Overview

“Retrospective Monte Carlo Methods” aim to exploit algorithmic design (in particular, the ordering of the steps in Monte Carlo algorithms) to address classically intractable classes of statistical model. Retrospective approaches open avenues to develop reliable statistical inference for infinite dimensional problems, “Big Data” problems and other intractable likelihood modelling contexts, by incorporating unbiased estimates of key intractable objects into algorithmic design. This enables us to tackle problems which are either outside, or several scales of magnitude computationally outside that within the scope of conventional Monte Carlo algorithms.

“Retrospective Monte Carlo Methods for Diffusions” are focused on characterising, in a finite dimensional manner and without approximation error, entire sample paths of diffusions. Diffusions are widely used across a number of application areas to model phenomena (such as the economic, physical and life sciences, to name but a few) and there is a profound need to use Retrospective methods for computational reasons (diffusions are infinite dimensional objects), to ensure the model simulated is the model desired, and to evaluate certain classes of test functional.

There are considerable opportunities for the student(s) working on this project to pursue research in a methodological, theoretical and/or application-based direction. A number of topical problems in Retrospective Monte Carlo are self-contained and could form the basis of a good paper and/or ‘scale-up’ to a PhD project for an interested student. Possible topics could include:-

1. Topics in the Scalable Langevin Exact (ScaLE) Algorithm
   • The ScaLE Algorithm is a novel alternative to traditional Langevin based MCMC schemes which, by exploiting recent advances in the exact simulation of diffusions, has algorithmic cost which is conducive to tackling “Big Data / Big Model” problems. There are a considerable number of open problems relating to this algorithm, including (but not limited to) optimal algorithmic design, parameter tuning, parallelisation and its theoretical behaviour.

2. Topics in Retrospective Methods for Simulating Diffusions
   • “Exact Algorithms” are Monte Carlo methods for simulating sample paths for a class of diffusions at a finite collection of points (without approximation error) and characterising the remainder of the un-simulated sample path. These methods are based upon rejection sampling on diffusion path space, and hinge upon finding suitable proposal measures from which proposal sample paths can be drawn exactly. Possibilities exist to address open problems with existing exact algorithm
3. Retrospective Uncertainty Quantification

- Uncertainty Quantification recognises that the (complex mathematical) models used to describe real world processes of interest are imperfect, and so aims to characterise and quantify the uncertainty that arises from this imperfection. Similarly, solutions of many mathematical models can’t be drawn exactly due to model complexity, and so instead a further approximate model is used in practice as a proxy. Retrospective Uncertainty Quantification is a project which would determine the extent to which Retrospective Monte Carlo methods could be employed to quantify (in a probabilistic sense) the additional uncertainty introduced between the true model and the proxy model.

4. Topics in the $\epsilon$-Strong Simulation of (Jump) Diffusions

- Recent methodological work enables the simulation of upper and lower bounding processes which almost surely constrain certain classes of (jump) diffusion sample paths to any specified tolerance. Such constructions can be used in principle to design algorithms to simulate unbiasedly the first passage times of (jump) diffusions and allow the efficient simulation of (jump) diffusion bridge sample paths – but do they have finite expected computational cost?

Interested student(s) should speak to one or more of {Murray Pollock, Gareth Roberts, Paul Jenkins, Adam Johansen} for more details on the proposed topic(s) and whether the topic(s) align with their interests.

Selected References


