The Dangers of Granger: What They Didn't Tell you about Granger Causality (in fMRI)

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Topics

1. What is Granger Causality?
2. fMRI Critical Heuristics: Subsampling and HRF Filtering
3. From Heuristics to Theory:
   Forward, Inverse and Computational Problems
4. Computational Solutions via State Space
5. Theory of Subsampling re Granger Causality
6. Theory of Filtering re Granger Causality
7. Illustrations: Spurious Causality
8. Conclusions

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What is Granger Causality?

Gedanken Experiment on Dynamics:

Rainfall R_t $\longrightarrow$ Catchment $\longrightarrow$ River Level L_t

$$\underset{\text{R pushes L}}{F_{R \to L}} = ln \frac{\sigma^2_{L|L^-}}{\sigma^2_{L|R^-,L^-}} \qquad \underset{\text{L pushes R}}{F_{L \to R}} = ln \frac{\sigma^2_{R|R^-}}{\sigma^2_{R|R^-,L^-}}$$

GEMs: $F_{RoL} = F_{R \to L} + F_{L \to R} + F_{L.R}$

An Aside:
But can one discern (temporal) causal relations from observational (time series) data?

fMRI Heuristics

Subsampling

Neuronal Processes are on a $\sim$30ms-50ms time scale whereas fMRI is on a $\sim$1 sec time scale. It is too slow.

Filtering

The fMRI hemodynamic response differentially filters and so messes up the dynamical interactions.
(And then subsampling makes this even worse).

Other Issues

Measurement Noise
Nonlinearity
Data Reduction
Omitted Third Variable

From Heuristics to Theory

Forward Problems Does $A \Rightarrow B$?
If GC relations exist on a fast time-scale, are they preserved under subsampling?

What does HRF filtering do to GC relations?

Inverse Problems Does $B \Rightarrow A$?
If GC relations are found on a slow time-scale do the same GC relations exist at a faster time-scale?

Can effects of HRF filtering be undone?

Computational Problems
$\longrightarrow$

Computational Solutions

How to get GEMs Reliably?

Need induced submodels e.g. R_t model from $\begin{bmatrix} L_t \\ R_t \end{bmatrix}$ model

Also e.g. submodel of VAR is VARMA $\equiv$ state space.
Solution: State Space Models + Ricatti Equations

How to get Subsampled Models reliably and so GEMs?
Solution: State Space Models + Ricatti Equations

GEMs can be decomposed by frequency and above state space solution method ensures reliable computation of the frequency domain GEMs.

Theorems: Subsampling

Forward Problem $A \Rightarrow B$
Strong unidirectional Granger causality
$(F_{L \to R} = 0, F_{R.L} = 0)$ is preserved under subsampling

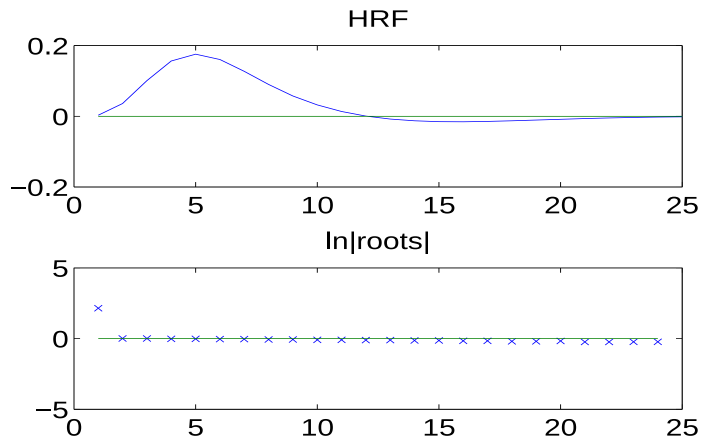
Inverse Problem $B \nRightarrow A$
Granger causal relations can be manipulated nearly arbitarily under subsampling.
NB. This is not so simple to do since $F_{L \to R}, F_{R \to L}$ depend nonlinearly on model parameters.

Theorems: Filtering

Minimum-phase filtering preserves GC

Nonminimum-phase filtering does not preserve GC

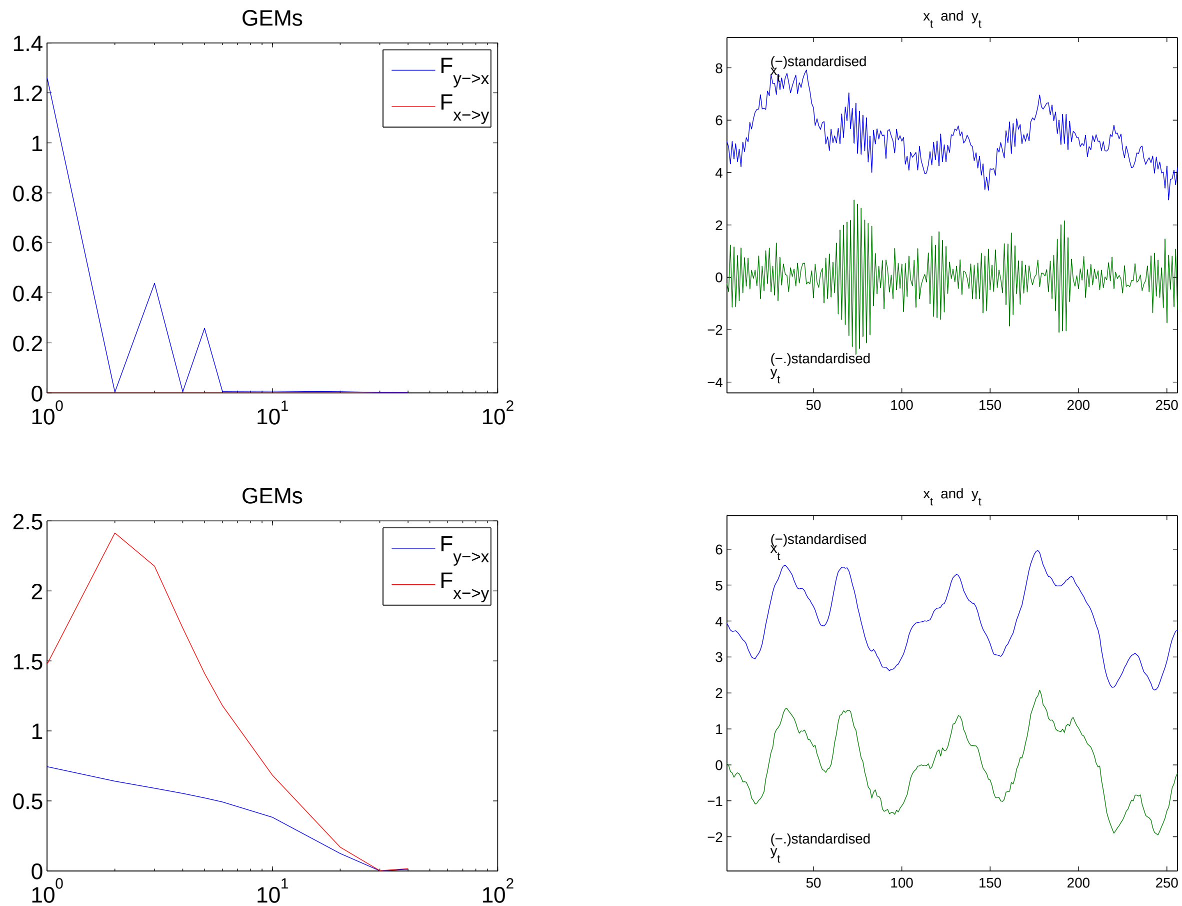
But HRFs are non-minimum phase!



Double Gamma Hemodynamic Response Function & Roots

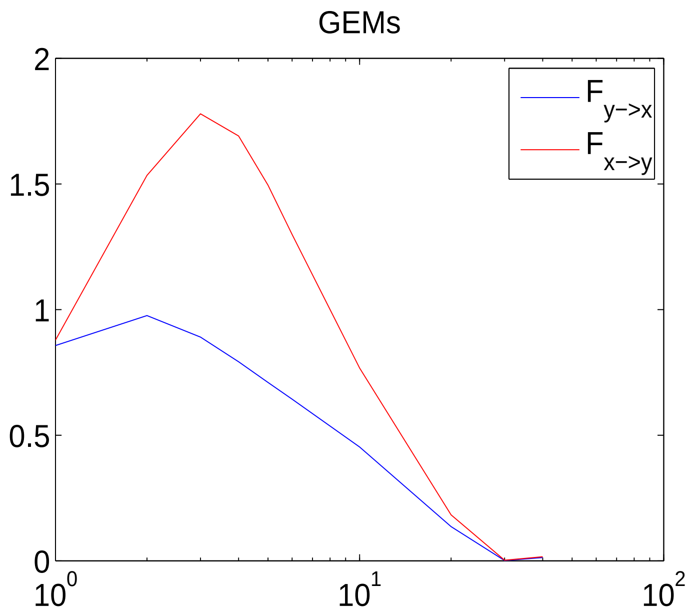
Spurious Causality I

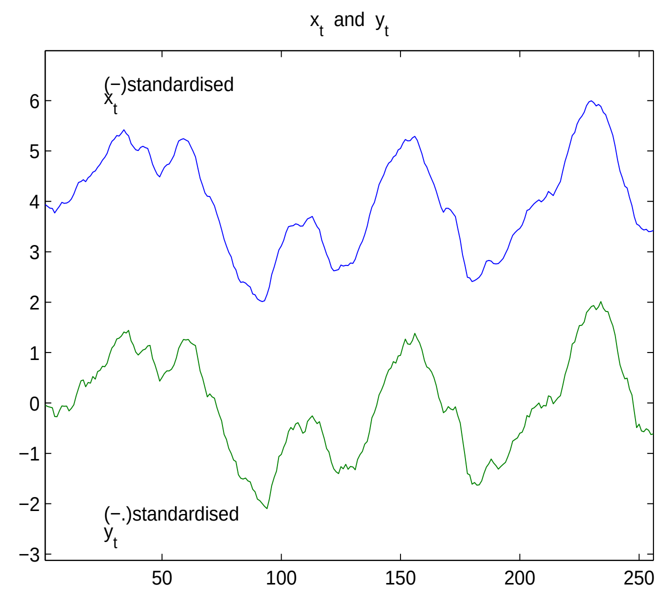
Unidirectional GC is preserved: but degrades semi-regularly



GC degrades irregularly

Spurious Causality II

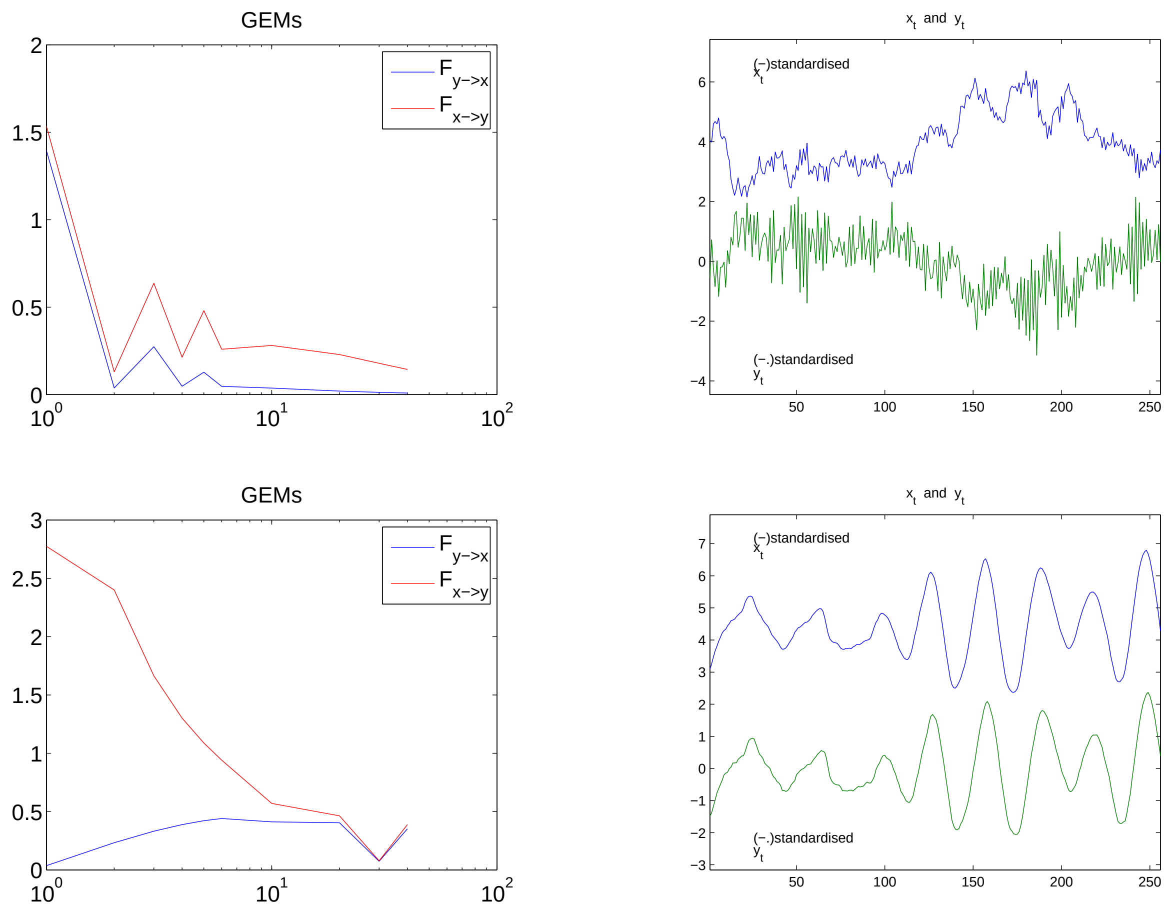




Equal GEMs Reverse Strongly

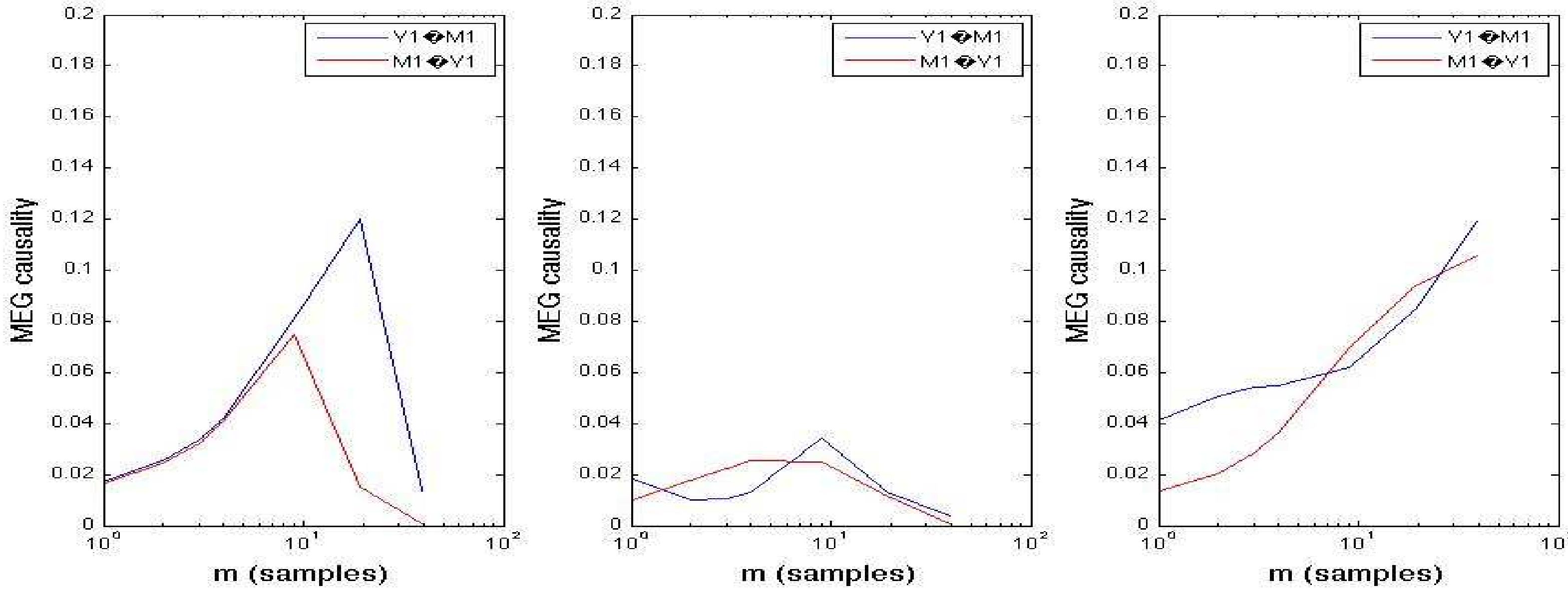
Spurious Causality III

Near equal GEMs become nearly unidirectional



Near unidirectional GEMs become nearly equal

Subsampled MEG Source Signals



Subsampling Reconstructed MEG Sourcce Signals

Conclusions

- Subsampling irretrievably destroys possibility of inverse GC recovery in fMRI.
- Non-minimum phase filtering, hence HRF filtering, destroys possibility of GC recovery in fMRI. Although higher order methods could help.
- New state space based computational methods provide reliable computation of submodels, subsampled models, GEMs and frequency domain GEMs.
- Need ms time-scale measurements e.g. MEG/EEG to pursue dynamic causality.