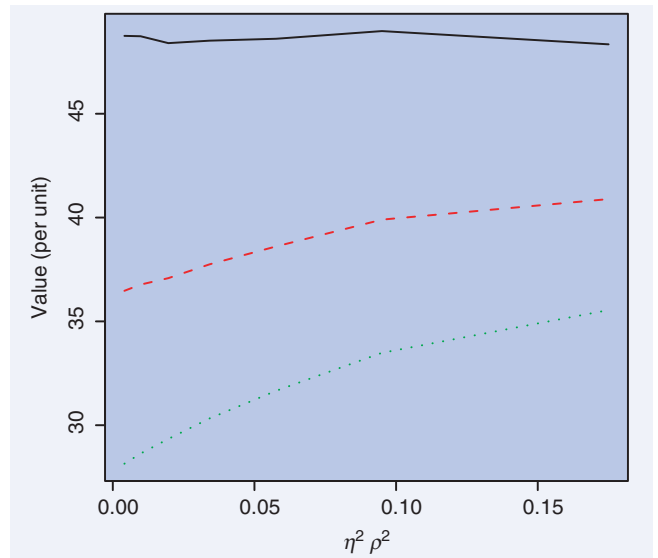

Figure 5. Effect of market risk on option value, fixing firm-specific risk. The three lines represent in-the-money ($S_0 = 100$, $K = 60$), at-the-money ($S_0 = 100 = K$) and out-of-the-money ($S_0 = 100$, $K = 140$) options from highest to lowest. Each uses $q = \lambda\gamma = 4/1500$. Other parameters are those used in figure 1.

firm-specific risk when market risk is fixed, although for out-of-the-money options it is possible for the value to increase when firm-specific risk is very low. The convexity does have an effect at low values of firm-specific risk in this case.

A final graph will fix firm-specific risk and vary market risk $\eta\rho$. Increasing market risk $\eta^2\rho^2$ whilst keeping firm-specific risk constant involves increasing both η and ρ. The same three options appear in figure 5 as those used in figure 4. For this example, as market risk increases, the value to the executive rises for the at

and out-of-the-money options. For the far in-the-money option (or stock) however, the value seems to be fairly constant as market risk increases. This is really a combination of the effects in figures 2 and 3 and therefore the value to the executive may increase or decrease with market risk, contrary to intuition.

Thus depending on which parameters are held fixed, differing but not inconsistent conclusions may be drawn about the effect of risks on the executive's valuation of the options. In particular, options do not give incentive to increase total risk, but rather to increase the proportion of market relative to firm-specific risk. If firm-specific risk is fixed however, the effect of market risk on value can be ambiguous.

5. Optimal choice of strike

We will now turn to the incentive benefits of stock-based compensation, focusing on the impact of the market portfolio. Following Hall and Murphy (2002) and others, we consider incentives to be the options' effect on motivation of a manager to increase the company's stock price. The idea is that by giving stock-based compensation to executives, they will act like an owner of the company. In common with Hall and Murphy (2002) and Jenter (2001), we define incentives as the derivative of the utility value with respect to stock price. However in contrast to these papers, we allow for trading in the market portfolio in continuous time. Using the expression for value in (8), we differentiate to obtain

$$\frac{\partial}{\partial S} p_\lambda^e(t,s) = -\frac{\exp[-r(T-t)]}{\gamma(1-\rho^2)} \frac{W_s(t,s)}{W(t,s)}. \qquad (10)$$

where the details of the expression for W_s are given in the Appendix, Calculation of incentives.

We see in figure 6, just as the company cost overstates the value of the options to managers, the Black–Scholes delta overstates their incentive effects. The plot shows incentives for an option with stock price 100, for two values of $q = \lambda\gamma$. Incentives decrease with risk aversion level, the lower line uses the larger of the two parameters. Thus options give much less incentive to increase stock price than is reflected by the naive Black–Scholes delta. Another observation that can be made from figure 6 is that incentives are greatest for in-the-money options (and the extreme case of stock) on a per-unit basis.

A caveat is in order here. To compare incentives as measured by (10) across different strikes K, we must assume that the initial stock price and dynamics of S are the same in each case. That is, we cannot have a feedback effect from the option grant to the stock price, see third footnote on p. 3 earlier. Clearly this is an approximation, but one that is widely accepted.

We now ask the question 'what is the best strike K for the company to set so as to maximize incentives to executives for a fixed cost?' For the same amount of money, the company could grant a larger number of out-of-the-money options or fewer stocks, with differing incentives from each. We allow the company to grant
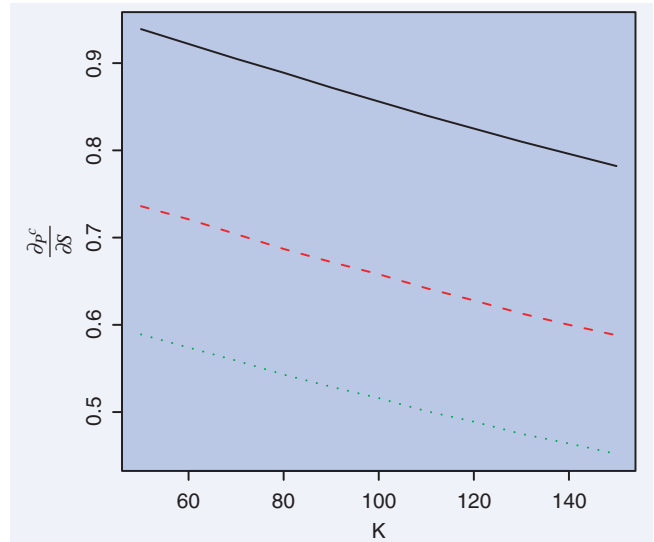

Figure 6. Option incentives (the change in the executive value (per option)p^e/λ for a 1 unit change in the stock price). The solid line corresponds to the risk neutral delta whilst the two lower lines use the utility model with $q = \lambda\gamma = 1/1000$ and $q = \lambda\gamma = 4/1500$ (lowest line). Parameters are those used in figure 1. with $S = 100$.

differing numbers of options and stocks and compare to see which strike maximizes incentives for a fixed cost. Equivalently, incentives could be fixed, and compensation chosen to minimize cost, as in Jenter (2001). Note in our formulation we do not allow for the situation where an executive receives both stock and options, although the analysis could be extended to cover this case.

Our analysis of this problem is based on that of Hall and Murphy (2002) earlier. We extend their work to include the market asset and ask the same question that they consider in their Case 2. It is worth noting here that neither their nor our analysis takes place in a principal-agent framework. We assume, as do Hall and Murphy (2002), that stock-based compensation is to be granted and then solve for the best strike to maximize incentives for a fixed-cost.

As in Case 2 of Hall and Murphy (2002), we will make an assumption, made in efficient contracting theory, that when stock-based compensation is given, the executive's base salary is reduced. The relevant amount of cash to be deducted from salary is exactly the executive's valuation of the additional options or stock. If we think of the executive starting with $100 000 cash, then the option grant must be such that the value of the options p_λ^e plus cash pay a total $100 000. In this scenario, the executive is indifferent between additional stock or options (and less cash) and the status quo. We also fix the cost to the company of the compensation, which is the Black–Scholes value of the options plus cash pay.

Returning to our illustration, assume the company paying the executive $100 000 cash decides to grant stock or options. Say the company gives options with Black–Scholes value $10 000, but the executive values these at only $6000. The company gives the executive $94 000 cash and the options, leaving the total value to him unchanged

at \$100 000. The total cost to the company is now \$104 000. When deciding on whether to grant stock, in-the-money, at-the-money or out-of-the-money options, the company keeps the \$100 000 value to the executive constant and the \$104 000 cost to themselves constant.

We now observe that the two constraints mentioned above can be combined into one single constraint. The first constraint denoted (i), is that the value to the executive is equal to the cash pay plus $p_\lambda^e(K)$, and this is kept constant. The second constraint labelled (ii), is that the company cost (also kept constant) equals the cash pay plus the Black–Scholes value of λ options. Subtracting (i) from (ii) eliminates cash pay and gives a single constraint — to keep fixed the difference in the Black–Scholes and executive value of the compensation. In the example above, it is equivalent to keeping a difference of \$4000 constant.

If we denote the executive value of this compensation by $p_\lambda^e(K)$ and the Black–Scholes value (of λ options) by λBS^K (where in both cases the K refers to the dependence on the strike) then we maximize incentives by

$$\max_K \frac{\partial p_\lambda^e(K)}{\partial S} \quad \text{such that} \quad \left[\lambda BS^K - p_\lambda^e(K)\right] = c, \quad (11)$$

where c is a fixed constant.

In the following analysis we choose a fixed c such that for typical parameters and an at-the-money option, λ is around 10 000 when $\gamma = 4/1.5 \times 10^7$ and $\eta = 0.4$. Once c is selected, the choice of λ satisfying the constraint in (11) will vary with choice of K. We will fix risk aversion, option expiry, μ, volatility of the market, riskless rate and initial stock value. The remaining parameters of stock volatility and correlation are varied in a number of ways. This enables us to investigate the conditions under which different option strikes may be optimal and the robustness of our conclusions.

Figure 7 fixes correlation and varies the stock volatility η and strike K to plot incentives for receiving λ options. We fix c as described. For each combination of volatility and strike, the constraint in (11) is solved for λ. When $\eta = 0.4$, the company can grant around 10 000 at-the-money options or more than 10 000 out-of-the-money options but fewer than 10 000 options if they are in-the-money. For the moment, we are interested in the behaviour of incentives across various strikes, for a given η. Later in section 6, we will also consider behaviour across different η for a given strike to analyse the relationship between risk and incentives.

The figure shows that for a fixed volatility η, incentives (for λ options or stock) are decreasing in strike K. To maximize incentives according to the optimization in (11), the company should grant stock rather than options. We also verified that this conclusion was robust to changes in risk aversion. We can also verify this is the case when we consider varying market and firm-specific risks, both together and in turn. First we fix stock volatility η and vary the correlation and strike. Again, this is done such that c is fixed and (11) is solved for λ for each combination of correlation and strike. This implies total risk is fixed but the proportion of firm-specific and market
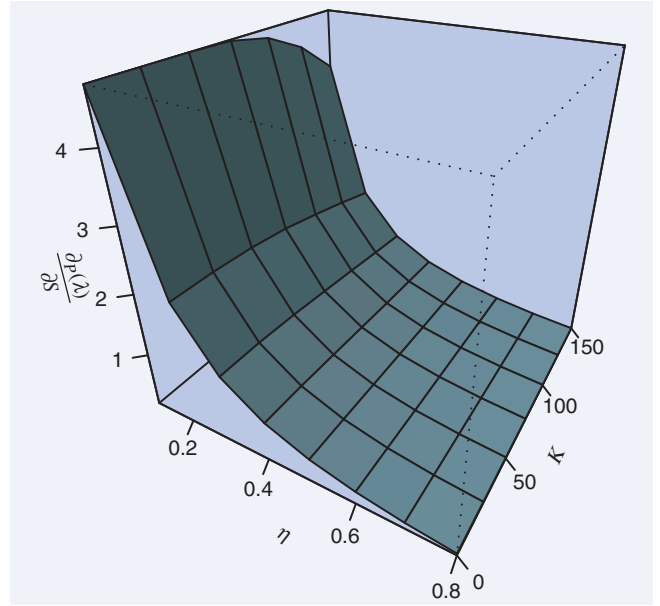

Figure 7. Incentives for λ options or stock across strike K and total volatility η, with fixed $\rho = 0.8$. A fixed c is chosen and λ determined for each K, η pair such that (11) is satisfied. Fix $\gamma = 4/1.5 \times 10^7$, $S_0 = 100$ and other parameters same as figure 1.
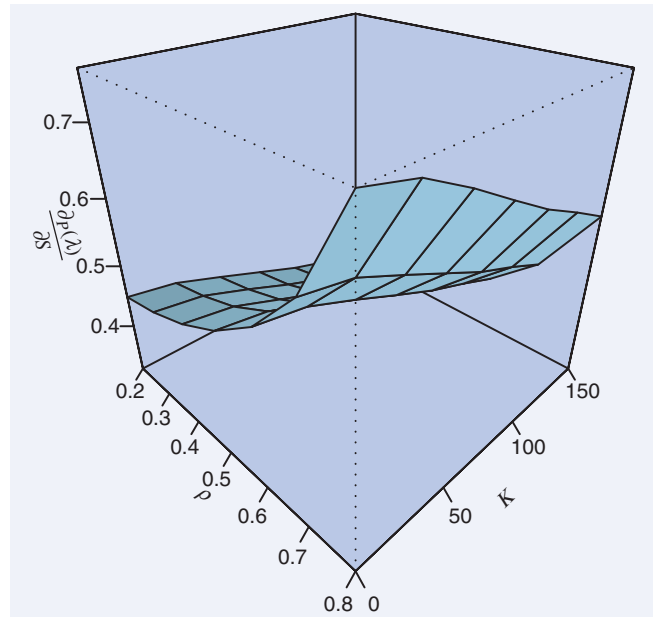

Figure 8. Incentives for λ options or stock across strike K and correlation, with fixed $\eta = 0.4$. A fixed c is chosen and λ determined for each K, ρ pair such that (11) is satisfied. Fix also $\gamma = 4/1.5 \times 10^7$, $S_0 = 100$ and other parameters same as figure 1.

risk changes with ρ. Figure 8 shows that for fixed ρ, incentives are still maximized by the choice $K = 0$. Figures 9 and 10 fix market risk and firm-specific risk in turn, whilst varying the remaining risk. Again, $K = 0$ is optimal to maximize incentives in both cases. Thus for typical volatilities the model concludes that stock compensation is optimal and options are an inefficient means of creating incentives, in agreement with Jenter (2001) and Hall and Murphy (2002). Our result is also
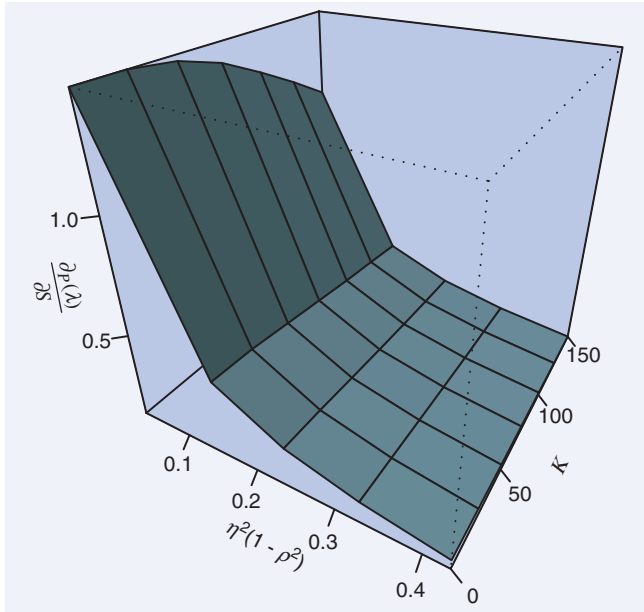
Figure 9. Incentives for λ options or stock across strike K and firm-specific risk, with fixed market risk $\eta^2\rho^2$. A fixed c is chosen and λ determined for each $K, \eta^2(1-\rho^2)$ pair such that (11) is satisfied. Fix $\gamma = 4/1.5 \times 10^7$, $S_0 = 100$ and other parameters same as figure 1.
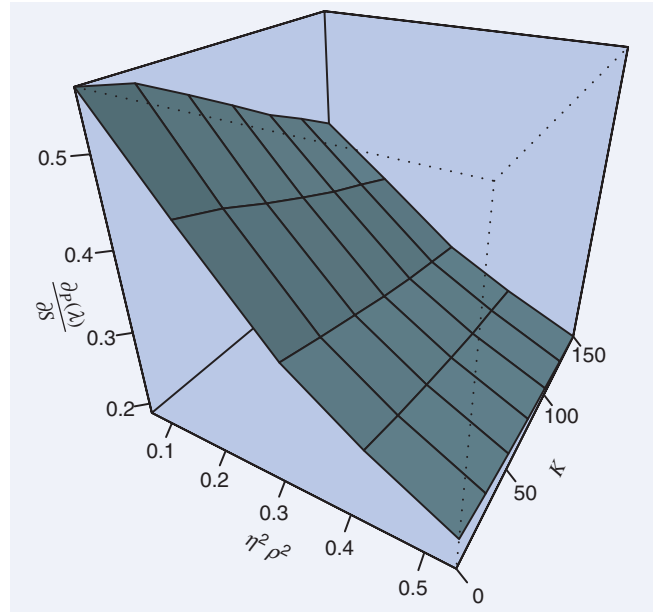
Figure 10. Incentives for λ options or stock across strike K and market risk, with fixed firm-specific risk $\eta^2(1-\rho^2)$. A fixed c is chosen and λ determined for each $K, \eta^2\rho^2$ pair such that (11) is satisfied. Fix $\gamma = 4/1.5 \times 10^7$, $S_0 = 100$ and other parameters same as figure 1.

consistent with that of Hemmer *et al.* (2000) who find in a principal-agent framework, introducing options with CARA preferences is not optimal.

6. The impact of market and firm-specific risk on incentives

We can analyse the effect of the various risks on incentives in our framework with some interesting new results. Firstly however, we discuss the existing literature. The trade-off between risk and incentives is at the centre of agency theory. The principal agent model predicts that executive's pay-for-performance sensitivity is decreasing in the variability of the firm's performance. The empirical evidence testing this hypothesis is mixed. Aggarwal and Samwick (1999) and Lambert and Larcker (1987) both find executives at riskier firms have lower incentive levels, supporting the conclusion of the principal agent model. Many papers however find no significant relationship, or even a positive one, see table 1 in Prendergast (2000). A possible explanation for the weak empirical link is that the literature focuses on total risk, when the effects of firm-specific and market risks are potentially very different. More seriously, the theoretical models also attempt to draw conclusions based solely on total risk, which (at least in the context of allowing the executive to trade) is not the only way to examine the link between risks and incentives. There is one exception to this. In a static principal agent framework, Jin (2002) finds optimal incentives decrease with firm-specific risk and are unaffected by market risk, when considering stock rather than option compensation. Empirically, he finds it is the

firm-specific risk driving the relation between risk and incentives.

We extend the investigation of these issues to options in a continuous time utility model. The model makes predictions about the optimal compensation structure and we may analyse how this changes with risk. Potentially different conclusions, not inconsistent with each other may be drawn if we consider firm-specific and market risks separately.

Returning to figures 7–10, we now consider how incentives vary with risk for any strike, where risk can be total, market or firm-specific. Figure 7 shows option incentives decrease with total volatility η when other parameters, including correlation are kept constant. Recall we have imposed constraint (11), which means the incentives are those achievable with varying numbers of options giving a fixed c. Optimal incentives (where incentives are maximized at $K=0$) also decrease with total volatility, consistent with the conclusions of principal agent theory.

Fixing total volatility and for any strike K, figure 8 shows incentives are increasing with correlation. Thus incentives are higher when more of the total risk is market risk, and lower when firm-specific risk makes up a larger proportion of the overall risk. Relating this to a company's beta, incentives are higher for a company with higher beta, when all other variables are the same. This is also true for optimal incentives, seen on the graph when $K=0$, so optimal incentives are higher when market risk makes up a larger proportion of total risk, or equivalently, when the company has a higher beta. However, since total risk is being held constant, increasing market risk (via increasing correlation) is also simultaneously reducing firm-specific risk, and market risk is reduced when firm-specific risk

is increased. Thus we cannot distinguish the true effect of the individual risks separately.

In figure 9, market risk is held constant whilst firm-specific risk varies. For all strikes, incentives are seen to decrease with firm-specific risk. Optimal incentives (when $K = 0$) also decrease with firm-specific risk. This result recovers the conclusion of Jin (2002) for the case of stock and extends his conclusions to options also.

We now attempt to isolate the effect of market risk by holding firm-specific risk constant, refer to figure 10. Incentives across all strikes and therefore optimal incentives actually decrease with market risk when firm-specific risk is fixed and for the chosen parameters. This conclusion differs from that made in Jin (2002) for stock compensation, that market risk has no effect. One explanation is that for stock compensation in our model, market risk does not affect value significantly, see figure 5. Figure 5 showed market risk could increase option value to the executive (and was flat for stock), and we commented that for other parameters and ranges it was possible that market risk decreased value. This effect appears to carry over to optimal incentives so that market risk decreases optimal incentives, at least for the examples we examine.

7. Conclusions

This paper proposes a continuous time utility model to value stock and option compensation, under the assumption that executives cannot trade in the stock of the company. Trading in the market ensures that executives may diversify away market risk, but remain exposed to firm-specific risk.

The model exhibits the well-known result that the cost of the compensation to the company and the resulting incentives overstate the value and incentives that executives perceive, due to restrictions faced by executives in trading their own company's stock. However unlike many one asset models, we can examine the effect of market and firm-specific risks separately on value. By distinguishing between the two types of risk, it is clear that the drop in value compared with Black–Scholes or company cost is due to the executive not being able to trade away firm-specific risk. There are two opposing forces at play. On the one hand, the board aims to tie managers' reward to company performance. However, this exposure reduces the private value they place on the stock-based compensation as they are risk averse and exposed to firm-specific risk.

Our model predicts that options do not encourage executives to increase total risk, but rather to increase the proportion of market relative to firm-specific risk, or the beta of the company. If firm-specific risk is fixed, the effect of market risk on option value is ambiguous. Future empirical work could test this implication.

The paper draws conclusions about the relationship between various types of risk and incentives. We find systematic relationships exist between the executives' incentives to increase the stock price, and total, market

and firm-specific risks. First, consistent with principal agent theory, incentives and optimal incentives decrease with total volatility or risk. Second, we find optimal incentives decrease with firm-specific risk but may increase or decrease with market risk, depending on which parameters are held fixed. This second conclusion differs from that made by Jin (2002) in the case of stocks. Empirical testing of these conclusions would involve an extension of the work of Jin (2002) to cover non-linear payoffs.

Although the calculations in the paper rely heavily on the assumption of exponential utility, we would expect using power utility that many qualitative results would remain unchanged. See Henderson (2002) for approximate option prices under power utility. Finally, future directions for research include incorporating American style exercise into the framework of the paper and explicitly modeling the executive's effort. Agliardi and Andergassen (2003) have attempted to extend the model in this paper to include an effort variable.

Appendix

Derivation of value function

From (3) the value function is written as

$$V(t, X_t, S_t; \lambda) = \sup_{(\theta_u)_{u \geq t}} \mathbb{E}_t[U(X_T + \lambda(S_T - K)^+)]$$

$$= -\frac{1}{\gamma} \exp(-\gamma X_t \exp[r(T - t)])$$

$$\times g(T - t, \log S_t),$$

where $g(0, \log s) = \exp[\lambda \gamma (S - K)^+]$ and for $U(x) = -1/\gamma \exp(-\gamma x)$. We know V is a supermartingale under any strategy θ and a martingale under the optimal strategy. Applying Ito's formula to V and optimizing

over values of θ gives

$$-\dot{g} + (v - \tfrac{1}{2}\eta^2)g' + \tfrac{1}{2}\eta^2 g'' - \frac{(\sigma\rho\eta g' + (\mu - r)g)^2}{2\sigma^2 g} = 0 , \tag{A1}$$

a non-linear pde. The optimal strategy is given by

$$\theta^* = \frac{\mu - r}{\gamma\sigma^2 \exp[r(T - t)]} + \frac{g'\eta\rho}{\gamma g\sigma \exp[r(T - t)]}. \tag{A2}$$

Using the approach in Henderson (2002), we consider the transformation $g(\tau, z) = \exp(\alpha\tau)G(\tau, z + (\delta - \eta^2/2)\tau)^b$. If we take $b = (1 - \rho^2)^{-1}$, $\alpha = -(\mu - r)^2/2\sigma^2$ and $\delta = r + \eta((v - r)/\eta - [\rho(\mu - r)/\sigma])$ we get the heat equation

$$\dot{G} = \frac{1}{2}\eta^2 G''$$

subject to $G(0, z) = \exp[-\gamma\lambda(e^z - K)^+/b]$.

The solution is

$$G(\tau, x) = \mathbb{E}G(0, x + \eta Z_\tau)$$

for Brownian motion Z and hence

$$g(T - t, \log S_t) = \exp[-(\mu - r)^2(T - t)/2\sigma^2]$$
$$\times \left[\mathbb{E}^0 \exp[-\lambda\gamma(1 - \rho^2)(S_T - K)^+]\right]^{(1-\rho^2)^{-1}} \tag{A3}$$

giving (4).

We define $W(t, S) = \mathbb{E}^0 \exp[-\lambda\gamma(1 - \rho^2)(S_T - K)^+]$ and hence using (A3) obtain the relationship

$$g(T - t, \log S) = \exp[-(\mu - r)^2(T - t)/2\sigma^2]W(t, S)^{(1-\rho^2)^{-1}}.$$

Differentiating and substituting into the pde (A1), gives the following pde for W

$$\dot{W} + \frac{1}{2}\eta^2 S^2 W_{ss} + SW_s\delta = 0 \tag{A4}$$

with $W(T, S) = \exp[-\lambda\gamma(1 - \rho^2)(S - K)^+]$.

Effect of correlation on executive value

We use a comparison argument. Take $(\hat{\rho})^2 < (\tilde{\rho})^2$. Denote by $\hat{f}$ (and $\tilde{f}$) the value p_λ^e evaluated with $\hat{\rho}$ (or $\tilde{\rho}$) respectively. Fix δ. The function $\hat{f}$ solves $\hat{\mathcal{L}}\hat{f} = 0$ subject to $\hat{f}(T, s) = \lambda(s - K)^+$ where

$$\hat{\mathcal{L}} = -r + \frac{\partial}{\partial t} + \frac{1}{2}\eta^2 S^2$$
$$\times \left[\frac{\partial^2}{\partial S^2} - \gamma(1 - \hat{\rho}^2) \exp[r(T - t)]\left(\frac{\partial}{\partial S}\right)^2\right] + \delta S \frac{\partial}{\partial S}.$$

Similarly, $\tilde{f}$ solves $\tilde{\mathcal{L}}\tilde{f} = 0$ with $\tilde{f}(T, s) = \lambda(s - K)^+$, where $\tilde{\mathcal{L}}$ is obtained from $\hat{\mathcal{L}}$ by replacing $\hat{\rho}$ with $\tilde{\rho}$. As a consequence, for any g, we have

$$(\tilde{\mathcal{L}} - \hat{\mathcal{L}})g = \frac{1}{2}\eta^2 S^2 \gamma \exp[r(T - t)](\tilde{\rho}^2 - \hat{\rho}^2)\left(\frac{\partial g}{\partial S}\right)^2. \tag{A5}$$

Consider $h = \tilde{f} - \hat{f}$ so that $h(T, s) = 0$. Then, using (A5)

$$\hat{\mathcal{L}}h = \hat{\mathcal{L}}(\tilde{f} - \hat{f}) = \tilde{\mathcal{L}}\tilde{f} - \hat{\mathcal{L}}\hat{f} - (\tilde{\mathcal{L}} - \hat{\mathcal{L}})\tilde{f}$$
$$= -\frac{1}{2}\eta^2 S^2 \gamma \exp[r(T - t)](\tilde{\rho}^2 - \hat{\rho}^2)\left(\frac{\partial\tilde{f}}{\partial S}\right)^2.$$

By the Feynman–Kac representation (see Duffie (1996))

$$h(t, s) = \tilde{f} - \hat{f} = \hat{\mathbb{E}}\left[\int_t^T \frac{1}{2}\eta^2 S^2 \gamma \exp[r(T - t)](\tilde{\rho}^2 - \hat{\rho}^2)\left(\frac{\partial\tilde{f}}{\partial S}\right)^2\right.$$
$$\left. \times \mathrm{d}u | S_t = s\right] \geq 0.$$

We have shown the executive's value is increasing in $|\rho|$, up to the Black–Scholes value or company cost.

Calculation of incentives

We begin with (8) and differentiate to obtain (10):

$$\frac{\partial}{\partial S}p_\lambda^e(t, s) = -\frac{\exp[-r(T - t)]}{\gamma(1 - \rho^2)}\frac{W_s(t, s)}{W(t, s)}.$$

Using the pde (A4) for W we can develop an expression for W_s. Setting $z(t, s) = W_s(t, s)$ we see z satisfies

$$\dot{z} + \tfrac{1}{2}\eta^2 S^2 z_{ss} + Sz_s(\eta^2 + \delta) + \delta z = 0$$

with
$z(T, s) = -\lambda\gamma(1 - \rho^2)I_{(s \geq K)} \exp[-\lambda\gamma(1 - \rho^2)(s - K)^+]$.
Define $\mathrm{d}\mathbb{P}^z/\mathrm{d}\mathbb{P}^0 = \exp(\eta\bar{B}^0 - (1/2)\eta^2 t)$, where $\bar{B}^0 = \rho B^0 + \sqrt{1 - \rho^2}W$ is $\mathbb{P}^0$ Brownian motion and $W^z = \bar{B}^0 - \eta t$. Then S is an exponential Brownian motion with volatility η and drift $(\delta + \eta^2)$ under $\mathbb{P}^z$. Now we may write (using the Feynman–Kac representation, see Duffie (1996))

$$z(t, s) = W_s(t, s) = \exp[\delta(T - t)]\mathbb{E}_t^{\mathbb{P}^z}(-\lambda\gamma(1 - \rho^2)I_{(S_T \geq K)}$$
$$\times \exp[-\lambda\gamma(1 - \rho^2)(S_T - K)^+]) \tag{A6}$$

and thus combining (A6) with (10), we have an explicit expression for incentives. Note that as $W_s < 0$, $\partial p_\lambda^e/\partial S$ is always greater than zero.

Optimal strategy

From (A2) we have an expression for θ^* in terms of g. Using the above relationship between g and W, we see $g'/g = SW_s/(W(1 - \rho^2))$ and

$$\theta^* = \frac{\mu - r}{\gamma\sigma^2 \exp[r(T - t)]} + \frac{\eta\rho SW_s}{\gamma \exp[r(T - t)]\sigma W(1 - \rho^2)}. \tag{A7}$$

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