The Multiregression Dynamic Model (MDM) is a multivariate graphical model for a multidimensional time series that allows the estimation of time-varying effective connectivity (Costa et al., 2013). An MDM is a state space model where connection weights reflect the instantaneous interactions between brain regions. Because the marginal likelihood has a closed form, model selection across a large number of potential connectivity networks is easy to perform. With application of the Integer Programming Algorithm, we can quickly find optimal models that satisfy acyclic graph constraints. Here, this constraint was violated in the method called a Directed Graph Model (MDM-DGM). For instance consider a search problem for 3 variables. The Table 1 shows the local scores (LPL). The MDM-DGM finds the directed graph that maximizes the LPL for every node independently. Therefore, the best scoring model takes node 1 to have no parents and the nodes 2 and 3 with the each other node plus node 1 as its parents, see Figure 2(a). Note this is a cyclic graph.

The MDM-IPA also considers the acyclic constraints. Here, this constraint was violated in the cluster formed by nodes 2 and 3. The best solution leaves node 2 with the same set of parents as before, while node 3 becomes parentless; see Figure 2(b).

First considering only acyclic models, we use an Integer Programming (IP) algorithm (gobnilp system; Bartlett and Cussens, 2013), to quickly find the DAG with the optimal LPL: we call this the MDM-IPA model. Without the acyclic constraint, we call our method a Directed Graph Model (MDM-DGM).

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Figure 1 (top) shows the LPL versus discount factor (DF) at level 

\[ Y_t = f(Y_{t-1}, r_t), \]

where \( r_t \) is the vector of time-varying intercept and connectivity parameters; \( f(r) \) are the “regressors” at time \( t \), a vector comprised of the value \( 1 \) (for the intercept) and values of data for the parent nodes of region \( r \) at the current time \( t \). Note that state space models do not assume nonstationarity, though when state covariance \( W_t(r) = 0 \) for all \( r \) and \( t \), the usual static regression model is obtained. When \( W_t(r) \) is unknown, it can be defined via a scalar discount factor (DF), such that a \( DF=1 \) produces a static model. The conditional forecast distribution of \( Y_t(r) \) given the past up to time \( t-1 \) at its parents is a Student \( \nu \) distribution, and thus the joint log predictive likelihood (LPL) can be calculated as the sum of this distribution over nodes.

Figure 2 (bottom) illustrates a 3-node DAG MDM, considering region 1 as the parent of region 2 and region 2 as the parent of region 3. The orange ovals represent the (time-varying) effective connectivity strength between two regions, the purple ovals the intercepts, and the blue circles the observation noise. The MDM is a state space model where connection weights reflect the instantaneous interactions between brain regions. Because the marginal likelihood has a closed form, model selection across a large number of potential connectivity networks is easy to perform. With application of the Integer Programming Algorithm, we can quickly find optimal models that satisfy acyclic graph constraints. Here, this constraint was violated in the method called a Directed Graph Model (MDM-DGM). For instance consider a search problem for 3 variables. The Table 1 shows the local scores (LPL). The MDM-DGM finds the directed graph that maximizes the LPL for every node independently. Therefore, the best scoring model takes node 1 to have no parents and the nodes 2 and 3 with the each other node plus node 1 as its parents, see Figure 2(a). Note this is a cyclic graph.

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Figure 3 (top) shows the LPL versus discount factor (DF). Note that when \( DF=1 \) (a static model), it is possible to distinguish the estimated DAG (solid line) and a non-Markov equivalent (NME) DAG (dotted line), but not the estimated DAG and a Markov equivalent (ME) DAG (dotted line). Moreover, Figure 3 (below) gives the smoothed posterior mean with 95% HPD interval for connectivity from Visual Cortex V2 to V1. As the DF for subject 1 (left; \( DF=0.67 \)) is smaller than for subject 8 (right; \( DF=0.80 \)), the connection estimates are more dynamic for the former.