1 - Overview

1.1 - The Goal...???
Evaluate with certainty whether or not a given jump diffusion sample path crosses a barrier.

1.2 - Main Difficulties...???
Sample paths are infinite dimensional random variables. Discretisation schemes introduce error and don’t sufficiently characterise sample paths to determine barrier crossing.

1.3 - Applications...Monte Carlo Integration, Option Pricing, Simulating First Hitting Times, Killed Diffusions, Rare Events...

2 - Summary of Key Methodology

2.1 - Pathspace Importance Sampling (PIMS)...A retrospective rejection sampler which characterises entire (accepted) sample paths in the form of a finite dimensional skeleton, composed of the sample path at a finite collection of intermediate points and spatial information.

2.2 - ε-Strong Simulation (εSS)...Methodology for constructing upper and lower convergent dominating processes (X^ε and X^ε'), which enfold almost surely sample paths over some finite interval.

2.3 - Sufficient Conditions... β ∈ C^1, γ ∈ C^2 and strictly positive, I locally bounded, linear growth and Lipschitz continuity conditions.

2.4 - Further Details...arXiv 1302.6964 or scan QR code

3 - Key Ideas

3.1 - Rejection Sampling

4 - PIMS

4.1 - Idealised Algorithm
1. Simulate X ~ P^ε_{0,θ}.
2. With probability \( P^ε_{0,θ}(X) = \frac{ε^T h}{\|h\|} \) \( \in [0,1] \), set \( I = 1 \).
3. \( X(I) = 1 \sim T_{0,θ}^ε \).

4.2 - Key Points
1. \( T_{0,θ}^ε \) - target law.
2. \( P^ε_{0,θ} \) - equivalent (ε tractable) proposal law.
3. \( \frac{ε^T h}{\|h\|} \) bounded (by \( M < \infty \)).

4.3 - Summary

Key Idea: Find and simulate some finite dimensional auxiliary random variable \( F \sim F(X) \sim ε^T h \), such that an unbiased estimator of the acceptance probability can be constructed which can be evaluated using only a finite dimensional subset of the proposal sample path...

Implementable Algorithm
1. Simulate \( X_I \sim y \sim θ \).
2. Simulate \( F \sim ε^T h \).
3. Simulate \( X_I' \sim P^ε_{0,θ}(X|F) \).
4. With probability \( P^ε_{0,θ}(X|F) \) accept, else reject and return to 1.
5. \( \sim \) ※ Simulate \( X' \sim \mathcal{N}_θ(F) \) as required. ※

5 - ε-Strong Simulation

5.1 - Adaptive EA Skeleton

5.2 - Intermediate Augmentation

5.3 - Layer Dissection

5.4 - Layer Refinement

5.5 - Augmented Skeleton

6.1 - Example 1: ε-Strong Simulation of Jump Diffusions

6.2 - Example 2: Nonlinear Two Sided Barrier

6.3 - Example 3: Jump Diffusion Intersection

6.4 - Example 4: Circular Barrier

"O God, I could be boundless in a nut shell and count myself a king of infinite space, were it not that I have bad dreams." — William Shakespeare, Hamlet Act II, Scene III

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6.1 - Example 3: Jump Diffusion Intersection